ON THE DEFINITION OF LOGARITHMIC REGION IN ZERO PRESSURE GRADIENT TURBULENT BOUNDARY LAYERS

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Summary We defined an exact fitting region applied to Clauser chart method as Quasi Logarithmic Region (QLR) in the low Reynolds number, 1100 < Re_θ < 6010. Adopting Clauser chart method in QLR, one can derive a reasonable friction velocity that agrees well with previous experiments. In this region, the velocity fluctuation shows the statistics similar with Gaussian. Furthermore, we determined the best combination Karman constant and additive constant characterizing log-law at low Reynolds number.

INTRODUCTION

Many studies of turbulent boundary layers on a zero pressure gradient smooth surface have been reported by the experimental and numerical methods. Enormous experiments had been performed so far, however a consideration of logarithmic region of mean velocity profiles is not sufficiently understand, especially in the low Reynolds number range, at Re_θ below 6000 (Re_θ is a Reynolds number based on a momentum thickness, θ).

It is understood well that the logarithmic region could appear between a buffer layer and a wake region. In order to discuss these regions, we need the friction velocity and the distance from the wall should be normalized by wall variables. One of the methods of evaluating the friction velocity is the Clauser chart, which is easy to adopt and the friction velocity is obtained from the mean velocity profiles. But several claims such as [1] showed that Clauser chart did not present the reasonable value and the friction velocity should be obtained independent of mean velocity measurement.

Österlund studied the friction velocity by an oil film interferometry and showed that logarithmic region appeared at about y^+ > 200 in the range of Reynolds number 2530 < Re_θ < 27300 [2]. Furthermore, the logarithmic region can not appear in the low Reynolds number like Re_θ < 6000. In this study, we newly define the Quasi Logarithmic Region (QLR) and suggest that the Clauser chart is still useful to evaluate the friction velocity in the low Reynolds number range. The friction velocity agrees sufficiently with the values obtained by the oil film interferometry. In this research, we focus on the difference between the traditional logarithmic region and the QLR. Karman constant, κ, and additive constant, B, characterizing the logarithmic mean velocity profile are determined in QLR, and discuss the Reynolds number dependence of these values.

EXPERIMENTAL SETUP

The experiment was performed in the boundary layer wind tunnel in the department of Energy Engineering and Science at Nagoya University. The wind tunnel is an open-return suction type of conventional design, with test section dimensions of 4 m long, 0.7 m wide, and 0.35 m high. It had three fixed walls and an adjustable straight wall which was used to set a nominal zero pressure gradient. The wind tunnel has 4 measurement points along the streamwise direction. The free stream turbulence level was about 0.26 % for a free stream velocity of 12.7 m/s. we adopted two different type of tripping devices. One is the single wire of 1mm in diameter and the other is a combination of the single wire and 7 tapes printed a capital letter of V. A constant-temperature hot-wire anemometer was used for all of the measurement of velocity fluctuation. A single hot wire, with 5 μm in diameter and 1.0 mm etched length, is operated. We measured velocity profile by a traversing system from the wall into the free stream region. The free stream mean velocity is adjusted as a unit Reynolds number, U_0/ν =3.75, 4.42, 5.27, 6.38, 7.49, 8.48, 9.46$\times 10^5$, U_0 is a free stream velocity and ν is a kinetic viscosity, where the Reynolds number based on momentum thickness varies from 1100 to 6010.

ANALYTICAL METHOD

In order to define the Quasi Logarithmic Region, we calculate a local slope of mean velocity profile, which is defined as the slope of interpolated line with discrete 5 locations of measurement points. If the local slope is plotted against the distance from the wall, it is found that the local slope has a local minimum value between the buffer layer and the wake region. Around the local minimum value, the local slope becomes almost constant. Thus, we defined the QLR as a region where the local slope variation can be neglect and QLR includes the local minimum point. The beginning and the end points of QLR are expressed as y_1 and y_2, respectively. We investigated the Reynolds number dependence of $y_1$and y_2. We also calculated the difference between log-law formula and the profiles fitted in QLR, and then determined which combination is the best one with varying the Karman constant and the additive constant. The discrepancy is defined as FD, $FD = (\ln(y^+) - \kappa(U^+ - B))/\ln(y^+)\times 100$, where $U^+$and y^+ represent the mean velocity and the distance from the wall scaled by the friction velocity. Therefore, FD=0 represents the region which is equivalent to QLR. First selecting an arbitrarily Karman constant κ and additive constant B and then fitting the Clauser chart over

QLR, we obtain the friction velocity i.e. mean velocity profile $U^+(y^+)$. We calculate 'True' Karman constant, κ_T, and additive constant, B_T. The result of the Karman constant is shown in Fig.2. In this figure, we selected the arbitrarily Karman constant equal to 0.384, and the additive constant equal to 4.17. Furthermore, we tried to understand the physical meaning of QLR. We calculate the higher-order moments of velocity fluctuations, such as skewness and flatness, and then analysed these statistics across the boundary layers.

RESULTS

We obtained three main results; first, a comparison of the friction velocity obtained by Clauser chart method applied to the QLR and the previous results reported by other researchers are plotted in Fig.1. Here, c_f is a local skin friction coefficient, defined as $c_f = 2 \times (u_\tau / U_0)^2$, u_τ is the friction velocity. QLR represents the values obtained by Clauser chart method for adopting Quasi Logarithmic Region. OFI is the values obtained by the oil film interferometry in our laboratory. The friction velocities obtained by the present method have a consistency with the oil film interferometry. And also they agree well with the results reported so far (Schlatter by a direct numerical simulation, Österlund by the oil film interferometry [2] and Osaka by a floating element). From these results, it is clear that the present method is useful to evaluate the friction velocity.

Second, the Reynolds number dependence of Karman constant is obtained and it is plotted in Fig.2. In the range of $\mathrm{Re}_\theta < 2000$, Karman constant has an obvious Reynolds number dependence. Increasing the Reynolds number, $\mathrm{Re}_\theta > 2000$, the Reynolds number dependence is negligible, and the Karman constant seems to asymptote to the constant. Therefore, over the Quasi Logarithmic Region, we conclude that Karman constant is to be 0.40 ± 0.01 in the range of $2000 < \mathrm{Re}_\theta < 6010$. The dashed-dotted line is the interpolation and it reaches 0.41. We are analyzing the data in higher Reynolds number cases, which will be reported in the presentation. Furthermore, we calculated FD defined as the discrepancy between the log-law formula and the fitted mean velocity profile. In a case we set $\kappa = 0.40$, and it is obvious that the profile fitted in QLR agrees rather well with the log-law profile. And we found that y_2 has clear Reynolds number dependence, which becomes larger value with increasing Reynolds number, but y_1 is almost constant.

Third, skewness of velocity fluctuation indicates almost zero and flatness is about 2.7 in QLR. This result shows that the Probability Density Function (PDF) is close to Gaussian distribution. From this fact, one can determine this Quasi Logarithmic Region by using the PDF.

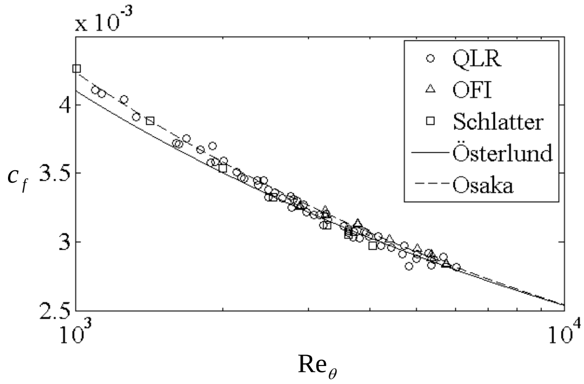

Fig.1 Local skin friction coefficient. '○' and '△' represent present experimental data by Clauser chart method in QLR and OFI, respectively. '□' represent DNS data by Schlatter. Solid line and dashed line represent empirical lines suggested by Österlund and Osaka, respectively.

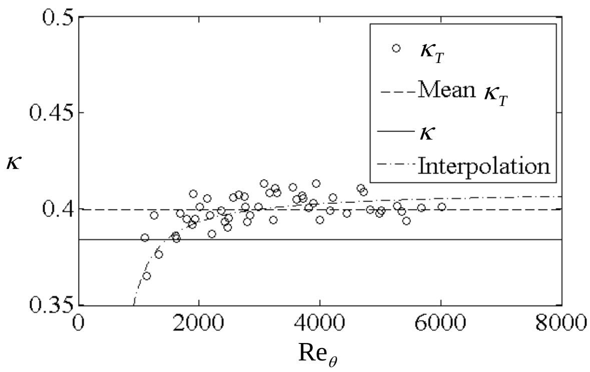

Fig.2 Variation of κ_T with Reynolds number. '○' represent 'True' κ in QLR. Dashed line represents mean 'True' κ. Solid line represents arbitral value $\kappa = 0.384$. Dashed-dotted line represents interpolation of 'True' κ.

CONCLUSIONS

In this paper, we present the definition of Quasi Logarithmic Region. If the Clauser chart is applied to this region, the reasonable friction velocity is derived. It agrees well with the previous experiments and recent direct numerical simulation. Following this procedure, Karman constant and the additive constant characterizing the log-law profile are evaluated to be 0.40 and 4.63, respectively, in the range of $2000 < \mathrm{Re}_\theta < 6010$. In the lower Reynolds number range, $\mathrm{Re}_\theta < 2000$, Karman constant indicates the clear Reynolds number dependence. In the QLR, the velocity fluctuation shows the statistics similar with Gaussian.

References

[1] Tie Wei, Rodney Schmidt, Patrick McMurtry: Comment on the Clauser chart method for determining the friction velocity, Experiments in Fluids 38: 695-669, 2005.

[2] Jens M. Österlund, Arne V. Johansson: Turbulence Statistics of Zero Pressure Gradient Turbulent Boundary Layers, 1999.