NONLINEAR STABILITY OF THE SUPERSONIC BOUNDARY LAYER UNDER THE CONSIDERATION OF RECEPTIVITY TO SLOW ACOUSTIC WAVES

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Summary The nonlinear stability of the supersonic boundary layer on a flat-plate with Mach number 4.5 is studied under the consideration of receptivity to slow acoustic waves by using nonlinear parabolized stability equations (NPSE). It is shown that the first mode wave could be exicted by slow acoustic wave directly. The second mode wave could be induced by the first mode wave downstream.

INTRODUCTION

The prediction of laminar-turbulent transition in hypersonic boundary layer is a very complicated problem, including receptivity phenomenon, which is the process of environmental initially entering the boundary layers and generating disturbances. Despite the decades of great efforts, the mechanism of receptivity phenomenon is less understood. For supersonic boundary layer, according to Zhong[1-3] and Balakumar[4], the slow acoustic wave can possibly excite Tollmien-Schlichting(TS) waves inside the boundary layer directly, however the fast acoustic wave cannot. So, only the receptivity of the boundary layer to slow acoustic wave will be taken into consideration in this study.The present paper aims to answer two questions: a) the excitation and nonlinear evolution of TS wave according to slow acoustic wave; b) whether or not the second-mode wave might be generated.

NUMERICAL METHOD

Assuming that the disturbances are periodic in time and spanwise. The disturbance function $\hat{\phi}$ can be expressed by the following truncated Fourier series:

$$\phi(x,y,z,t)=\sum_{m=-M}^{M}\sum_{n=-N}^{N}\hat{\phi}_{mn}(x,y)e^{i\left(\int_{x0}^{x}\alpha_{mn}(\bar{x})d\bar{x}+n\beta z-m\omega t\right)} \tag{1}$$

where ϕ is the disturbance vector, $\hat{\phi}$ the shape function vector, α_{mn} the wave number of streamwise, β the wave number of spanwise, ω the frequency. Substituting Eq.(1) into disturbance equations and neglecting the terms of order $1/Re_0$. The nonlinear parabolized stability equations can be arranged in the following form:

$$V_{yy}{}^{l}{}_{mn}\frac{\partial^2\hat{\phi}_{mn}}{\partial y^2}+\left(\hat{A}_{mn}\frac{\partial\hat{\phi}_{mn}}{\partial x}+\hat{B}_{mn}\frac{\partial\hat{\phi}_{mn}}{\partial y}+\hat{D}_{mn}\hat{\phi}_{mn}\right)=\hat{F}_{mn}, m=0,\pm1,\pm2,\cdots\cdots,\pm M, n=0,\pm1,\pm2,\cdots\cdots,\pm N \tag{2}$$

where V^l_{yy}, $\hat{A}_{mn}$, $\hat{B}_{mn}$, and $\hat{D}_{mn}$ are coefficients matrices, the vector function $\hat{F}_{mn}$ on the right-hand represents all the nonlinear terms.

The receptivity mechanism of the boundary layer to the slow acoustic waves is assumed that the phase speed of a Tollmien-Schlichting wave is equal to that of the slow acoustic wave, the Tollmien-Schlichting wave is excited.

$$C_s = 1-\frac{1}{M_e}=C_{TS}=\frac{\omega}{\alpha_r} \tag{3}$$

where M_e is the Mach number, C_S is the phase speed of the acoustic wave, C_{TS} is the phase speed of the TS wave.

NUMERICAL RESULT

This research focused on a flat-plate with M=4.5. The free stream parameters are same as those of Zhong [3]. Blasius similarity solution is used as basic flow at the location x^*=0.138.

Table 1 Free stream parameters

M_e	T_e^*	Re_{unit}	x_0^*	Δx^*	yl^*	Wall condition
4.5	65.15	7200000	0.138	0.00216625	0.0722	adiabatic

In fact, TS waves satisfying the Eq. (3) can always be found as shown in Fig. 1. It means that the first mode waves are excited directly by slow acoustic waves. In the current case, all the induced TS waves are unstable (see in Table 2). For discussed in detail, the evolution curves of tow TS waves, with same frequency but different spanvise wave number, are plotted and compared. The initial amplitude is 0.001.

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Table 2 Parameters of initial disturbances for two cases

Case	$f^*(Hz)$	$\beta^*(m^{-1})$	$\alpha_r^*(m^{-1})$	$-\alpha_i^*(m^{-1})$
One	16055.23	96.25	178.06	1.11
Two	16055.23	553.12	178.06	9.57
Three	17660.75	120.76	201.13	1.69
Four	17660.75	536.09	201.13	10.11
Five	19266.28	150.88	213.67	2.49
Six	19266.28	512.96	213.67	10.43
Seven	20871.79	189.87	231.48	3.59
Eight	20871.79	480.69	231.48	10.47
Nine	22477.32	247.13	249.21	5.28
Ten	22477.32	430.35	249.21	9.89

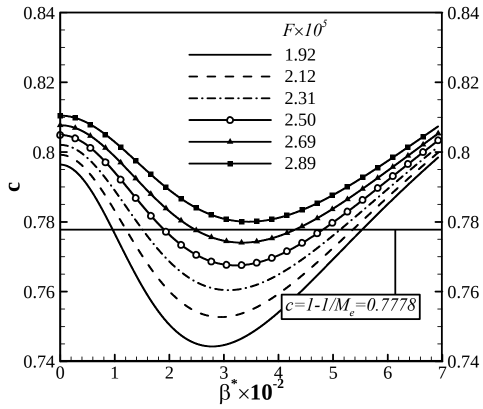

Figure. 1 Each frequency curve vs. β

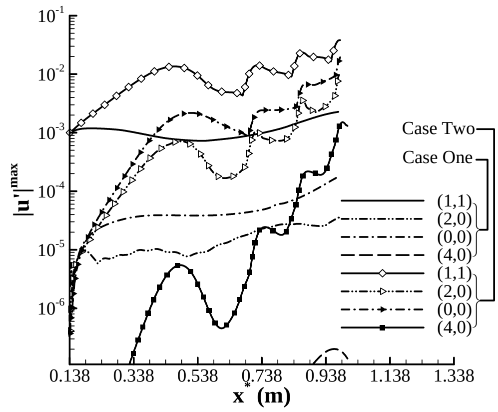

Figure. 2 The amplitudes of each mode for case on and case two

As shown in Fig.2, the TS waves with short spanwise wavelength grow faster than those with long spanwise wavelength. Some disturbances are evoked by the nonlinear interactions. The frequency of mode (4,0) is in accord with that of second mode wave. The growth rate of mode (4,0) tends to decrease with increase of the spanwise wave number. This is the characteristic of the second mode waves. The most unstable waves is two dimensional instability waves. Comparing local wave structure with eigenfunction of the second mode base on the linear stability theory(LST), it is in a good agreement between the results from present calculations and LST. It is indicated that second Mack mode waves could be induced by first Mack mode waves.

CONCLUSION

The first mode waves are generated directly by slow acoustic waves as long as the phase speed of TS wave matches that of slow acoustic wave. The growth rates of first mode waves vary with spanwise wavelength. The instability wave with short spanwise wavelength grows faster than that of long spanwise wavelength. The result demonstrates that the second mode waves could be generated further downstream by first mode waves via nonlinear interactions.

ACKNOWLEDGEMENTS

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