QUANTIFYING MACH NUMBER SIMILARITY OF COMPRESSIBLE TURBULENT BOUNDARY LAYERS

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Summary Analysis of direct numerical simulation (DNS) of spatially developing compressible turbulent boundary layers (CTBL) is performed using ratios of the components of Reynolds stress tensor to its trace,$\Gamma_{u_iu_j}$. It is found that the bulk flow is characterized by four *universal* constants:$\Gamma_{uu}^{bulk}\approx 0.5$,$\Gamma_{vv}^{bulk}\approx 0.2$,$\Gamma_{ww}^{bulk}\approx 0.3$,$\Gamma_{uv}^{bulk}\approx 0.15$,which we view as manifestations of *universal similarity* of vortex motions. Two critical locations are identified:δ, where Γ_{vv} exceeds Γ_{uu},defining the edge of the CTBL and δ_1 where Γ_{vv} exceeds Γ_{ww},the inner boundary of an entrainment layer. It is shown that δ is close to δ_{99} for incompressible flow, and that the ratio $\delta/\delta_1\approx 1.2$ is universal-independent of the Mach number (Ma),the Reynolds number (Re) and the wall temperature. We also present a quantitative description of a set of Reynolds stress length $\ell_{u_iu_j}$(RSLs), and find a multi-layer description for its Ma-independent expression.

IDENTIFYING THE MACH NUMBER SIMILARITY FOR CTBL

The ratios between the components of Reynolds stress tensor to its trace,$\Gamma_{u_iu_j}$, are computed for a spatially developing CTBL field obtained by direct numerical simulation (DNS) up to $Ma = 2.25, 4.50$ and 6.00[1]. It is found that $\Gamma_{u_iu_j}$ is *independent* of how the Reynolds stresses are defined, e.g. by Reynolds averaging or Favre averaging, and with or without local density weighted. In fully developed turbulence zone, four *universal* plateaus are found in the bulk zone with values of $\Gamma_{uu}^{bulk}\approx 0.5$,$\Gamma_{vv}^{bulk}\approx 0.2$,$\Gamma_{ww}^{bulk}\approx 0.3$,and $\Gamma_{uv}^{bulk}\approx 0.15$,for all values of Ma. The collapse, hence a result of Ma-similarity,of $\Gamma_{u_iu_j}$(see Figure 1) is obtained by a careful comparison between simulation data at different Ma with the same $\mathrm{Re}_\delta = \delta\bar{u}_\tau/\bar{\nu}_w$ or $\mathrm{Re}_{\delta 1} = \delta_1\bar{u}_\tau/\bar{\nu}_w$ (but not $\mathrm{Re}_\tau = \delta_{99}\bar{u}_\tau/\bar{\nu}_w$), where a boundary layer thickness δ is defined by the condition $\Gamma_{vv} = \Gamma_{uu}$, and δ_1 by $\Gamma_{vv} = \Gamma_{ww}$. It turns out that $\delta/\delta_1\approx 1.2$ at all Ma. The similarity property also holds for several boundary conditions of wall temperatures. This finding provides an answer to a key question regarding the application of the Morkovin hypothesis[2]: what kind of outer length (or Re) should we use to perform the comparison among different Ma?

The collapse extends to a modified mixing length defined from the Reynolds shear stress, see Figure 2. It also extends to the whole profiles of transformed mean velocity profile (MVP) that is viscosity-weighted (see Figure 3), or Reynolds stresses, skewness and flatness for Ma up to 6. When the y-coordinate is scaled by δ_1 (or δ), the maximum vertical deviation for transformed MVP is 1.8% at the same Re_δ. In contrast,the maximum deviations is 5% when δ_{99} and the same Re_τ are used. Therefore, the observation of the onset of intermittency nearer to wall with increasing Ma[3] is in fact due to inappropriate choice of the outer length scale, but not the failure of Morkovin's hypothesis for high order moments.

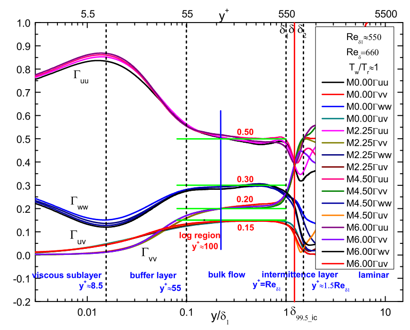

Figure 1. (color online).The collapse of $\Gamma_{u_iu_j}$ for three Ma with the same Re_δ and wall temperature condition.Viscous sublayer,buffer layer,bulk flow,intermittent layer and laminar layer are identified.

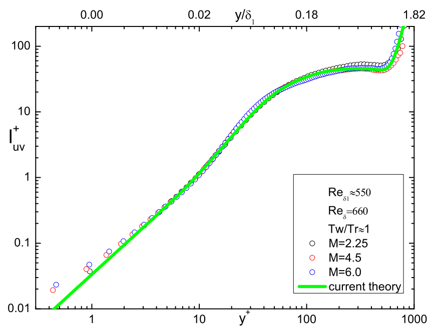

Figure 2. (color online).The Ma invariance of modified mixing length ℓ_{uv}^+ for three Ma with the same Re_δ and wall temperature condition. The green line is a multi-layer model (Eq.1).

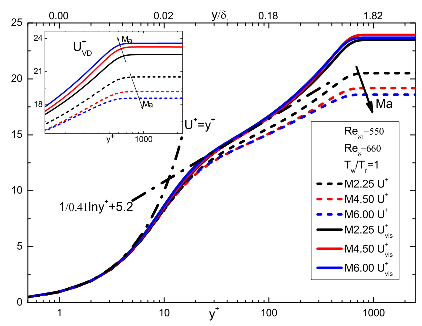

Figure 3. (color online).Viscosity-weighted transformed MVPs for three Ma with the same Re_δ and wall temperature condition. The insert is the result using the van Driest transformation.

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QUANTIFYING THE MACH NUMBER INDEPENDENCE OF LENGTH SCALES

The turbulence dissipation rate $\bar{\varepsilon}=\overline{\tau_{ik}^{'}\partial_{x_k}u_i^{'}}$ is proportional to dynamical viscosity $\bar{\mu}$. Compared with (wall unit normalized) $\bar{\varepsilon}_{ic}^{+}$ of incompressible TBL, we argue-verified also by DNS database-that the influence of normal variation of mean thermodynamic quantities in a CTBL on $\bar{\varepsilon}_c^{+}$ should be proportional to the relative magnitude of local viscosity, namely $\frac{\bar{\varepsilon}_c^{+}(y/\delta)}{\bar{\varepsilon}_{ic}^{+}(y/\delta)}=\frac{\bar{\mu}_c(y/\delta)/\bar{\mu}_c|_w}{\bar{\mu}_{ic}(y/\delta)/\bar{\mu}_{ic}|_w}=\bar{\mu}_c^{+}(y/\delta)$. This means that $\bar{\varepsilon}^{+}/\bar{\mu}^{+}$ is an invariant of Ma. In the bulk region, the quasi-balance of turbulence production and dissipation $\bar{\rho}^{+}\widetilde{u''v''}^{+}\frac{\partial\tilde{u}^{+}}{\partial y^{+}}\approx\bar{\varepsilon}^{+}$, combined with the Ma-independent Morkovin scaling $\bar{\rho}^{+}\widetilde{u''v''}^{+}$ and $\bar{\varepsilon}^{+}/\bar{\mu}^{+}$, suggests that $\frac{1}{\bar{\mu}^{+}}\frac{\partial\tilde{u}^{+}}{\partial y^{+}}$ is a Ma-invariant quantity. Those invariants immediately imply that modified viscosity coefficients $\nu_{t\_u_iu_j}=\bar{\rho}^{+}\widetilde{u''v''}\Big/\left(\frac{1}{\bar{\mu}^{+}}\frac{\partial\tilde{U}}{\partial y}\right)$, and RSLs $\ell_{u_iu_j}=\sqrt{\bar{\rho}^{+}\widetilde{u''v''}}\Big/\left(\frac{1}{\bar{\mu}^{+}}\frac{\partial\tilde{U}}{\partial y}\right)$ are relevant Ma-invariant quantities (in wall units). Those invariants address another key question regarding Morkovin's hypothesis[2]: what is the quantitative effect of compressibility on length scales of turbulent fluctuations. Fig.2 shows the Ma invariant of modified Prandtl mixing length ℓ_{uv}^{+}. Consequently, a Ma-invariant transformation to collapse the *whole* profiles of MVP at different Ma but the same Re_δ is derived:$\bar{u}_{ic}^{+}=\int_0^{\tilde{u}_{c\infty}^{+}}\frac{1}{\bar{\mu}^{+}}\left(\frac{-1+\sqrt{1+4\ell_{uv}^{+2}}}{-\bar{\mu}^{+2}+\sqrt{\bar{\mu}^{+4}+4\ell_{uv}^{+2}}}\right)^{p}d\tilde{u}_c^{+}$ with $p=1/(1+e^{y^{+}-40})$, which improves the traditional van Driest transformation[4] (see Fig. 3). Note that the original van Driest transformation can only recover the classical log-law of incompressible TBL,and the deviation above the log layer increases significantly with increasing Ma(see the insert of Fig. 3). A multi-layer expression of ℓ_{uv}^{+}, as well as $\ell_{u_ku_k}^{+}$,is formulated by following She et al.'s Lie group analytical procedure [5] and reads:

$$\ell_{uv}^{+}=\frac{5}{6}(\frac{y^{+}}{8.5})^{(3/2)}\left(1+(\frac{y^{+}}{8.5})^{4}\right)^{(1/2)/4}\left(1+(\frac{y^{+}}{47.5})^{4}\right)^{-1/4}\frac{1-\mathrm{sgn}(z)\,|z|^{4}}{4\,(1-z)},z=1-y^{+}/0.5\mathrm{Re}_\delta. \quad (1)$$

$$\ell_{uu}^{+}=\frac{7}{2}(\frac{y^{+}}{8.5})^{1}\left(1+(\frac{y^{+}}{8.5})^{4}\right)^{(2/3)/4}\left(1+(\frac{y^{+}}{31})^{4}\right)^{(-13/15)/4}\left(1+(\frac{y^{+}}{31})^{4}\right)^{(1/5)/4}\frac{1-\mathrm{sgn}(z)\,|z|^{4}}{4\,(1-z)}. \quad (2)$$

$$\ell_{vv}^{+}=\frac{1}{88}(\frac{y^{+}}{1})^{2}\left(1+(\frac{y^{+}}{1})^{4}\right)^{(-1/3)/4}\left(1+(\frac{y^{+}}{8.5})^{4}\right)^{1/4}\left(1+(\frac{y^{+}}{41})^{4}\right)^{(-5/3)/4}\frac{1-\mathrm{sgn}(z)\,|z|^{4}}{4\,(1-z)}. \quad (3)$$

$$\ell_{ww}^{+}=\frac{1}{4}(\frac{y^{+}}{1})^{1}\left(1+(\frac{y^{+}}{1})^{4}\right)^{(-1/4)/4}\left(1+(\frac{y^{+}}{8.5})^{4}\right)^{(5/4)/4}\left(1+(\frac{y^{+}}{45})^{4}\right)^{-1/4}\frac{1-\mathrm{sgn}(z)\,|z|^{4}}{4\,(1-z)}. \quad (4)$$

They are in good agreement with DNS data, see Fig. 2. Systematic determination of the parameter values involved in the model will be presented. With those expressions,the normal variation of $\Gamma_{u_iu_j}$ can be solved analytically.

CONCLUSIONS

We specify quantitatively,rather than qualitative description of Morkovin's hypothesis, the influence of compressibility on turbulent boundary layer. A suitable outer length,as well as Re, is determined to collapse statistical quantities for all Ma, and Ma-independent length scales are defined to consider the normal variation of mean thermodynamic quantities. The latter leads to a transformation that collapses the whole profiles of MVP. Further, those length scales are formulated analytically with a novel Lie group method recently proposed by She et al.

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