DISTURBANCE EVOLUTION IN A MACH 4.8 BOUNDARY LAYER WITH SEPARATION

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Summary Direct numerical simulations serve to investigate laminar-turbulent transition in a flat-plate boundary layer at Mach 4.8 with roughness-induced separation. Results for two different transition scenarios, oblique breakdown and subharmonic wave breakdown, are compared. It is found that the large-amplitude two-dimensional wave present only in the subharmonic wave breakdown scenario is not significantly enhancing the transition process. Hence, in both cases essentially the same breakdown mechanism is active.

INTRODUCTION

Predicting the location of laminar-turbulent transition on the surface of a vehicle moving at supersonic speeds is a critically important task. An accurate prediction of this location may allow to precisely define the dimensions of the vehicle's thermal protection system. Turbulent flow causes a more severe heat load compared to laminar flow, and insufficient thermal protection could lead to the failure of the vehicle's mission. At present, a general mechanism-based transition prediction method does not exist for surfaces with a localized two-dimensional roughness element [1]. These elements cause boundary-layer separation, which in turn influences the evolution of boundary-layer disturbances responsible for transition. A better knowledge of transition mechanisms may help to develop advanced transition-prediction methods.

The laminar-turbulent transition process can be divided into three stages: receptivity, linear disturbance evolution, and non-linear breakdown to turbulence. The two most important types of non-linear breakdown are wave breakdown and streak breakdown. The breakdown to turbulence occurs when the primary flow structure, the wave or the streak, reaches a sufficiently large amplitude to create a new instability. This secondary instability takes the form of a linear parametric instability of the boundary layer distorted by a finite-amplitude, non-linear streamwise wave [2] or of an instability of the streamwise streak [3]. A special case of streak breakdown, in which only a pair of oblique waves at moderately large amplitude is forced, is the so-called oblique breakdown in supersonic flow. Previous numerical simulations of unsteady secondary instability [4, 5, 6] or oblique breakdown [7, 8, 9] have only considered the flow over a smooth flat plate.

METHODOLOGY

We obtain numerical solutions of the compressible Navier-Stokes equations, applying sixth-order compact finite differences together with third-order explicit Runge-Kutta time stepping. The numerical discretization is constructed on a structured, curvilinear grid using staggered variables, based on an algorithm described in [10]. The fluid is assumed to be a calorically perfect gas, and the viscosity is computed from Sutherland's law. Details of the setup and a more comprehensive description of the numerical method can be found in [11]. Disturbances of fixed frequency F and fixed spanwise wave number γ=$2\pi/\lambda_z$=10.4, are triggered via blowing and suction at the wall close to the inflow boundary.

For our direct numerical simulations we used 1800 equidistant grid points in the streamwise dimension x for a domain extending from $R_x = 401$ to $R_x = 2620$ with $R_x = \sqrt{x \cdot Re_\infty}$, whereas in the wall-normal direction y, 200 grid points were used on a stretched grid for a domain height $y_{max} = 1.575$ (at the inflow: $y_{max}/\delta^* = 34.32$). In the periodic spanwise direction z, 160 equidistantly spaced grid points were used for a domain width of $4\lambda_z$=2.42. The Reynolds number is $Re_\infty = 10^5$, the free-stream Mach number is $Ma_\infty = 4.8$ and the free-stream temperature is $T_\infty = 55.4\,K$. The ratio between the boundary-layer thickness δ_{99} at the center location of the roughness ($x = 15$), and the roughness height is $h_R/\delta_{99} \approx 0.6875$. We assume an adiabatic wall boundary condition.

STEADY-STATE BASE FLOW AND LINEAR INSTABILITY

Figure 1 depicts the flow field by means of a numerical Schlieren image. A large-amplitude two-dimensional disturbance wave is present in the boundary layer. Ref. [11] gives a detailed description of the unperturbed laminar flow with a two-dimensional roughness. Boundary-layer separation occurs both upstream and downstream of the roughness.

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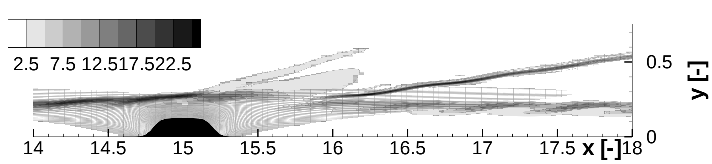

Figure 1. Instantaneous contours of the absolute wall-normal density gradient $|\partial\rho/\partial y|$ for $h_R = 0.125$ in the vicinity of the roughness.

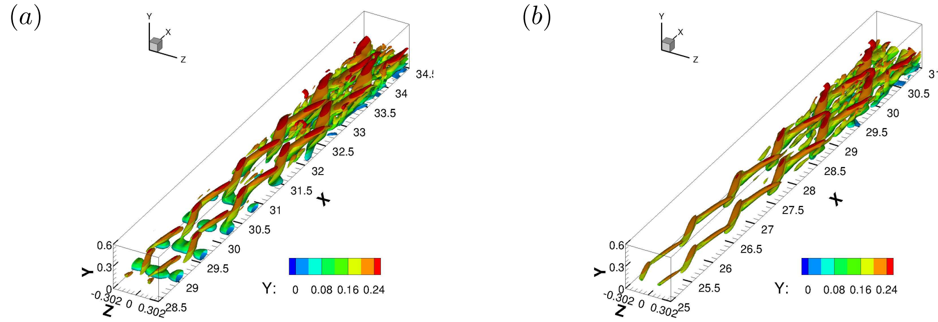

Figure 2. Vortex visualization (q-criterion) downstream of the roughness for subharmonic (a) and oblique (b) breakdown.

The amplification of two-dimensional waves is strongly increased by the presence of roughness-induced separation zones, while oblique waves are not significantly affected [11]. As a result, two-dimensional disturbances are expected to reach large amplitudes much earlier than oblique waves. Hence, wave breakdown is more likely to occur with roughness than without it, since the unsteady secondary instability strongly depends on the amplitude of the two-dimensional wave. On the other hand, oblique breakdown is expected to be the preferred transition mechanism at this Mach number if we have a smooth wall. It is therefore not clear a priori which mechanism will initiate the breakdown to turbulence. A disturbance frequency within the relevant range and its subharmonic, i.e. F_p=10^{-4} and F_s=0.5×10^{-4}, are considered here.

NON-LINEAR INSTABILITY AND BREAKDOWN TO TURBULENCE

In order to trigger subharmonic wave breakdown, a large-amplitude two-dimensional wave at F_p is forced together with a pair of oblique waves at F_s with a lower amplitude. If the amplitude of the subharmonic pair of oblique waves is very low, we observe indeed a significant increase of oblique-wave amplification caused by subharmonic resonance. However, low-amplitude oblique waves are unable to transition the flow to turbulence within the present integration domain. If the amplitude of the oblique waves is increased in order to initiate the breakdown to turbulence, we find that the amplitude of the two-dimensional wave decreases due to the emergence of streamwise streaks, which are generated by the oblique waves. As a result, the secondary amplification of the pair of oblique waves is reduced almost back to the linear amplification rate. A consequence of this effect is that the breakdown process is almost identical to the oblique breakdown scenario, in which only the pair of oblique waves at F_s with the same amplitude is forced. Indeed, vortical structures developing during the breakdown to turbulence are very similar for both breakdown scenarios, with the exception of two-dimensional structures present only in the subharmonic wave breakdown scenario (Figure 2).

CONCLUSIONS

Laminar-turbulent transition downstream of a two-dimensional roughness is found to occur due to the oblique breakdown scenario. Surprisingly, a two-dimensional wave with carefully selected frequency is not able to initiate an earlier breakdown in spite of a strong increase in linear amplification rate of such a wave. Hence, even in the presence of a two-dimensional roughness, oblique breakdown may be the most important mechanism to cause transition at Mach 4.8.

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