# A NEW RECEPTIVITY MECHANISM OF SUPERSONIC BOUNDARY LAYERS TO FREE-STREAM DISTURBANCES

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*Summary* Linear instability of a supersonic flat-plate boundary layer is investigated with the specific interest being in the dispersion properties relevant for receptivity. It is found that lower-branch neutral second modes all have, in the inviscid limit, the same speed $c = 1$ so that they are synchronized with, and are therefore receptive to, free-stream vortical disturbances. We show that vortical disturbances with an $O(\epsilon)$ amplitude generate, via quadratic interactions, an $O(\epsilon^2)$ pressure fluctuation. The latter then excites an $O(\epsilon^2 R^{1/2})$ second mode, where $R$ is the Reynolds number based on the local boundary-layer thickness. The coupling coefficient, i.e. the ratio of the amplitude of the excited second mode to that of the vortical disturbances, is thus of $O(\epsilon R^{1/2})$. A calculation indicates that the amplitude of the second mode excited is about 4 times that of the vortical disturbances for $R = 10^4$ and $\epsilon = 10^{-3}$.

## INTRODUCTION

Laminar-turbulent transition of boundary-layer flows involves a series of processes: receptivity, linear development of instability waves, nonlinear inter-modal interactions and breakdown into small-scale motions via rapid secondary instability. While transition in the subsonic and super/hypersonic regimes follows broadly the same sequence of stages, substantial differences exist. For subsonic (two-dimensional) boundary layers, transition is caused by viscous Tollmien-Schlichting modes, which have a small characteristic phase speed and concentrate in the wall region. Relevant receptivity mechanisms include (a) leading-edge adjustment (Goldstein 1983), and (b) interaction of free-stream acoustic/vortical disturbances with local (Goldstein 1985, Ruban 1984, Duck *et al.* 1996) or distributed roughness (Wu 2001).

In the supersonic regime, a multitude of instability modes arise including first modes, which have a viscous origin, and essentially inviscid Mack second and higher modes,whose phase speeds are fairly close to that of the free stream. Different receptivity mechanisms operate (Fedorov 2011). Fedorov & Khokhlov (1991) showed that upstream/downstream propagating acoustic waves in the free-stream are nearly synchronized with the so-called slow and fast instability modes within the boundary layer, and hence may be diffracted into the latter respectively through the weak nonparallel-flow effect; the slow mode then evolves into a first mode. This leading-edge receptivity process is of primary importance, and has been investigated experimentally (Maslov *et al.* 2001) and numerically (e.g. Ma & Zhong 2003). Surface roughness, on the other hand, does not appear to play a significant role in receptivity. Wu (1999) showed shat suitable vortical and acoustic disturbances may interact with each other to excite oblique first modes in a flat-plate boundary layer. In this paper, we will present a new and effective mechanism by which free-stream vortical disturbances may generate second modes.

## FORMULATION OF THE THEORY

We consider the boundary layer over a flat plate. The usual Cartesian coordinates $(x, y, z)$ are used, where $x$, $y$ and $z$ are normalized by $\delta^*$, the boundary-layer thickness at the location of interest. The streamiwse variation of the base flow will be described by the slow variable $x_2 = x/R$, where $R = U_\infty \delta^*/\nu_\infty$ is the Reynolds number. The base flow is perturbed by a disturbance

$$(\tilde{u}, \tilde{p}) = \epsilon(\hat{\mathbf{u}}, \hat{p})E + \text{complex conjugate} \quad \text{with} \quad E = \mathrm{e}^{\mathrm{i}(\alpha x + \gamma y + \beta z - \omega t)}. \tag{1}$$

The linear stability formulation leads to an eigenvalue problem consisting of the compressible Orr-Sommerfeld equations and homogeneous boundary conditions for $\hat{\mathbf{u}}$ and $\hat{p}$. In order to reveal the essential instability and to describe the receptivity mechanism in a self-consistent manner, we take $R \gg 1$, in which case, the eigenvalue problem simplifies to

$$\mathcal{L}\,\hat{p} = 0; \qquad \hat{p}'(0) = 0, \qquad \hat{p} \to 0 \ \text{ as } \ y \to \infty, \tag{2}$$

where $\mathcal{L}$ is the familar compressible Rayleigh operator. Both the inviscid and full viscous instability problems are solved numerically for the Mach number $M = 6$. Figure 1a shows growth rates of two-dimensional modes in a range of frequency $\omega$. The family of second modes represents the dominant instability as expected. Calculations for $\beta > 0$ find the neutral curves in the $\beta - \omega$ plane (figure 1b), and the corresponding phase speeds $c$ of the neutral modes (figure 1c). The most important observation is that lower-branch second modes have the same phase speed $c = 1$.

The fact that $c = 1$ means that any lower-branch second mode within the boundary layer is completely synchronized with a vortical disturbance in the free stream. However, a vortical disturbance does not excite a neutral mode with the same frequency because it has no pressure fluctuation at leading order whereas the latter is, in contrary, characterized by a pressure. The important fact is that any two generic vortical components, represented say by

$$\tilde{\mathbf{u}}_c = \epsilon(\hat{\mathbf{u}}_1 E_1 + \hat{\mathbf{u}}_2 E_2 + c.c.) \quad \text{with} \quad E_j = \mathrm{e}^{\mathrm{i}\{\alpha_j(x-t)+\gamma_j y+\beta_j z\}} \ \ (j = 1, 2),$$

do interact to generate an $O(\epsilon^2)$ pressure with a wavenumber $(\alpha_1 + \alpha_2)$. This pressure is in resonance with the neutral mode $(\alpha, \omega)$ provided that $\alpha_1 + \alpha_2 = \alpha$. The resonance occurs in an $O(R^{-1/2})$ vicinity of the neutral position $x_n$,

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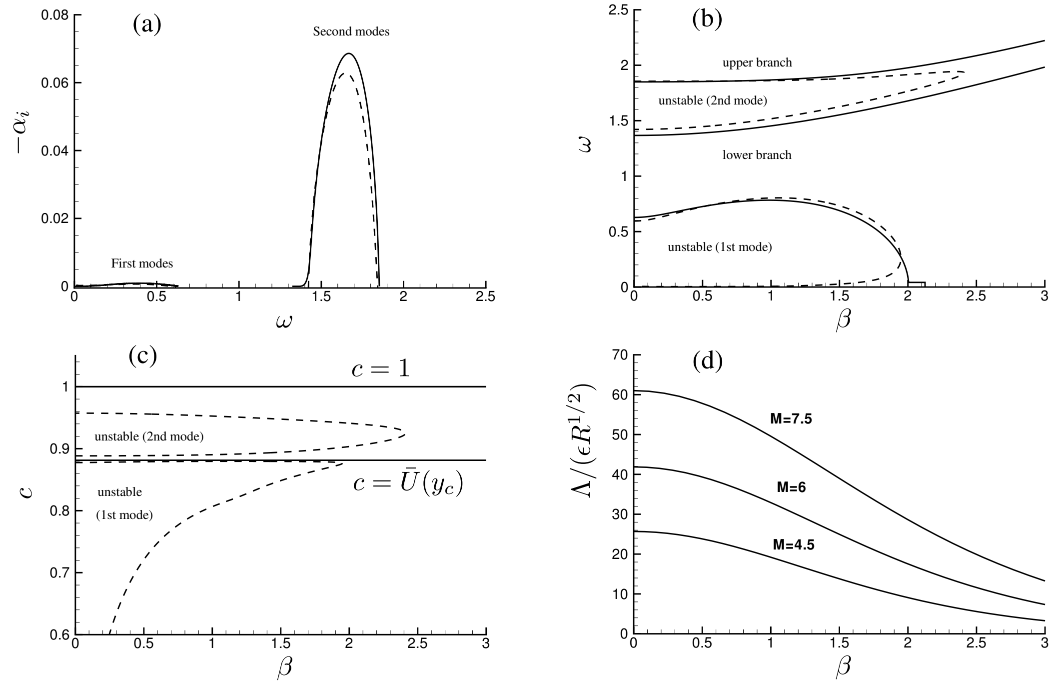


**Figure 1.** (a) Growth rates of two-dimensional modes; (b) neutral curves of first and second modes; (c) phase speeds of neutral modes; (d) coupling coefficients. Solid lines: inviscid limit; Dashed lines: viscous results with $R = 10^5$.

generating an $O(\epsilon^2 R^{1/2})$ eigen mode, and the solution for the pressure fluctuation thus takes the form

$$\tilde{p} = \epsilon^2 R^{1/2} A(\bar{x})\hat{p}_0 E + \epsilon^2 \hat{p}_1 E + \ldots \qquad \text{with} \quad \bar{x} = R^{1/2}(x_2 - x_n) = O(1), \tag{3}$$

where $A(\bar{x})$ and $\hat{p}_0$ represent, respectively, the amplitude and eigenfunction of the nearly neutral mode. The function $\hat{p}_1$ is governed by an inhomogeneous boundary-value problem, the solvability of which leads to the amplitude equation

$$A' = (\sigma\bar{x} + \mathrm{i}\alpha_1)A + \mathcal{F}, \tag{4}$$

where $\sigma$, $\alpha_1$ and $\mathcal{F}$ can be determined numerically. The appropriate initial condition is $A \to -\mathcal{F}/(\sigma\bar{x}+\mathrm{i}\alpha_1)$ as $\bar{x} \to -\infty$, representing asymptotic matching with the pre-resonant regime upstream (cf. Wu 1999, 2001). As $\bar{x} \to \infty$,

$$A \to A_0 \exp\{\tfrac{1}{2}\,\sigma\bar{x}^2 + \mathrm{i}\alpha_1\bar{x}\}, \qquad \text{with} \quad A_0 = \mathcal{F}(2\pi/\sigma)^{1/2}\exp\{-\alpha_1^2/(2\sigma)\}, \tag{5}$$

where $A_0$ represents the scaled amplitude of the second mode excited by the vortical-vortical interaction.

## A SAMPLE OF RESULTS AND EXTENSION OF THE THEORY

The coupling coefficient may be defined as $\Lambda = (\epsilon R^{1/2} A_0 \max_y |\hat{v}|)/|\hat{v}_c|$. Figure 1d shows the value $\Lambda/(\epsilon R^{1/2})$ for different $\beta$. At a given $M$, two-dimensional modes are more receptive than oblique ones. The receptivity becomes stronger as $M$ increases. For $4.5 \le M \le 7.5$, $\Lambda/(\epsilon R^{1/2}) \approx 40$. If the free-stream disturbance level $\epsilon = 0.1\%$ and $R = 10^4$, then $\Lambda \approx 4$, that is, the magnitude of the excited second mode is about $0.4\%$, or 4 times as large as the free-stream vortical disturbances. A lower-branch neutral point is an optimal receptivity site in that the excited mode does not undergo decay, nor does it skip any linear amplification. Therefore, the present mechanism is particularly effective.

Lower-branch second modes may also be generated by the interaction between a downstream and an upstream propagating acoustic waves with an equal streamwise wavenumber as well as by suitable acoustic-vortical and acoustic-entropic interactions. Such extensions of the theory and further results will be reported at the Congress.

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