FRONT PROPAGATION IN COMPRESSIBLE FLOWS

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Summary We investigate the relevance of the compressibility for the front propagation in laminar flows. We study two classes of imposed model flows in which the compressibility is controlled parametrically: shear and cellular compressible flows. We focus on regimes at high Peclet and Damköhler number and we observe that in both shear and cellular flows the mean asymptotic front speed does not change very much at varying the intensity of the compressibility. On the contrary the instantaneous shape of the concentration field can be visibly altered.

INTRODUCTION

Transport of reacting species advected by laminar or turbulent flows described by an advection reaction diffusion equation (ARD) is an issue of interest in many fields, e.g., population dynamics, propagation of plankton in ocean, reacting chemicals in the atmosphere, combustion, etc.
While this issue has been widely studied in the case of incompressible passive flows, the reaction diffusion problem in compressible media is much less discussed in literature.
Our aim is to investigate the effects of the compressibility to the bulk burning rate of the reaction by studying the following PDE:

$$\rho\left[\frac{\partial\theta}{\partial t}+u_i\frac{\partial\theta}{\partial x_i}\right]=D_0\frac{\partial^2\theta}{\partial x_i^2}+\dot{\omega} \tag{1}$$

The scalar field θ represents the mass fraction of a species of a premixed binary mixture, while u_i is the i^{th} component of a given compressible flow, $D_0=\rho D$ is the diffusion coefficient (supposed to be constant), $\dot{\omega}$ the rate of production of one of the two species and ρ the density of the carrying fluid.
Assuming an auto-catalytic irreversible law $A+B\rightarrow 2A$, the source term is defined as follow:

$$\dot{\omega}=\frac{\rho^2}{\rho_0}\alpha\theta(1-\theta) \tag{2}$$

where α is the characteristic time of reaction and ρ_0 a density reference.
We will consider two flows. The first is a 2D steady state compressible shear flow:

$$\rho(x)=\rho_0+\epsilon\rho_0\sin\left(\frac{2\pi x}{\lambda}\right) \qquad \bar{u}(x,y)=\left(\frac{U_0\sin\left(\frac{2\pi y}{L}\right)}{1+\epsilon\sin\left(\frac{2\pi x}{\lambda}\right)},0\right) \tag{3}$$

which is a Kolmogorov flow with amplitude U_0 and wavelength L perturbed by a steady compressional wave of wavelength λ and magnitude $\rho_0\epsilon$. The perturbation is oriented along the direction of propagation of the reactive front i.e. the x axis. In the same manner, we consider a compressible cellular flow:

$$\rho(x)=\rho_0+\epsilon\rho_0\sin\left(\frac{2\pi x}{\lambda}\right) \qquad \bar{u}(x,y)=\left(\frac{U_0\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi y}{L}\right)}{1+\epsilon\sin\left(\frac{2\pi x}{\lambda}\right)},\frac{U_0\cos\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)}{1+\epsilon\sin\left(\frac{2\pi x}{\lambda}\right)}\right) \tag{4}$$

in which the 2D steady flow of amplitude U_0 is composed by square closed cells of dimension $L/2$ and it is perturbed by the presence of a steady compressional wave as above. Both flows, in the limit case of $\epsilon=0$, are well studied, see for instance [1][2][3].
Let define the characteristic advection, reaction and diffusion times respectively $\tau_a=L/U_0$, $\tau_r=1/\alpha$ and $\tau_d=\rho_0L^2/D_0$. We will focus our attention in regimes in which the diffusion time is always very large compared to the others. Since most of reaction of interest have time scales comparable or faster than the advection time, apart from slow biological reactions, or when the stirring by the velocity field is extremely intense, we will also consider $\tau_r\geq\tau_a$.

NUMERICAL RESULTS

We define the instantaneous bulk burning rate:

$$v_f(t)=\int_0^L\int_{-\infty}^{+\infty}\frac{\rho^2}{\rho_0}\alpha\theta(1-\theta)dxdy=\int_0^L\int_{-\infty}^{+\infty}\dot{\omega}(\theta)dxdy \tag{5}$$

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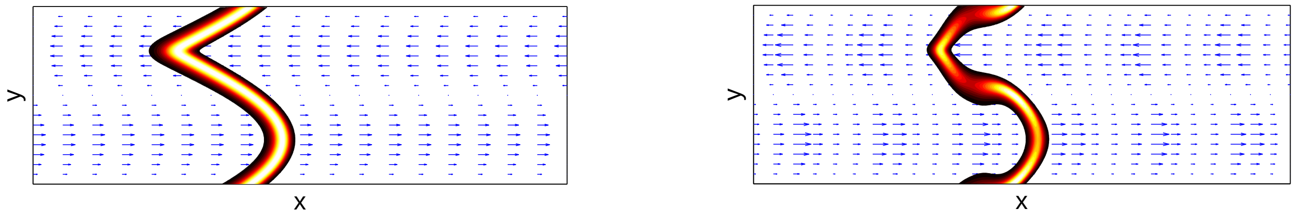

Figure 1. Instantaneous snapshot of the reactive front in a shear flow test case ($Pe = 100$ and $Da = 100$). Left panel shows the incompressible case ($\epsilon = 0$). Right panel the compressible case ($\epsilon = 0.5$ and $\lambda = 0.5$)

A convenient parameter of comparison, is the percentage difference of the mean asymptotic bulk rate between a compressible and an incompressible test case:

$$\Delta v_{\%} = 100 \cdot \frac{\langle v_f \rangle - \langle v_f \rangle}{\langle v_f^0 \rangle} \tag{6}$$

where $\langle \, . \, \rangle$ indicates a time average on a suitable long interval and $\langle v_f^0 \rangle$ the mean bulk rate for the incompressible case ($\epsilon = 0$).
In terms of adimensional quantities we study regimes at high Peclet number ($Pe = U_0 L \rho_0 / D_0 \gg 1$) varying the Damköhler number ($Da = L\alpha/U_0$). Thus, for a fixed Peclet we perform a parametric analysis to shed some light to the role played by the characteristic length and magnitude of the compressibility (λ, ϵ) and how this can vary by varying the Damköhler number.
Equation (1) has been solved using an eighth-order explicit finite difference scheme in space and a fourth-order Runge-Kutta integration in time moreover, since long times of integration are needed to observe the asymptotic behavior of front propagation, the computational domain is remapped following the front.
In the presence of a relevant compressibility (i.e. $\epsilon = 0.5$) the instantaneous front speed (and shape) are visibly altered by the perturbation (see figure 1). Nevertheless the differences in terms of mean asymptotic propagation are quite modest (see figure 2). For instance, in a compressible shear flow we obtain $\Delta v_{\%} \sim \epsilon^2$. Moreover, for a fixed ϵ, the wavelength of the perturbation (λ) does not affect the asymptotic value of the bulk burning rate.

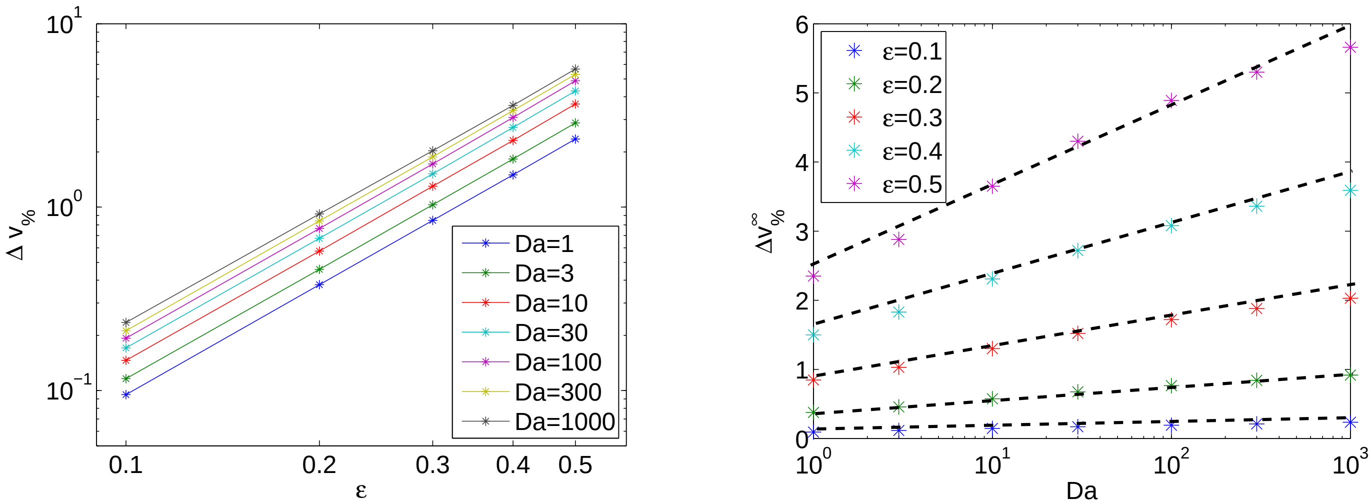

Figure 2. Percentage difference between compressible and incompressible shear flow at different compressibility magnitude (ϵ) and for different Damköhler numbers (Da). $Pe = 100$ and $\lambda = 1$.

References

[1] M. Abel, A. Celani, D. Vergni and A.Vulpiani, *Front propagation in laminar flows*, Physical Review E **64** 6307 (2001).
[2] M. Abel, M. Cencini, D. Vergni and A.Vulpiani, *Front speed enhancement in cellular flows*, Chaos **12** p.481.
[3] N. Vladimirova, P. Constantin, A. Kiselev, O. Ruchayskiy, L. Ryzhik, Combustion theory and modelling **7** p.487 (2003).
[4] T. Poinsot and D. Veynante, *Theoretical and numerical combustion* Edwards (2005)
[5] R. Proud'homme, *Flows of reactive fluids* Springer (2010)
[6] S. Berti, D. Vergni and A. Vulpiani,*Combustion dynamics in steady compressible flows* ,EPL **83**, 54003 (2008).
[7] A. Kolmogorov, I. Petrovsky, N. Piscounoff, Moscow Univ. Bull. Math., **1** (1) (1937).