RECEPTIVITY OF THE BOUNDARY LAYER TO ELASTIC VIBRATIONS OF THE WING SURFACE

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Summary This paper is concerned with the process of generation of Tollmien-Schlichting waves in the boundary layer due to elastic vibrations of the wing surface. It is assumed that the Reynolds number, Re, is large and the flow around the wing is studied by means of asymptotic analysis of the Navier-Stokes equations. We were able to deduce an explicit formula for the receptivity coefficient which determines the amplitude of the generated Tollmien-Schlichting wave as a function of the amplitude and form of the wing vibration.

Introduction

The Receptivity Theory is twenty years old now, and has been developed hand in hand with experimental observations, primarily, in low turbulence wing tunnels. It deals with the process of transformation of external disturbances into internal natural instability modes of the boundary layer, Tollmien-Schlichting waves, Cross-Flow Vortices, Görtler Vortices, etc. It is known that some disturbances easily penetrate into the boundary layer and turn into an instability mode, others do not. In the former category are acoustic waves and free-stream turbulence. Still even these have to satisfy rather restrictive resonance conditions. Experimental observations (see, for example, [1]) show that unlike in a simple mechanical system, say, a pendulum, where the resonance is observed when the frequency of the external forcing is close to the natural frequency of the pendulum, in fluid flows effective transformation of external disturbances into instability modes of the boundary layer is only possible if in addition to the frequency the wave number of external perturbations is in tune with internal oscillations of the boundary layer. In most cases such tuning can only be achieved if two or more external perturbations of different physical nature come into interaction with one another.

A classical example is the generation of the Tollmien-Schlichting waves by acoustic noise. If the noise has wide enough spectrum, then the receptivity process "extracts" from it a harmonic whose frequency is in tune with the frequency of the Tollmien-Schlichting wave. Of course, under this condition the wave length of the "chosen" acoustic wave appears to be much longer than the wave length of the Tollmien-Schlichting wave, which means that the acoustic wave alone is incapable of generating the Tollmien-Schlichting wave. However, if the surface of the wing is not absolutely smooth (which, of course, is the case in all practical applications), then one also need to look at perturbations produced by the wall roughnesses. These perturbations are steady and have a short length scale necessary for the resonance. Non-linear interaction of the acoustic wave with the steady perturbations due to the wall roughnesses is one of the routes through which the instability modes of the boundary layer are generated. The theory of this process was developed by Ruban [2] and Goldstein [3]. Both authors used the triple-deck model for their studies. It should be noted that it was Terent'ev [4] who first applied the triple-deck theory to the receptivity analysis. In his paper Terent'ev proposed a theoretical model of early experiments by Schubauer and Skramstad [5]; these authors devised a technique of generating the Tollmien-Schlichting waves in wind tunnel experiments using a vibrating ribbon. Later the theory was extended to other forms of external perturbations. Duck *at al* [6] used it to study the generation of the Tollmien-Schlichting waves by free-stream turbulence interacting with local roughness on the body surface, and Wu [7] demonstrated that the resonance conditions may be satisfied when the free-stream convective gusts come into interaction with acoustic waves.

In this paper we present a theoretical analysis of the process of generation of Tollmien-Schlichting waves in the boundary layer due to elastic vibrations of the wing surface. Our study shows that the corresponding receptivity coefficient is as high as in the case of acoustic noise (see [2], [3]) which make it surprising that this receptivity mechanism has not been considered before by either experimentalists or theoreticians.

PROBLEM FORMULATION

We assume that the aerofoil equation has the form

$$y = f_0(x) + \varepsilon e^{i\omega t} f_1(x). \tag{1}$$

Here we use suitably scaled non-dimensional variable with (x, y) being Cartesian coordinates, and t time. The aerofoil is placed in uniform flow of a perfect gas with free-stream Mack number $M_\infty < 1$. In order for the aerofoil vibrations to be in resonance with Tollmien-Schlichting waves we choose the frequency of oscillations, ω, large, namely, $\omega = O(Re^{1/4})$. Since the sound propagates through the wing structure significantly faster than through air, it is assumed in (1) that the wave length of elastic vibrations of the wing is comparable with the aerofoil cord.

To study the flow past the aerofoil we use the asymptotic (large Reynolds number) analysis of the Navier-Stokes equations. Of course, the applicability of this approach in studies of stability of fluid flows is open to question especially close to the critical Reynolds number. However, the asymptotic methods proved to be invaluable in identifying various receptivity mechanisms.

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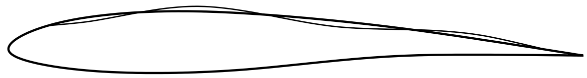

Figure 1: Vibrating aerofoil.

SOLUTION PROCEDURE

Our first task is to determine the pressure perturbations produced by the wing vibrations in the inviscid part of the flow. We found that the governing parameter for the flow outside the boundary layer is $\chi = M_\infty Re^{1/4}$. If $\chi \ll 1$, then the pressure distribution on the aerofoil surface may be found with the help of incompressible version of unsteady thin aerofoil theory. Since the frequency, ω, is large, the wake behind the aerofoil can be disregarded leading to the following equation for the complex conjugate velocity,

$$u - iv = -\frac{i\omega}{2\pi}\int_0^1 \frac{f_1^+(\zeta) - f_1^-(\zeta)}{\zeta - z}d\zeta + \frac{\omega}{2\pi\sqrt{z(z-1)}}\int_0^1 \sqrt{\zeta(1-\zeta)}\,\frac{f_1^+(\zeta) + f_1^-(\zeta)}{\zeta - z}d\zeta.$$

Here u and v are the longitudinal and lateral velocity components, respectively; $z = x + iy$ defines the position of the of point in space where the complex conjugate velocity is calculated. Once u and v are found the pressure on the aerofoil surface is calculated using the Cauchy-Lagrange integral.

If, on the other hand, $\chi \gg 1$, then the piston theory appears to be applicable with the pressure on the aerofoil surface given by the following simple formula,

$$p = i\omega a\rho f_1(x).$$

Here a is the speed of sound in the air, and ρ is the air density.

As the pressure perturbations penetrate into the boundary layer, the Stokes layer is created on the aerofoil surface. Its thickness is estimated as an $O(Re^{-5/8})$ quantity. The velocity in the Stokes layer is given by

$$u = U_S = \frac{ie^{i\omega t}}{\omega}\frac{dP}{dx}\Big(1 - e^{(1+i)\sqrt{\frac{\omega}{2}}Y}\Big), \qquad Y = Re^{5/8}y, \tag{2}$$

where dP/dx is the amplitude of the pressure gradient at the chosen position on the aerofoil surface. From (2) it follows that the wave length of the velocity perturbations in the Stokes layer is much larger than that required for the resonance with the Tollmien-Schlichting waves. In order satisfy the resonance conditions the Stokes layer should interact with a wall roughness. These are, of course, plentiful on a real aircraft wing. We assume that the roughness is very small and given by the equation

$$y = Re^{-5/8}hG\Big(\frac{x - x_0}{Re^{-3/8}}\Big), \tag{3}$$

which ensures that the flow in the vicinity of the roughness is described by the triple-deck equations.

These may be solved analytically if the roughness is "shallow", i.e. parameter h in (3) is small. In this case the velocity is written as

$$u = U_S + hU_R + U_{TS},$$

where U_S is the velocity profile in the Stokes layer, U_R represents the steady flow perturbations produced by the roughness. Our main interest is in the third term, U_{TS}, which satisfies the linearised triple-deck equations with a forcing produced by the interaction between the Stokes solution, U_S, and the roughness solution, U_R.

The equations for U_{TS} are solved using the Fourier Transform with respect to the coordinate along the aerofoil surface, and then the asymptotic behaviour of the solution downstream of the roughness is analysed. As a result the amplitude of the Tollmien-Schlichting wave forming in the flow is determined.

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