A FLAME DESCRIBING FUNCTION ANALYSIS OF COMBUSTION INSTABILITY LIMIT CYCLES SUSTAINED BY MULTIPLE MODES

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Summary Combustion instabilities have considerable practical consequences and raise many fundamental issues. Much of the research effort has focused on the linear growth of oscillations but it is well recognized that a nonlinear approach is required if one wishes to understand limit cycles, mode switching and triggering. Many of these features can be deduced from a flame describing function (FDF) framework. This can be used to predict the amplitude and frequency of combustion oscillation when the limit cycle is sustained by a single mode and features a stable amplitude (SALC). Recent experiments exhibit other types of limit cycles with variable amplitude (VALC) with regular or irregular amplitude modulation. These oscillations also designated as galloping limit cycles (GLC) are characterized experimentally and the FDF framework is used to infer conditions leading to this behavior.

Introduction

Dynamical flame modeling constitutes an essential component in the prediction of thermo-acoustic instabilities. In many studies the flame response is characterized by a Flame Transfer Function (FTF) but this only provides a linear insight into the combustor dynamics. It was however recognized in early studies and most notably in the area of aerospace propulsion that combustion nonlinearity was responsible for amplitude saturation. This was observed in model scale experiments [1] and envisaged theoretically for example by Dowling [2] in an analysis of afterburner instability. It was shown that the describing function (DF) methodology from nonlinear control could be used to represent the flame gain saturation corresponding to flow reversal [3]. It was shown more recently that a unified framework using a "Flame Describing Function" (FDF) [4] provides reliable predictions of frequency and amplitude of self-sustained combustion oscillations in a multiple flame burner without confinement of the combustion zone and that this could also be used to predict limit cycles in generic systems including an upstream manifold, an injection element and a flame tube confining the combustion region [5]. It is found however, that in certain ranges the limit cycles feature a variable amplitude which is not well rendered with the FDF methodology. Problems arise when the amplitude evolves as a function of time. Galloping limit cycles (GLC) of this type were not observed in the unconfined situation investigated in [4] but they arise in the generic system considered in the present study in a limited range of the bifurcation parameter L_1.

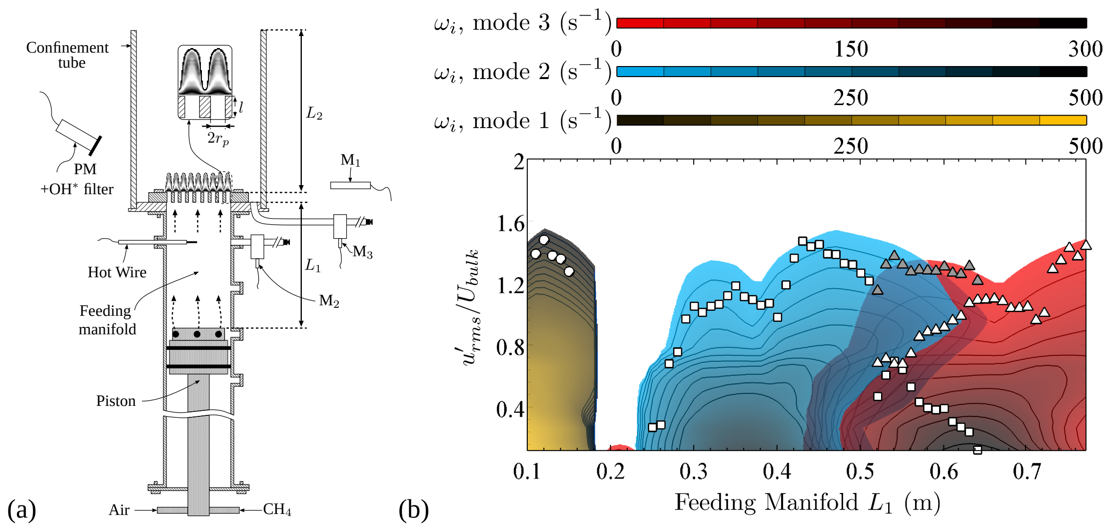

Figure 1. (a) Experimental setup and diagnostics used to characterize self-sustained instabilities. (b) Calculated growth rate maps obtained by varying the upstream manifold length L_1 from 0.11 to 0.77 m. Growth rates correspond to first (yellow), second (blue) and third (red) eigenmodes. Scales are shown above to graph. L_2 = 0.1 m. Amplitudes of oscillation are displayed as as circle symbols (○) for the first mode, open box symbols (□) for the second mode, triangle symbols (△) for the third mode. Gray triangle symbols correspond to the sum of the two modal amplitudes in the multiple frequency range (from L_1 = 0.52 to 0.64 m).

Experimental results

Experiments are carried out in a configuration including a variable upstream manifold, an injection element and a short flame tube L_2 = 0.1 m (Fig. 1(a)). This system exhibits various types of dynamics depending on the feeding manifold

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length L_1 (Fig. 1(b)). By varying the bifurcation parameter L_1, it is possible to observe stable combustion or self-sustained combustion oscillations which are found to belong to two groups. In the first group, oscillations occur at a nearly constant amplitude and yield stable amplitude limit cycles (SALC). In the second class, oscillation exhibits variations in amplitude which may be regular or irregular. These variable amplitude limit cycles (VALC) also designated as Galloping Limit Cycles (GLC) are characterized by a specific sound radiation signature which can be qualified as “modulated buzzing” combustion. In the present experiments, GLCs feature a regular modulation but in other circumstances this can be irregular and even chaotic.

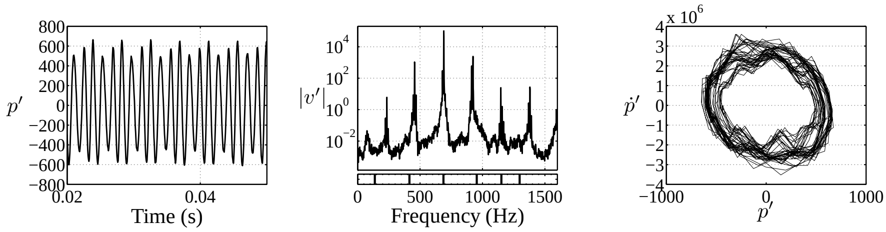

Figure 2. Pressure evolution recorded by microphone M_2 (left) for L_2 = 0.1 m and L_1 = 0.58 m. The spectrum is displayed in the center, a phase portrait is shown on the right. Acoustic eigenmodes calculated without the flame but by assuming different temperatures in each cavity are drawn as vertical lines below the pressure spectrum.

Theoretical interpretation

Using the unified framework devised in [4, 5] it is shown that stable amplitude limit cycles observed in different ranges of L_1 could be suitably retrieved theoretically and that triggering, frequency shifting and mode switching can be predicted quite reliably. The main focus of the present investigation concerns GLCs. Detailed analysis of the range $0.52 < L_1 < 0.64$ m indicates that these variable amplitude limit cycles are sustained by a combination of two modes (Fig. 2). When this occurs, it is also found to lead to frequency “heterodyning" where the nonlinear mixing of two frequencies gives rise to a component at the frequency difference. Some theoretical calculations [6] indicate the possibility of double mode oscillations. Culick and his co-workers show for example that this can happen when one mode is linearly stable while the other is linearly unstable but this requires that the growth rates belong to some specific ranges of values. These conditions are not observed in the present experiments but it is found that oscillations sustained by two modes arise when a different set of conditions is fulfilled. The analysis relies on FDF calculations which provide solutions of the nonlinear dispersion relation (NDR) describing the system dynamics, for each oscillation amplitudes u'_{rms}/U_{bulk} where u'_{rms} is the root mean square velocity fluctuation at the base of one flame and U_{bulk} is is the bulk velocity. Roots of the NDR are calculated for each amplitude of oscillation. The solution $\omega = \omega_r + i\omega_i$, provides the growth rate ω_i and the angular frequency ω_r ($2\pi f$) as function of amplitude. The growth rate contours obtained for the first three modes are plotted in Fig. 1(b). In the stable amplitude limit cycle cases, when the growth rate ω_i vanishes, the limit cycle is reached. The measured amplitudes are found to be close to the $\omega_i = 0$ boundary in this case (see Fig. 1(b)). GLCs arise when there is an overlap between a linearly unstable mode and a nonlinearly unstable mode as exemplified in Fig. 1(b) where modes 2 and 3 overlap. In some cases this will simply lead to mode switching and a single mode with the highest growth rate will prevail. A detailed analysis of growth rate trajectories of the overlapping modes indicates that the two modes coexist when the rates of change of the growth rates corresponding to the two modes (determined from the FDF framework) are simultaneously negative, a condition which may be written in the form $(d\omega_{i3}/da)_{a_0} < 0$ and $(d\omega_{i2}/da)_{a_0} < 0$ where a_0 is the amplitude where the growth rate of the linearly unstable mode vanishes $\omega_{i3}(a_0) = 0$.

In conclusion, it is shown that the FDF framework can be used to calculate the stable amplitude limit cycles (SALC) and to define ranges where one expects galloping limit cycles (GLC) sustained by two modes. Experiments indicate that this happens when an overlap exists between a linearly unstable mode and a nonlinearly unstable mode and when an additional condition on the modal growth rates is fulfilled.

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