TRANSITION ROUTE OF KLEBANOFF TYPE IN HYPERSONIC BOUNDARY LAYERS

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Summary The present study is concerned with identifying dominant nonlinear modal interactions causing rapid amplification of three-dimensional disturbances in hypersonic transition. We carried out direct numerical simulations (DNS) of the nonlinear development of a time-harmonic but spanwise localized disturbance. The result indicates that spectral components close to a preferred finite spanwise wavenumber acquire large amplitudes, and correspondingly the flow field exhibits the pattern characteristic of the well-known Klebanoff type (K-type) of transition. In order to seek the underlying mechanism, further DNS were performed for the interaction between a planar and a pair of oblique second modes, all having the same frequency. It is found that the planar mode acts as a *catalyst* to promote the growth of oblique modes. Theoretically, that the K-type transition occurs may be attributed to a remarkable dispersion property of upper-branch second modes that planar and oblique modes with the same frequency (i.e. frequency-locked) also have nearly the same phase speed (i.e. phase-locked). An effective interaction thus takes place in their common critical layer. The evolution equations for such interacting modes were derived, and their solution indicates that the planar mode causes the oblique modes to amplify super-exponentially, thereby offering a theoretical explanation for the findings by the DNS.

INTRODUCTION

Reliable prediction of laminar-turbulent transition of hypersonic boundary-layer flows is of crucial importance for calculating accurately the drag and thermal loading of a vehicle at high speeds. While transition follows broadly the same sequence of processes as in the subsonic regime, namely, receptivity, linear development of instability waves, nonlinear inter-modal interactions and breakdown into small-scale motions via rapid secondary instability, transition in the hypersonic regime is much more complex. This is because different families of instability modes exist, and possible receptivity mechanisms and nonlinear interactions are numerous, and remain to be identified and understood.

At supersonic speeds, possible instabilities consist of first modes, which have a viscous origin, and second and higher modes, which are essentially inviscid (Mack 1987). First modes amplify faster when three-dimensional, and represent the dominant instability for the Mach number $M < 4$, whilst second modes are dominant for $M > 4$. For $M < 4$, the interaction between a pair of most amplified oblique first modes is the key mechanism, causing the so-called 'oblique breakdown' (Fasel, Thumm & Bestek 1993). For $M > 4$, planar second modes have greater growth rates and thus may gain a larger amplitude first, but their growth itself cannot lead to transition. It is therefore crucial to seek nonlinear interactions through which planar modes enhance the growth of certain oblique modes. Possible candidates include resonant triads (Mayer, Wernz & Fasel 2011) and fundamental interaction. However, an appropriate triad requires rather restrictive conditions, which may not be satisfied at high Mach numbers with wall cooling. In this paper, we investigate the fundamental interaction, first by direct numerical simulations of suitable forms of disturbances. A theoretical description of the interaction will then be provided to explain the numerical results.

DIRECT NUMERICAL SIMULATIONS AND RESULTS

The DNS were performed using the code developed during the last ten years by the Tianjin University Transition Research Group. The code is validated by repeating the calculations of Mayer *et al.* (2011). All simulations were carried out for $M = 6$ and the Reynolds number (based on the displacement thickness at the inlet) $R = 2.23 \times 10^5$.

DNS were first performed for a time-periodic but spanwise-localized disturbance: $A\,\mathrm{e}^{-az^2}\,\hat{\mathbf{u}}\,\mathrm{e}^{\mathrm{i}(\alpha x-\omega t)} + c.c.$ Figure 1a shows contours of the instantaneous streamwise velocity $\tilde{u}$. The initially single-peaked spanwise wavepacket splits. Streaks are formed on each side, and remain aligned in the streamwise direction as in the K-type transition. Fourier transforming $\tilde{u}$ with respect to z, we obtain $\widehat{u}(x,\beta)$. Contours of $\widehat{u}(x,\beta)$ (figure 1b) indicates that the disturbance is dominated by components centred at $\beta \approx 1.8$, which are preferentially amplified by nonlinear interactions.

In order to probe into the mechanism for the amplification of oblique modes, further simulations were performed for an initial disturbance consisting of a planar and a pair of oblique second modes:

$$[A_0\hat{\mathbf{u}}_0 + A(\mathrm{e}^{\mathrm{i}\beta z} + \mathrm{e}^{-\mathrm{i}\beta z})\hat{\mathbf{u}}]\exp\{\mathrm{i}(\alpha x - \omega t)\} + c.c; \tag{1}$$

they all have the same frequency $\omega = 1.78$, and are near the upper branch of second modes (see figure 1c). Figure 1d displays the development of oblique modes $(1, n)$ with different $\beta = 0.45n$. Their amplification is all enhanced by the planar mode. The enhanced growth is strongest and earliest for the mode $(1, 4)$, which has $\beta = 1.8$, coinciding with the wavenumber developed from the spanwise wavepacket. This is also the unstable mode having the largest β.

We also simulated the interaction of the planar mode $(\omega, 0)$ with the oblique modes having the same β but different frequencies $n\omega/6$, designated as $(n, 1)$. The result is shown in figure 1e. Among these, mode $(6, 1)$, which has the same frequency as the planar mode, exhibits the most rapid amplification due to the fundamental resonance. Next is mode $(2, 1)$, which happens to be a nearly neutral upper-branch first mode, and its growth is promoted most likely by the phase-locked interaction (Wu & Stewart 1996). Mode $(3, 1)$ forms a subharmonic resonant triad with $(3, -1)$ and the planar mode, but the enhancement is not as strong as the fundamental, probably because modes $(3, \pm 1)$ are damped.

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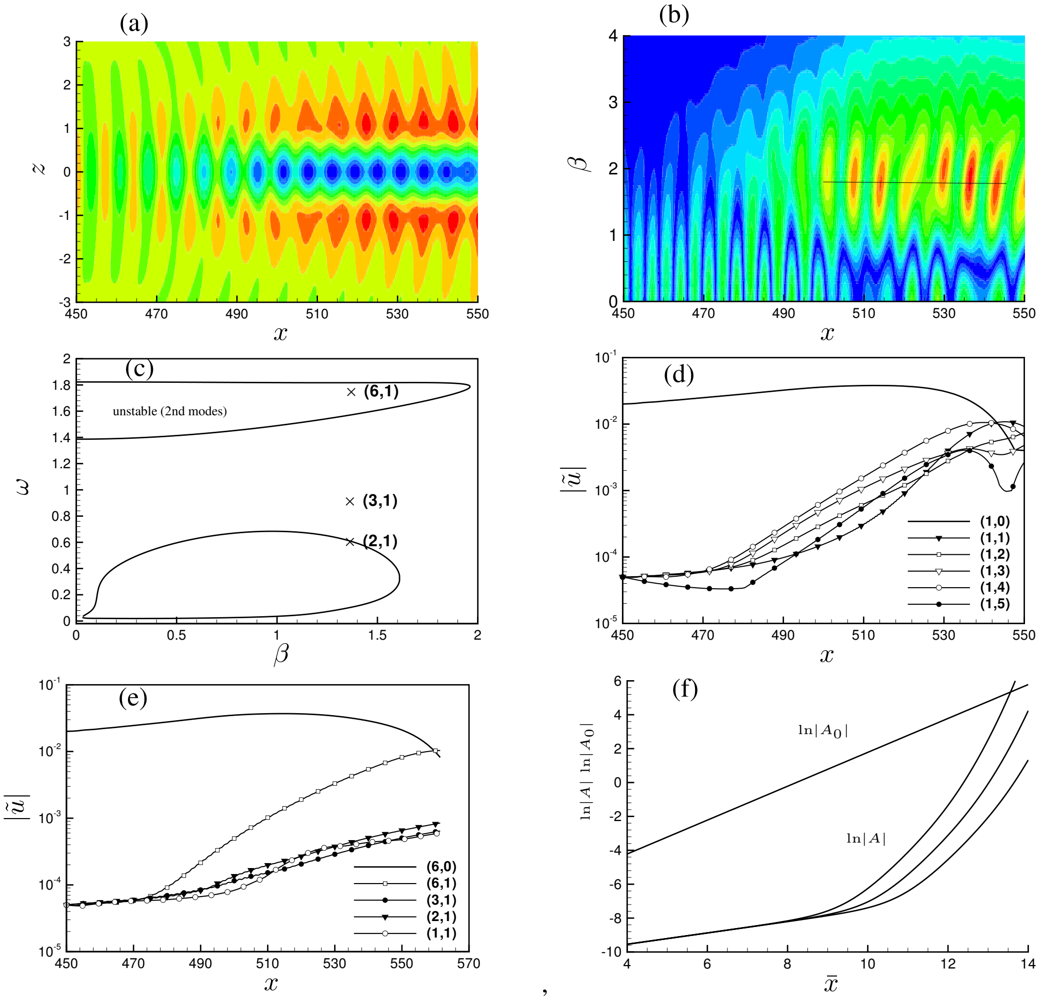

Figure 1. (a) Contours of $\tilde{u}(x, z)$; (b) contours of $\widehat{u}(x, \beta)$; (c) the neutral curve of second modes; (d) development of modes $(1, n)$; (e) development of modes $(n, 1)$; (f) enhanced growth predicted by the theory for frequency-phase-locked modes.

THEORY: FREQUENCY-PHASE-LOCKED INTERACTION

Upper-branch second modes share the same phase speed, i.e. the base-flow velocity at the generalized inflection point, but a remarkable feature is that their frequencies are almost constant (figure 1c), which means that nearly neutral modes of the same frequency also have nearly the same phase speed. Effective interaction takes place in their common critical layer. This is studied theoretically for the two modes represented by (1). A scaling argument shows that once the planar mode acquires a threshold of $O(R^{-1})$, the interaction influences the evolution of the oblique modes. The analysis yields the amplitude equations

$$A'(\bar{x}) = \kappa A + \gamma \int_0^\infty K(\xi, \eta) A_0(\bar{x} - \xi) A_0^*(\bar{x} - \xi - \eta) A(\bar{x} - 2\xi - \eta) \mathrm{d}\xi \mathrm{d}\eta, \qquad A_0'(\bar{x}) = \kappa_0 A_0, \tag{2}$$

where $\bar{x} = R^{-1/3}x$ and $\gamma \sim \beta^2$. The planar mode evolves linearly. but it causes the oblique modes to amplify much more rapidly, as is indicated by numerical solutions depicted in figure 1f. It can be shown analytically that as $\bar{x} \to \infty$,

$$A \sim \exp\{a_\infty \, \mathrm{e}^{\kappa_0 \bar{x}/3} + b_\infty \bar{x}\} \qquad \text{with } \; a_\infty \sim \gamma^{1/6}, \tag{3}$$

i.e. the oblique modes grow super-exponentially. The theoretical prediction is in qualitative agreement with DNS results. Further quantitative comparison will be reported at the Congress.

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