A NEW METHOD FOR MEASURING SPECIFIC HEAT RATIO OF COMBUSTION GASES IN HYPERSONIC FLOW[a)]

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Summary We present a novel efficient technique measuring the specific heat ratio of combustion gases, γ, utilizing Pitot tube and laser beam tracking system. The key benefit of the new technique is the elimination of the complicated gas chromatography apparatus. Pressure ratio and the angle of refraction of a laser beam across the normal shock wave generated by Pitot tube are concurrently measured. The angle of refraction of the laser beam are supposed to be accurately obtained by the laser tracking system consisting of two CCD cameras and the mobile spatial coordinate measuring system (MScMS), and the density ratio across the shock wave is obtained with Gladstone–Dale relation and Snell's law. γ is accordingly derived from its relation to the pressure ratio and the density ratio across the shock wave. This technique may effectively reduce the time taken to obtain γ. Uncertainty estimation shows that the uncertainty brought by the laser beam tracking system and the Pitot tube measurement are limited within 2%. Feasibility analysis shows that the technique is physically realizable but a few technical issues are need to be addressed before implementing it. This technique obtaining precise γ can be used in a broad range of applications, for example, the realtime flow control in hypersonic flows.

INTRODUCTION

High temperature gases as the combustion reaction products of pure oxygen or air with the fuels such as hydrogen, methane, are always adopted as the working media in hypersonic experiments. However, the complexity of combustion process challenges accurate measurements of the quantity of the combustion gases, and also eliminates the possibility of predetermination of the gas properties. The specific heat ratio γ is a critical property of the combined gases in calculating Mach number in a hypersonic wind tunnel. There are two common techniques in measuring γ: one is *the empirical method* usually adopted in the flow field calculation of engine exhaust system; another is *the composition analysis method* used in calibration of the thermal structure of a hypersonic wind tunnel nozzle. The empirical method calculates γ of the combustion gases based on an empirical combustion efficiency measured from the similar combustion conditions. Thus the resultant γ is only an approximation from a similar flow condition and the uncertainty of the value is intrinsically unpredictable. The composition analysis method obtains γ according to the quantities of the components of combustion gases by analyzing the gas chromatography at different temperatures. Though gas chromatography may provide the accurate γ, the complicated facilities still hinder its applications. This paper presents a new technique for efficiently obtaining accurate γ of the combustion gases in hypersonic flow field. In this technique, γ is derived from the relation of fluid density before and after a normal shock wave induced by the Pitot tube and the pressures behind the shock. Feasibility and uncertainty estimation of this technique is testified and discussed.

PRINCIPLE OF MEASURING SPECIFIC HEAT RATIO

The relation of gas properties across a normal shock wave is applied to estimate the features of the combustion gases: $\frac{p_2}{p_1} = \left[\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$ and $r \equiv \frac{1}{r_a} = \frac{\rho_2}{\rho_1} = \left[(\gamma+1)M_1^2\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\frac{\gamma+1}{2}}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$.
where p_1 and p_2 are total pressures just upstream and downstream of the normal shock wave, respectively; ρ_1 is the static density upstream of the normal shock wave, ρ_2 is the total density downstream of the normal shock wave; γ is the specific heat ratio of combustion gases, and M_1 is the Mach number of hypersonic flow upstream of the normal shock wave. For the hypersonic nozzle with multi-components, both M_1 and γ are the quantities to be measured. Thus it is necessary to combine the above two equations with the measured results of p_1, p_2 and r, to calculate the specific heat ratio, γ. p_1 and p_2 can be directly measured with Pitot tube. The schematic for measuring $\frac{\rho_2}{\rho_1}$ is presented in Figure 1(c) and 1(b). According to Snell's law of light and the Gladstone-Dale formula of refraction ratio and gas density, the relation between gas densities and angles of incidence and refraction for the laser beam is obtained as follows [1]:

$$\frac{\rho_1}{\rho_2} = 1 + \frac{\frac{sin\alpha_2}{sin\alpha_1} - 1}{1 - \frac{1}{n_2}} \tag{1}$$

where α_2 is a known quantity. Hence, r is obtained with α_1 and n_2 being measured by the laser beam tracking system.

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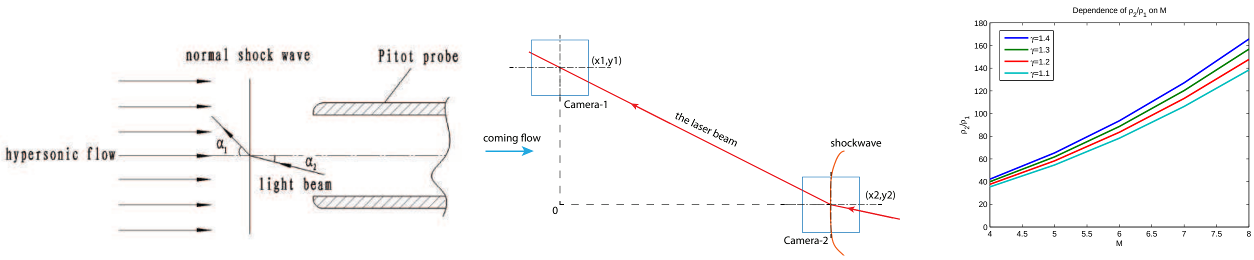

(a) Schematic of the technique (b) Refraction angle measurement (c) Density ratio vs. the Mach number

Figure 1. (a) Schematic of measurement of γ; (b)The schematic graph for the laser beam tracking system. The dash line indicates the coordinate system supporting CCD cameras, Camera-1 and Camera-2. Camera-1 is located at $(x1, y1)$ in the coordinate system of MScMS to capture the image of the refracted laser beam. Camera-2 is placed at $(x2, y2)$ in the coordinate system of MScMS to record the position where the laser beam is refracted by the normal shock wave; (c) The relation of density ratio and the Mach number.

FEASIBILITY STUDY AND UNCERTAINTY ESTIMATION

We take the apparatus in CAAA for example to examine the feasibility of the measurement system, as shown in Figure 1(b). The uncertainty for the recorded laser beam is estimated below $15\mu m$. The mobile spatial coordinate measuring system (MScMS) is going to localize the two camera with the position uncertainty of $2\mu m$ [2]. Thus the system uncertainty for measuring the position of the laser beam is limited within $\pm 20\mu m$. The test section is $300mm$(width)$\times 40mm$ (height) $\times 70mm$ (depth) [3]. Thus the distance between two cameras is supposed to be $150mm$, the uncertainty of the measured angle of α_1 is lower than 1.5×10^{-4} in radian number, and the corresponding relative error is under 0.01%. With primary analysis of the relation between the Mach number and density ratio (also and pressure ratio), we found that the measurement is effective at $4 < M < 8$ and $1.1 < \gamma < 1.4$.

Figure 1(c) shows the profiles of density ratio, r, vs. the Mach number, M. Note that the density ratio r remarkably increases with M and γ. Hence, the measurement of r and M determines the accuracy of γ. Since r is extracted from the measurement of the angle of refraction, α_1, of the laser beam lit from the tube (see Figure 1(a)), we have the estimation of error propagation for r_a, $|\frac{\delta r_a}{r_a}| \sim \frac{\delta \alpha_1}{\alpha_1}|r'_a|\frac{\alpha_1}{r_a}$. Considering the relative error of α_1 about 0.01%, the relative error of the density ratio is then estimated about 0.5%. The accuracy of the density-ratio-measuring system is satisfactory. For estimating the uncertainty brought from r and M_1, the relative error for the specific heat ratio $\frac{\delta\gamma}{\gamma}$ is derived as follows,

$$|\frac{\delta\gamma}{\gamma}| \leq \frac{|\frac{\delta r}{r}| + |\frac{M_1}{r}\frac{dr}{dM_1}\frac{\delta M_1}{M_1}|}{|\frac{\gamma}{r}\frac{dr}{d\gamma}|} \tag{2}$$

where $\frac{\delta\gamma}{\gamma}$ and $\frac{\delta M_1}{M_1}$ are the relative error of density ratio r and that of the Mach number M_1, respectively. Estimation of the relative error of γ is obtained from eq. (2). If the uncertainty of r and M_1 are supposed to be $\frac{\delta r}{r} \sim \frac{\delta M_1}{M_1} \sim 0.5\%$ in the range of $4 < M_1 < 8$ and $1.1 < \gamma < 1.4$, $\frac{\delta\gamma}{\gamma}$ is then estimated less than 2.0%. Hence, the present method may provide an acceptable accuracy. The uniqueness of the present technique is the ability to obtain the real time signals of the Mach number, the density ratio of a hypersonic flow and the specific heat ratio of gases over the whole flow field with a physically realizable system. The post processing of these results is more efficient and more extensive than gas chromatography. This technique is expected to have a wide range of applications in high speed flow measurements, though a few technical issues are yet to be addressed.

CONCLUSIONS

A novel, accurate and simple -measuring method utilizing laser beam tracking and pressure measurement techniques has been presented. The pressures before and after the normal shock wave formed by Pitot tube are measured concurrently with the density ratio by laser beam tracking system. This improves ease of use and reduces the time to obtain final data for calibration of a hypersonic wind tunnel. The uncertainties of the technique are acceptable - the upper bound of the uncertainty is assured below 2%. Feasibility analysis shows that the technique is physically realizable.

References

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