ANALYSIS OF CONTINUOUS ADJOINT CONTROL OF COMPRESSIBLE PLANE JETS

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Summary Continuous adjoint optimal aeroacoustic control of compressible plane jets using direct numerical and large-eddy simulation for the 2D and 3D case, respectively, is presented. The gradient of the cost functional obtained using the adjoint is compared to that using finite differences. It is shown that the error increases drastically with increasing control horizon. Preliminary tests applying a second order Newton based conjugate gradient method show a faster convergence and indicate usefulness for future applications.

INTRODUCTORY REMARKS

Although much progress has been made in the control and understanding of aeroacoustic sound emission, there is still a lack of simple, yet sufficient models for typical flow control applications and there are still open questions regarding the sensitivity and controllability of such flows [1, 2, 3]. One approach is to find the optimal control strategy by making use of an iterative approach, based on the adjoint state equations. Here, a cost functional constraint by a differential equation is introduced, measuring for example the acoustic intensity in a target region. Determining the minimum of the cost functional numerically involves calculation of its gradient with respect to the control. A finite-difference approach would involve $\mathcal{O}(n)$ primal solutions (here: full solutions of the Navier-Stokes equations) for n design variables and is therefore impractical for non-stationary turbulent flows involving many control parameters. Therefore the gradient is calculated by making use of the so-called adjoint equations, offering the possibility to determine the gradient *independently* of the number of control variables [4]. The accuracy and succession relies, however, on the accuracy of the calculation of the gradient. Inconsistencies due to modelling, boundary condition treatment and differences in the numerical treatment of the adjoint and primal solution leads to deviations of the gradient direction from the optimal direction, especially for in-stationary flow problems. In the following, we present results obtained by comparing the gradient obtained using finite differences and the adjoint for the configuration of a plane jet with Reynolds number based on the diameter $Re_D = 2000$, Mach number $M = 0.9$ and domain lengths of $L_x/D = L_z/D = 30$. The grid resolution was 512×640 and $320 \times 64 \times 384$ for the 2D (DNS2D) and 3D (LES3D) case, respectively. Furthermore, a receding horizon algorithm was used to extend the control horizon beyond the limitation imposed by the gradient accuracy.

The compressible Navier-Stokes equations are solved in cartesian coordinates. Errors in the adjoint optimization are minimized by applying a low-dispersion- dissipation fourth-order Runge-Kutta scheme [5] and an optimized explicit DRP-SBP (dispersion- relation-preserving summation by parts) finite-difference scheme of 6th-order for the spatial derivatives [6]. For subgrid scale modeling a variant of the approximate deconvolution method originally introduced by Stolz & Adams [7] is used, which is based on explicit filtering alone [8, 9]. Non-reflecting boundary conditions and sponge layers are imposed on the domain boundaries. The transition to turbulence was generated by convecting disturbances obtained in separate precursor simulations of plane temporally evolving mixing layers into the jet shear layers [10]. The adjoint equations were obtained similarly as in [2] and solved using the same numerical schemes and grid as the primal flow solution. Analogously to the boundary treatment of the primal flow solution, characteristic boundary conditions were developed for the adjoint equations. As a cost functional, we chose a measure for the far field sound pressure

$$\Im = \int_{\Omega} \int_{T} \left(p(\mathbf{x}, t) - \overline{p}(\mathbf{x})\right)^2 dt\, d\Omega, \tag{1}$$

with control interval T, the target volume in the farfield of the jet Ω and the temporal average of the pressure over the interval T, $\overline{p}$. The flow is controlled via two small regions within the jet shear layers, where heating and cooling is applied. The gradient is obtained using the adjoint flow variables and used in a conjugate-gradient optimization routine with Brent's line minimization [11] to update the control from the previous conjugate gradient iteration, $\phi^{new} = \phi^{old} - r\, g(\phi^{old})$, where r denotes a generalized distance in control space. The code was verified by calculating an anti-sound test case as detailed in [2].

RESULTS

The accuracy of the calculated gradient for the jet simulations DNS2D and LES3D is estimated by comparing the difference in the cost-functional, induced by a small perturbation $\mathbf{\Delta u}$, $F_{FD}(\mathbf{u}, \mathbf{\Delta u}) = \Im(\mathbf{u}+\mathbf{\Delta u}) - \Im(\mathbf{u})$ to the cost-functional difference estimated by the linear approximation $F_{grad}(\mathbf{u}, \mathbf{\Delta u}) = \frac{d\Im(\mathbf{u})}{d\mathbf{u}} \cdot \mathbf{\Delta u}$. The perturbations were modelled assuming a Gaussian shape in space and time with an amplitude small enough to remain in the linear regime. Owing to the localization in time the gradient accuracy can be estimated at several distinct times. Comparing F_{FD} with F_{grad}, $F_{FD} \cdot F_{grad} < 0$ can be considered as a worst case scenario as it implies that the calculated gradient isn't an ascent direction. Fig. 1 shows F_{FD} and F_{grad} for the 3D-LES-jet with a control length of $\Delta T \approx 65$, for various perturbations and for different controls, obtained after different conjugate-gradient optimization steps. The gradient appears to be acceptable within the time $64 \geq t \geq 18$, beyond that time the gradient direction may even be in the opposite direction and the optimization process becomes ineffective. This situation may be avoided by using a so called receding horizon approach (see e.g. [12]). Here,

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the full control interval is split into several subintervals and the control is obtained for these subintervals by solving the primal and adjoint until convergence. This control is then used to advance forward to some larger time and the procedure is iterated until the desired control-length is obtained. Fig. 2 shows that this approach is successful for long time intervals.

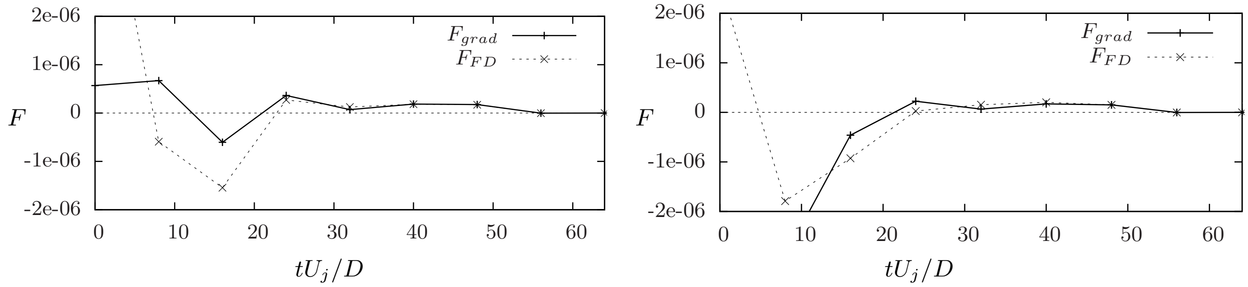

Figure 1. Comparison of F_{FD} and F_{grad} (see text) for the LES3D-case for (a) the first and (b) the second CG-iteration.

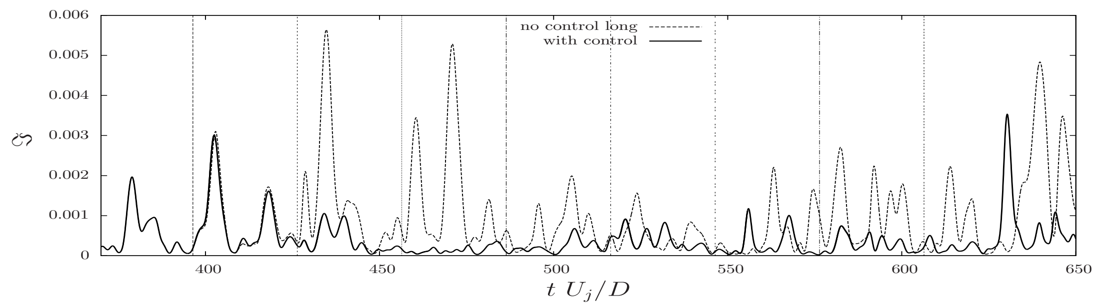

Figure 2. Cost-function over time for the uncontrolled and controlled DNS2D-case using the receding horizon procedure. Note, the cost-functional don't differ at the beginning of the simulation as the control needs a finite time-interval to effect the sound in the farfield.

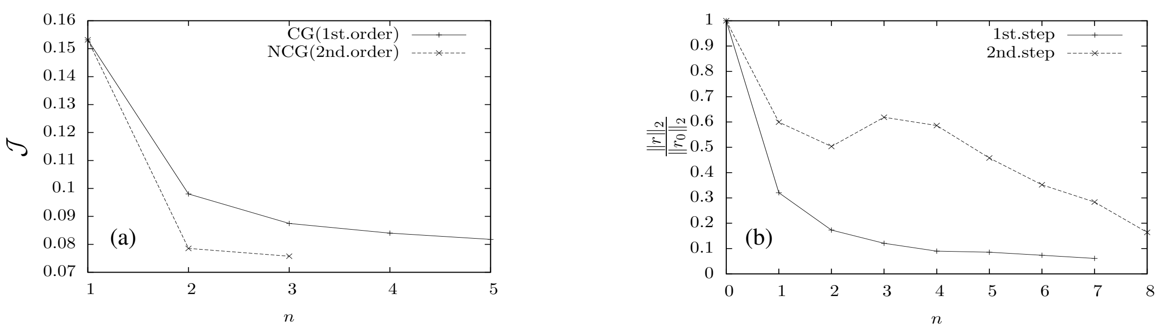

Figure 3. (a) Comparison of the cost functional reduction of the anti-sound test case [2]. The computational effort is chosen to be equal for both schemes; NCG: Newton Conjugate Gradient method, CG: Conjugate Gradient method (b) Convergence history of NCG method for solving the Newton equation. $|r|_2/|r_0|_2$ is the normalized L_2 norm of the residual, r_0 denotes the residual at the starting point and the convergence criteria was chosen to be $min(0.05, |\nabla\mathcal{J}|_2$, where $\mathcal{J}$ is the cost functional.

CONCLUSIONS

Results for continuous adjoint optimal aeroacoustic control simulations were presented using heating and cooling within the jet shear layers. The gradient accuracy was shown to degrade with increasing optimization time owing to numerical and modeling inconsistencies between the adjoint and primal flow solution. These inconsistencies may be reduced and the optimization extended to long time horizons using, although more expensive, the receding horizon approach. Currently, a Newton conjugate gradient method is developed using the second order adjoint equation. First results (figure 3) look promising and may improve the applicability of the continuous adjoint.

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