A MULTI-LAYER MODEL FOR MEAN VELOCITY AND DENSITY PROFILE IN COMPRESSIBLE CHANNEL FLOW

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Summary A turbulent compressible channel flow is studied by direct numerical simulation (DNS) at Re_τ=430 with Ma=1.5, 3.0, 6.0 and modelled by multi-layer structure theory. As an extension of the mixing length for incompressible flow, an effective Reynolds stress is defined and a closure new length order function L_M^+ in streamwise mean momentum equation is introduced to describe the effect of density and velocity fluctuations. The new model yields an accurate description of both the mean velocity and density profile for the compressible channel flows, and suggests an improved transformation leading to Ma-independent profile.

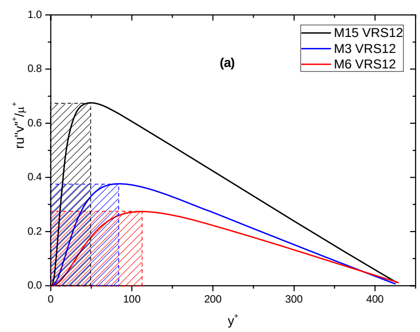
INTRODUCTION

Compressible channel flow is the simplest wall-bounded turbulence where effects of compressibility interact with shear to produce highly non-equilibrium dynamics and structures. In 1951, van Driest [1] proposed a density scaling, $\overline{U}_{VD} = \int_0^{\overline{U}} \sqrt{\frac{\overline{\rho}}{\rho_w}} dU$ for compressible wall-bounded flow, and obtained: $\frac{\overline{U}_{VD}}{u_\tau} = \frac{1}{\kappa} \ln y^+ + B$. Huang et al. [2] suggests a semi-local scaling as $y^* = \overline{\rho} y u_* / \overline{\mu}$ & $u_* = \left(\tau_w / \overline{\rho}\right)^{1/2}$. Wilcox (1992) has devised a function in the ω equation's closure coefficient as $\beta^* = \beta_i^* [1 + \xi^* F(M_t)]$ for modelling compressible wall-bounded flows [7]. Foysi et al. (2004) investigate compressibility effects on the turbulent stresses and explain the reduction in the near-wall pressure-strain correlations by a Green's-function-based analysis of the pressure field.

More recently, a new approach is introduced to integrate the effects of fluctuations into prediction of mean flows for wall-bounded turbulent flows. Several length functions are introduced by She et al. in [6] [8], which have successfully described the mean velocity profiles (MVP) and mean kinetic-energy profile (MKP) in incompressible channel and pipe flow. We extend the latter analysis to study the compressible channel flow with a new length function for describing group invariant solutions to the unclosed MME which to representing effects of density variation.

Streamwise mean momentum equation [2] in compressible channel flow is written as:

$$\overline{\mu}^+ \frac{\partial \overline{U}^+}{\partial y^+} + \frac{\overline{\mu' \partial U^{+'}}}{\mu_w \partial y^+} - \overline{\rho u'' v''}^+ = 1 - \frac{1}{H} \int_0^y \frac{\overline{\rho}}{\rho_m} dy$$



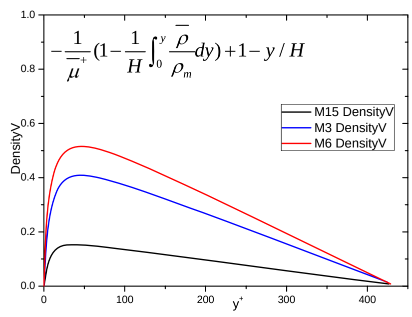


Figure1. Black, blue and red lines indicate Ma=1.5, 3.0 and 6.0 cases. The left figure shows the peak position goes away from the wall and $-\frac{\overline{\rho u'' v''}^+}{\overline{\mu}^+}$ is decreasing with increasing Mach number. The area has little Mach number effect.

The right figure is mean density and viscous term. It increases with Mach number.

It can be rewritten in a similar form as the incompressible channel flow:

$$\frac{\partial \overline{U}^+}{\partial y^+} + \underbrace{[\frac{1}{\overline{\mu}^+}(\overline{\frac{\mu'}{\mu_w}\frac{\partial U'^+}{\partial y^+}} - \overline{\rho u''v''}^+ - 1 + \frac{1}{H}\int_0^y \frac{\overline{\rho}}{\rho_m}dy) + 1 - y/H]}_{\equiv W_c^+} = 1 - y/H$$

This equation leads to the definition of an efficient Reynolds stress containing three terms as viscous fluctuation, viscous Reynolds stress and mean density & viscous term.

THE MULTI-LAYER MODEL FOR MEAN VELOCITY

With the effective Reynolds stress, we could closure the RANS equation by the closure length order function $L_M^+ \equiv \sqrt{W_c^+}/S^+ = \frac{\sqrt{1-y-S^+}}{S^+}$, where $S^+ = \frac{\partial \overline{U}^+}{\partial y^+}$. The closure length order function L_M^+ directly yields S^+, which integrate to give the mean velocity profile of compressible channel flow. In the viscous sublayer, the length order function has a scaling of $y^{+1/2}$, which is different from incompressible channel flow due to compressibility effect near the wall. In buffer layer, the closure length order function has a scaling of y^{+2}, similar to incompressible flow, because the density variation is mainly restricted near the wall. However, the density variation which increases the thickness of viscous sublayer and buffer layer. The outer region scale has little Mach number effect and one can use the result of the incompressible flow in the model directly. These considerations and the measurements of scaling and thickness lead to a multi-layer model, similar in structure as the incompressible channel but different in details:

$$L_M^+ = \Theta(\frac{y^+}{y^+_{sub}})^{0.5}\left(1+(\frac{y^+}{y^+_{sub}})^4\right)^{1.5/4}\left(1+(\frac{y^+}{y^+_{buf}})^4\right)^{-1/4}\frac{1-r^4}{4(1-r)}\left(1+(\frac{r}{0.27})^{-2}\right)^{0.5/2}.$$

As shown in figure 2, we obtain a satisfactory description of the simulation data:

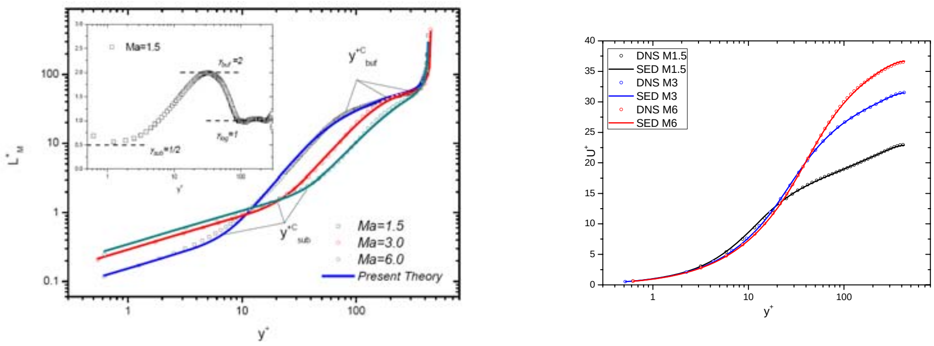


Figure2. Dots are DNS data and lines are results of multi-layer structure model for compressible channel flow. Inset of the left figure shows the diagnostic function $d[\ln(L_M^+/G)]/d(\ln y^+)$, where G is an outer function, revealing inner three layers: namely, sublayer, buffer layer, bulk region. The right figure shows mean velocity profile which is the integral of $\frac{dU^+}{dy^+} = \frac{-1+\sqrt{4(1-y/H)L_M^{+2}+1}}{2L_M^{+2}}$.

The compressible mixing length model is can also used to describe in other cases as [4] by adjusting calculating a Mach-number dependent effect of thickness in of viscous sublayer and buffer layer. Results are shown in Figure 3.

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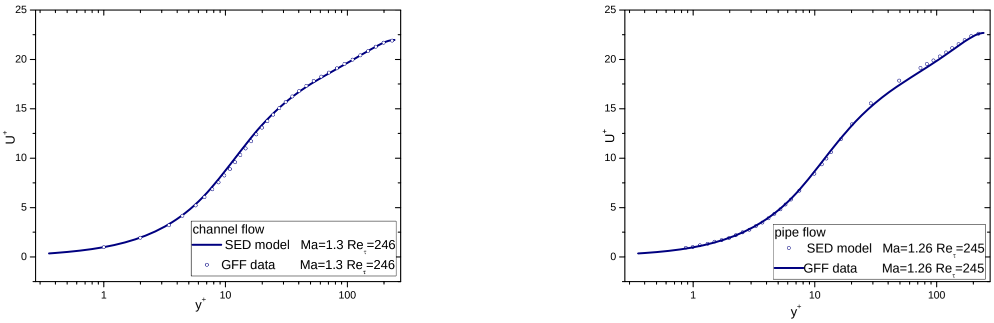

Figure3. The prediction of SED model (lines) for mean velocity profile at Ma=1.3 and Re_τ=246 in channel flow (left) and Ma=1.3 and Re_τ=246 pipe flow (right). Dots are DNS data calculated by [4].

AN IMPROVED TRANSFORMATION

Unlike classical formula which uses Taylor expansion of mean velocity to express mean density, we use the formula which uses the closure length function to obtain mean density.

We improve Van Driest Transformation $\frac{d\overline{U}_{In}^{+}/dy^{+}}{d\overline{U}^{+}/dy^{+}} \approx \sqrt{\frac{\overline{\rho}}{\rho_w}}$ by structure ensemble transformation $\frac{d\overline{U}_{In}^{+}/dy^{+}}{d\overline{U}^{+}/dy^{+}} \approx \frac{L_M^{+}}{l_m^{+}}$.

From the definition, we obtain $\frac{d\overline{U}_{In}^{+}/dy^{+}}{d\overline{U}^{+}/dy^{+}} = \frac{-1+\sqrt{1+4l_m^{+2}(1-y/H)}}{-1+\sqrt{1+4L_M^{+\,2}(1-y/H)}}\frac{L_M^{+\,2}}{l_m^{+2}} = \frac{\sqrt{1+4L_M^{+\,2}(1-y/H)}+1}{\sqrt{1+4l_m^{+2}(1-y/H)}+1}$. When y+>10,

$\frac{d\overline{U}_{In}^{+}/dy^{+}}{d\overline{U}^{+}/dy^{+}} \approx \frac{L_M^{+}}{l_m^{+}}$. The L_M^{+} and l_m^{+} can be calculated by the multi-layer model [6].

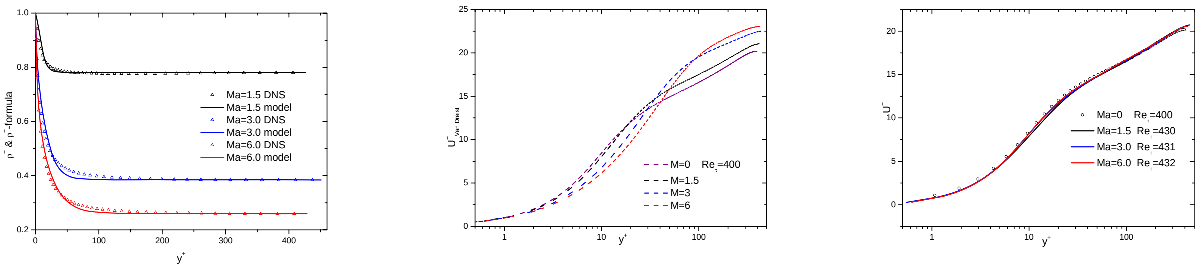

Figure4. On the right, compare the density formula with DNS data through the whole domain. The dots are DNS data of M1.5, 3.0 and 6.0 and the lines are calculated by $\frac{\overline{\rho}_{model-0}-\rho_w}{\rho_c-\rho_w} = L_M^{+\,3/2}(1+L_M^{+\,2})^{-3/4}$. In the middle shows Multi-layer length function transformation, which is shown much better than van Driest transformation on the right.

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