ANALYSIS OF AN AERO-ACOUSTIC FLOW FIELD USING DYNAMIC MODES

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Summary We present a dynamic mode decomposition of the composite vorticity-dilatation fields from linearized simulations of a compressible, axisymmetric jet emanating from a modeled nozzle. A subdomain analysis concentrates on the shear layer emanating from the nozzle wall and the farfield dominated by acoustic radiation. For the case of $Re = 1000$ and $Ma = 0.75$, a non-dimensionalized frequency of $f = 0.2934$ set by vortical fluid structures in the shear layer and a substantially weaker, but broader, spectral peak centered about $f = 0.9005$ associated with dilatational structures have been detected. The associated spatial modes exhibit elements reminiscent of shear layer roll-up and propagating wave-structures in the respective domains. These structures will form the basis for a reduced-order description of the overall dynamics.

INTRODUCTION AND MOTIVATION

The study of acoustic radiation from shear layers has a long history, dating back to early analytical work to more recent large-scale aero-acoustic simulations. The goal is the quantitative description of the radiated sound field, its prediction and its link to sound-generating structures in the shear flow. In this effort, we use linearized simulations of a compressible axisymmetric jet emanating from a modeled pipe-nozzle and analyze the data sequence using a model-free decomposition techniques that allows the necessary flexibility to examine isolated subdomains of the flow and determine the prevalence of fluid or acoustic processes in each. This study aims at identifying and quantifying the underlying mechanisms contained in the generated data sequence, but also to provide a low-dimensional reduced-order description of the principal dynamics in form of a dynamical system. A model of this type can subsequently be used in control applications where fast predictions of the dynamics constitute a critical component of reactive control strategies (such as in model-predictive control).

NUMERICAL SIMULATIONS AND DECOMPOSITION TECHNIQUE

A direct numerical simulation of an axisymmetric, compressible jet, linearized about a base flow, has been performed and subsequently decomposed to extract coherent structures as well as their dominant frequencies and decay rates.
A steady base flow, i.e. an exact solution of the nonlinear, compressible, axisymmetric Navier-Stokes equations, is first computed for a Reynolds number $Re = 1000$, Mach number $Ma = 0.75$, Prandtl number $Pr = 1$ and density ratio $S = 1$; the selective-frequency-damping (SFD) method [1] has been employed. The geometric setup is as follows: the jet exits from an idealized nozzle (at $x = 0$), modeled as an infinitely thin, adiabatic wall at $r = 1$ and $x \leq 0$. Only the upper half-plane $r > 0$ is resolved in the simulations, and appropriate symmetry conditions for axisymmetric flow are imposed on the jet axis $r = 0$. To maintain control over the jet profile at $x = 0$, velocity and temperature profiles are imposed at a specified streamwise location $x = x_0 < 0$, close to the jet nozzle. The axisymmetric, compressible Navier-Stokes equations are then solved downstream of x_0 using LODI boundary conditions [2] on the numerical boundaries at $x_{\min}, x_{\max}$ and $r_{\max}$. Once the SFD-calculations have converged to a steady base state, a parallel flow region is added upstream of x_0 in order to extend the base flow to the domain size which is later used for the linearized simulations.
In a second step, linearized Navier-Stokes simulations are performed using the base flow above. To this end, non-reflecting boundary conditions given by [3] are employed at the inflow, outflow and lateral boundary. In addition, all perturbation quantities are artificially attenuated in sponge layers [4] to further minimize spurious reflections. The evolution of only axisymmetric perturbations $(m = 0)$ are computed, and symmetry conditions on the jet axis are imposed accordingly.
In both cases, i.e. for the base flow calculations and the perturbation dynamics, an explicit 5^{th}-order finite-difference scheme in combination with a spatial filtering technique [5] has been used for spatial derivatives. The spatial filter merely suppresses numerical instabilities of the finite-difference scheme. Matrix-free time-stepping of the extended linear equations is performed using a 3^{rd}-order Runge-Kutta algorithm. For the subsequent decomposition, flow fields spanning the numerical domain $-40 \leq x \leq 60$ and $0 \leq r \leq 60$ and discretized with 1024×384 grid points have been produced at equispaced time intervals. A non-uniform grid distribution has been employed to accurately resolve structures in the shear layer emanating from the nozzle.
Based on the temporally equispaced data-sequence produced by the linearized simulations we now perform a decomposition into dynamic modes [6]. Within these assumptions, the structures extracted by the decomposition are equivalent to global modes. In general, the dynamic mode decomposition is a data-based algorithm that computes finite-dimensional representations of Koopman eigenfunctions [7] which describe invariants of observables governed by a nonlinear dynamical system. More specifically, a linear (high-dimensional) mapping is assumed between the computed snapshots, which is taken as constant for the entire data sequence. For sufficiently long data sequences it appears reasonable to assume a linear dependence of the individual snapshots. This is equivalent to adopting the notion of a low-rank dynamics that can be expressed by only a few coherent flow fields. Mathematically, this observation corresponds to representing the dynamics linking the snapshots by a low-dimensional mapping (in lieu of the high-dimensional one). This low-dimensional matrix,

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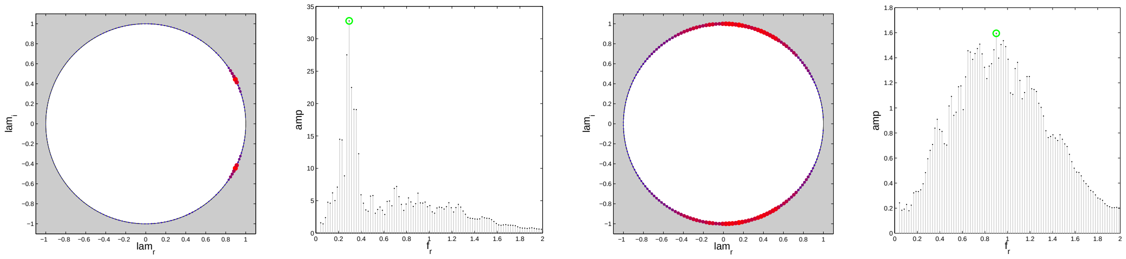

Figure 1. Shear layer analysis: (a) dynamic mode spectrum in time-stepper form showing dominance of an identified structure at a non-dimensionalized frequency of $f = 0.2934$ and (b) associated amplitude distribution. (c,d) Corresponding analysis of the far-field.

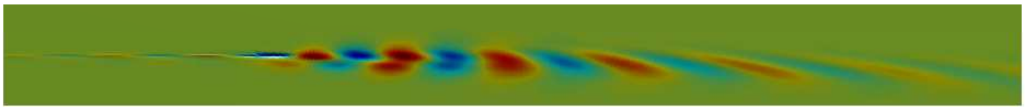

Figure 2. Azimuthal vorticity contours of the most dominant dynamic mode of the shear layer region (at a non-dimensionalized frequency of $f = 0.2934$. The displayed domain dimensions are $-5 \leq x \leq 15$ and $0 \leq r \leq 2$.

expressed in its simples form by a companion matrix, can easily be determined using a least-squares technique. Special care has to be exercised to treat rank-deficient (or nearly rank-deficient) data-matrices [6].

This type of decomposition, based merely on temporally-aligned data sequences, has various advantages over more common decompositions that are based on the availability of an underlying model (given by the discretized governing equations), such as the Arnoldi technique. Since the dynamic mode decomposition is not bound by governing equations, measured or computed variables can be combined at will to form a snapshot. Two specific applications of this feature concern the analysis of subdomains and the analysis of composite state-vectors. Both cases will be treated here. In the former case, we will analyze the dynamics of the acoustic jet by concentrating on isolated regions of the computational domain: first, the region near the centerline containing the jet's shear layer; then, the acoustic farfield containing the jet's sound field. In the latter case, we will form a composite state-vector containing the azimuthal vorticity and dilatation of the respective subdomain. In this manner, we can delineate regions of the computational domain that are dominated by the respective dynamics and identify the principal processes (fluid-based given by the vorticity, or acoustic-based given by the dilatation) within these domains.

RESULTS

A sequence of 200 snapshots, with an inter-snapshot spacing of $\Delta t = 0.25$, has been processed using the dynamic mode decomposition. In a first attempt the region downstream of the nozzle wall has been analyzed in an attempt to isolate the fundamental structures in the shear layer; the data consist of compound vectors based on the azimuthal vorticity and the dilatation. Figure 1 (a,b) shows the spectrum of the inter-snapshot map, colored by the amplitude of the respective frequency content in the data sequence. A dominant frequency component at $f = 0.2934$ has been detected and the associated spatial structure (Fig. 2) shows the typical vortical structures of a shear-layer mode. An analogous analysis of the far-field revealed a dynamics characterized by a broader spectrum at a substantially reduced amplitude, shown in subfigures 1(c,d). The peak of this broad spectrum is centered around a frequency of $f = 0.9005$. The corresponding mode (not shown) is distinct from the shear layer structure in the sense that it consists of only the dilatational components, with a negligible vortical element. A more complete analysis of the identified structures, together with a dynamic representation of the reduced-order dynamics, will be presented at the Congress.

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