NONSIMILAR MHD FALKNER-SKAN FLOW WITH LOCALIZED WALL HEATING (COOLING)

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Summary:This paper examines the effects of localized wall heating (cooling) on the steady, laminar boundary layer flow of electrically conducting viscous fluid over a wedge in the presence of a transverse magnetic field numerically, using an implicit finite–difference scheme, along with quasilinearization technique [1]. Comparison with previously published works, available in literature is performed and the results are found to be in very good agreement. The present results vindicate the role of localized wall heating or cooling in controlling the laminar boundary layer.

INTRODUCTION

The process of localized wall heating or cooling is of vital importance for various scientific and technological potential applications including the controlling of the laminar boundary layer flow separation. Indeed, localized wall heating or cooling promotes energizing of the inner portion of boundary layer in adverse pressure gradient and plays a key role in boundary layer control [2]. Within the field of computational fluid dynamics, the analysis of boundary layer problems for two-dimensional incompressible laminar flow passing a fixed wedge is a common area of interest. Historically, the steady laminar flow passing a fixed wedge was first analysed in the early 1930's by Falkner and Skan [3] to illustrate the applications of Ludwig Prandtl's boundary layer theory. Later, several investigators, have studied the classical Falkner-Skan problem employing various numerical methods for different flow as well as heat transfer situations.

MATHEMATICAL ANALYSIS

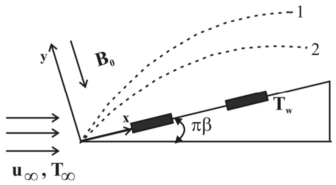

Fig.1. Physical model and co-ordinate system with localized slots for Falkner-Skan wedge flow where 1 and 2 represents edge of thermal and momentum boundary layers, respectively.

We consider the steady, two-dimensional laminar boundary layer flow of an incompressible electrically conducting viscous fluid flowing towards the wedge with velocity u_∞ with localized wall heating (cooling) (See Fig.1). The temperature of the wall T_w is uniform and constant, and is greater than the free stream temperature T_∞. A transverse magnetic field of strength B_0 is applied in $y-$direction normal to the wedge surface and it is assumed that the magnetic Reynolds number is small, so that the induced magnetic field can be neglected as compared to the applied magnetic field. Hence, the applied magnetic field contributes only to the Lorentz force which acts in the x - direction. The fluid is assumed to have constant physical properties. The surface of the wedge is maintained at a constant temperature T_o except in certain portions of the wedge $[x_i, x_j]$ where it varies slowly with the stream wise distance x. Under the aforesaid assumptions, the boundary layer equations governing the forced convection steady, MHD Falkner-Skan wedge flow are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \;.... \;(1); \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}(u-U) \;....\; (2); \qquad u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \;....\; (3)$$

with boundary conditions : $u(x,0)=0$; $v(x,0)=0$; $T(x,0)=T_w(x)$ *for* $x_i \le x \le x_j$; $T(x,0)=T_0$ *for* $0 \le x < x_i, x > x_j$;

$$u(x,\infty) = U(x);\; T(x,\infty) = T_\infty \text{ where } U(x) = u_\infty (x/L)^m \;\; (0 \le m < 1)\; ;\; T_w(x) = T_\infty + (T_0 - T_\infty)\,[1 + \varepsilon\,(x - x_i)(x_j - x)/(x_j - x_i)^2\,] \quad (4)$$

Nomenclature:	***Greek symbols***
f dimensionless stream function; Pr Prandtl number ; F dimensionless velocity; G dimensionless temperature; Re_L local Reynolds number ; L characteristic length; m Falkner-Skan power-law parameter T fluid temperature; M dimensionless magnetic parameter $U(x)$ velocity outside the boundary layer; u, v velocity components along the x - and y - directions. ***Subscripts:*** w condition at the wall; ∞ condition away from the wall; ξ derivatives with respect to ξ	ψ dimensional stream function; β pressure gradient parameter ; σ electrical conductivity; ρ density α thermal diffusivity; β thermal expansion coefficient ξ and η similarity variables ; ν kinematic viscosity ε constant, $\varepsilon > 0$ or < 0 wall heating or cooling resp. $\varepsilon = 0$ wall at constant temperature T_o ***Superscripts*** $(\,'\,)$ differentiation with respect to η

Introducing the following transformations:

$$u = \frac{\partial\psi}{\partial y}\;;\; v = -\frac{\partial\psi}{\partial x};\; f(\eta) = \sqrt{\frac{1+m}{2}\frac{L^m}{\upsilon u_\infty}}\left(\frac{\psi}{x^{(1+m)/2}}\right);\; \eta = \sqrt{\frac{1+m}{2}\frac{u_\infty}{\upsilon L^m}}\left(\frac{y}{x^{(1-m)/2}}\right);\; G(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\;;\; \xi = m\left(\frac{x}{L}\right) \qquad (5)$$

to Eqns. (1) - (3), we observe that continuity Eqn.(1) is identically satisfied and Eqns.(2) and (3) are, respectively, transformed to non-dimensional form as:

$$f''' + f\,f'' + M\,[1 - f'] + \beta\,[1 - f'^2 - \xi\, f''\, f_\xi + \xi\, f'\, f'_\xi] = 2\xi\,(f'\,f'_\xi - f''\,f_\xi) \qquad (6)$$

$$(1+\xi)\left(\mathrm{Pr}^{-1}\,G'' + f\,G'\right) = 2\xi\,(f'\,G_\xi - G'\,f_\xi) \qquad (7)$$

where $u/U = f' = F$; $v = -\sqrt{\left(2/(1+m)\right)\left(\upsilon u_\infty/L^m\right)}\; x^{(m-1)/2}\left\{f_\xi (x/L)m + \left((m+1)/2\right)f + \eta\left((m-1)/2\right)F\right\}$

$$\mathrm{Pr} = \upsilon/\alpha\;;\quad M = 2L\sigma B_0^2/\rho u_\infty (m+1)\;;\quad \mathrm{Re}_L = u_\infty L/\upsilon\;;\quad \upsilon = \mu/\mu_\infty \qquad (8)$$

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The transformed boundary conditions are: $F(\xi,0)=0.0$; $G(\xi,0)=1.0$ for $\xi<\xi_i$, $\xi>\xi_j$ and

$$G(\xi,0)=1+\varepsilon\left[(\xi-\xi_i)(\xi_j-\xi)/(\xi_j-\xi_i)^2\right] \text{ for } \xi_i\le\xi\le\xi_j;\ F(\xi,\infty)=1.0\ ;\ G(\xi,\infty)=0.0 \qquad (9)$$

We also note that in Eqn.(5), the parameter m is connected with the apex angle $\pi\beta$ by the relation $\beta=2m/(1+m)$. Further, the numerical computations have been carried out for entire range of realistic flow i.e., for the range $0\le\beta\le0.33$ (wedge angle ranging from 0^o to 60^o). It is remarked here that the dimensionless wall temperature G (ξ, 0) is a continuous function of the dimensionless stream wise distance ξ, with a small change in the interval $[\xi_i,\ \xi_j]$ over the constant value of 1 [See Eqn. (9)]. The increase or reduction in the wall temperature in the interval by a constant value introduces a finite discontinuity at the leading and trailing edges of the slot and this causes difficulties in the solution of the equations. In order to avoid these difficulties, we have taken a non-uniform distribution of wall temperature in the interval $[\xi_i,\ \xi_j]$ which varies slowly with ξ .The quantities of engineering interest are the local skin friction and heat transfer coefficient in the form of Nusselt number and these are expressed as:

$$C_f(\mathrm{Re}_L)^{1/2} = \frac{\tau_w}{\frac{1}{2}\rho U^2} = 2\sqrt{1/(2-\beta)}\,(F')_{\eta=0}; \qquad Nu(\mathrm{Re}_L)^{-1/2} = -k\left(\partial T/\partial y\right)_{y=0}\Big/(T_w-T_\infty) = -\sqrt{1/(2-\beta)}\,(G')_{\eta=0}$$

RESULTS AND CONCLUSIONS

The computations have been carried out for various values of M ($0\le M\le1.0$), ε ($-2.0\le\varepsilon\le2.0$) considering various wedge angles $\beta=0.0,\ 0.25,\ 0.33$ for air (Pr = 0.7) using the method as described in [1]. It may be noted that for computation purposes, two slots have been located in the intervals [0.5, 1.5] and [3.5, 4.5] and the wall is heated or cooled in these intervals only. In the remaining portions, the wall is at constant temperature $T_o\ (>T_\infty)$. To validate the accuracy of the numerical method used, particular cases of our results are compared with those of Watanabe [4] and White [5], and they are found to be in excellent agreement. To conserve space, details of comparison are omitted here.

The effect of magnetic field (M) on skin friction [$C_f(\mathrm{Re}_L)^{1/2}$] and heat transfer coefficient [$Nu(\mathrm{Re}_L)^{-1/2}$] is exhibited in Fig.2, for $\beta=0.25$, (wedge angle = 45^o). It is found that M increases both $C_f(\mathrm{Re}_L)^{1/2}$ and $Nu(\mathrm{Re}_L)^{-1/2}$ whenever there is wall heating (cooling) ($\varepsilon=\pm0.2$) or not (ε = 0.0). This is because when M increases, the Lorentz force produces more resistance to the transport phenomena which leads to the deceleration of the flow, enhancing the surface shear stress as well as heat transfer. During wall cooling, $Nu(\mathrm{Re}_L)^{-1/2}$ increases as the slot starts and attains its maximum value in the middle of the slot and decreases from its maximum value towards trailing edge of the slot. However, the effect of wall heating on $Nu(\mathrm{Re}_L)^{-1/2}$ is just opposite, but it is not a mirror reflection of the wall cooling [Fig 2(b)].

In figure 3, the effect of the wall heating or cooling on $C_f(\mathrm{Re}_L)^{1/2}$ and $Nu(\mathrm{Re}_L)^{-1/2}$ for different wedge angles $\beta=0.0,\ 0.25,\ 0.33$ when M = 1.0 are displayed. The effects of wall heating and cooling in two slots are found to be more pronounced on $Nu(\mathrm{Re}_L)^{-1/2}$ than on the $C_f(\mathrm{Re}_L)^{1/2}$, because the wall heating or cooling directly affects the thermal field, whereas its effect on the velocity field is indirect. Since the temperatures of the wall in the slots are changed, significant changes take place in the vicinity of the leading edge of the slots. Generally, the changes are more near to the trailing edge of the second slot, because it also carries the perturbation caused by the first slot. During wall heating, $C_f(\mathrm{Re}_L)^{1/2}$ is found to decrease sharply at the starting value of the first slot [0.5, 1.5] and decreases slightly at the starting value of the second slot [3.5, 4.5] [See Fig. 3(a)], and in the process, the values of $C_f(\mathrm{Re}_L)^{1/2}$ attains a new lower steady state value elsewhere, ensuring the delay in the boundary layer separation. Also, in Fig. 3(b), it is evident that $Nu(\mathrm{Re}_L)^{-1/2}$ attains a maximum or minimum value inside the slots, whenever ($\varepsilon=\pm0.2$) for all wedge angles, confirming the stabilization of the thermal boundary layer.

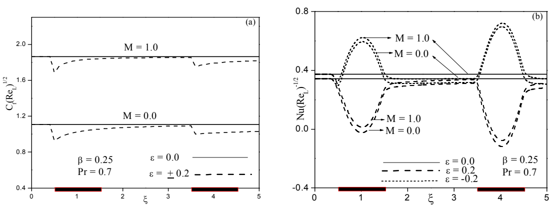

Fig.2. The effect of (M) and (ε) on $C_f(\mathrm{Re}_L)^{1/2}$ **and** $Nu(\mathrm{Re}_L)^{-1/2}$

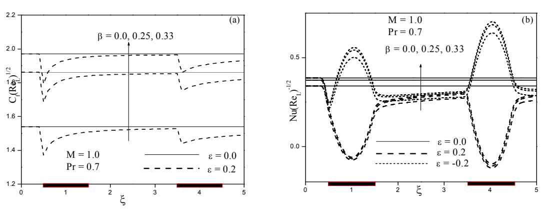

Fig.3. The effect of ε = 0.2 **and** ε = -0.2 **on** $C_f(\mathrm{Re}_L)^{1/2}$ **and** $Nu(\mathrm{Re}_L)^{-1/2}$

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