# IS THE WALL 'NECESSARY' FOR GENERATING TURBULENCE IN WALL-BOUNDED FLOWS?

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*Summary* The invariants $P$, $Q$ and $R$ of the velocity gradient tensor have been widely used to study turbulent flow structures in order to extract information regarding the scales, kinematics and dynamics of these structures, see [1], pg. 268 and [2]. These invariants are zero at a no slip wall and can no longer be used to identify and study structures at the no slip surface in any wall bounded flow. An alternative scheme for studying structures at a no slip wall is is to investigate the surface skin friction and the surface vorticity fields. The flow close to critical points in the surface skin friction field can be described by a no slip Taylor-series expansion and the topology is defined by the three invariants ($\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{R}$) of the 'no slip tensor'.

## THE NO SLIP TENSOR AND ITS INVARIANTS

If the origin of a Taylor-series expansion is located at any arbitrary point on a no slip wall, where the velocities $u_1$, $u_2$ and $u_3$ in the streamwise, spanwise and wall normal directions respectively are zero, i.e. $u_1 = u_2 = u_3 = 0$ when $x_3 = 0$, (here, $x_3$ is the wall-normal direction and $x_1$ and $x_2$ are the streamwise and spanwise components respectively), the velocity field at the wall can be described by

$$\begin{aligned} u_1 &= \dot{x_1} = dx_1/dt = A_{13}x_3 + (\mathcal{A}_{11}x_1 + \mathcal{A}_{12}x_2 + \mathcal{A}_{13}x_3)\,x_3 \\ u_2 &= \dot{x_2} = dx_2/dt = A_{23}x_3 + (\mathcal{A}_{21}x_1 + \mathcal{A}_{22}x_2 + \mathcal{A}_{23}x_3)\,x_3 \\ u_3 &= \dot{x_3} = dx_3/dt = \quad 0 \quad + (\mathcal{A}_{31}x_1 + \mathcal{A}_{32}x_2 + \mathcal{A}_{33}x_3)\,x_3 \end{aligned} \tag{1}$$

If $\overset{o}{x}_i$ is defined as

$$\overset{o}{x}_i = \frac{dx_i}{d\tau} \tag{2}$$

where $d\tau = x_3 dt$, the 'no slip velocity' field can be expressed as

$$\begin{aligned} \frac{u_1}{x_3} &= \overset{o}{x}_1 = dx_1/d\tau = A_{13} + (\mathcal{A}_{11}x_1 + \mathcal{A}_{12}x_2 + \mathcal{A}_{13}x_3) \\ \frac{u_2}{x_3} &= \overset{o}{x}_2 = dx_2/d\tau = A_{23} + (\mathcal{A}_{21}x_1 + \mathcal{A}_{22}x_2 + \mathcal{A}_{23}x_3) \\ \frac{u_3}{x_3} &= \overset{o}{x}_3 = dx_3/d\tau = \quad 0 \quad + (\mathcal{A}_{31}x_1 + \mathcal{A}_{32}x_2 + \mathcal{A}_{33}x_3) \end{aligned} \tag{3}$$

A critical point occurs on the wall when $A_{13} = A_{23} = 0$ and at this critical point

$$\begin{bmatrix} \overset{o}{x}_1 \\ \overset{o}{x}_2 \\ \overset{o}{x}_3 \end{bmatrix} = \begin{bmatrix} dx_1/d\tau \\ dx_2/d\tau \\ dx_3/d\tau \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{A}_{23} \\ \mathcal{A}_{31} & \mathcal{A}_{32} & \mathcal{A}_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{4}$$

$$\text{i.e.} \qquad \overset{o}{x}_i = \frac{dx_i}{d\tau} = \mathcal{A}_{ij}x_j. \tag{5}$$

$\mathcal{A}_{ij}$ will be referred to as the 'no slip tensor'. To satisfy boundary conditions on a no slip surface, the no slip tensor is given by

$$\mathcal{A}_{ij} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{A}_{23} \\ 0 & 0 & -\frac{1}{2}(\mathcal{A}_{11} + \mathcal{A}_{22}) \end{bmatrix} \tag{6}$$

By substitution of the above expansion into the Navier-Stokes equation, it can be shown that $\mathcal{A}_{11}$, $\mathcal{A}_{12}$, $\mathcal{A}_{21}$ and $\mathcal{A}_{22}$ are related to the vorticity gradients and $\mathcal{A}_{13}$ and $\mathcal{A}_{23}$ are related to pressure gradients. The no slip tensor has three invariants $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{R}$ which are given by

$$\begin{aligned} \mathcal{P} &= -\frac{1}{2}\left(\mathcal{A}_{11} + \mathcal{A}_{22}\right), \\ \mathcal{Q} &= -\mathcal{A}_{12}\mathcal{A}_{21} - \left[\frac{1}{2}(\mathcal{A}_{11}^2 + \mathcal{A}_{22}^2)\right], \qquad \text{and} \\ \mathcal{R} &= (\mathcal{A}_{11}\mathcal{A}_{22} - \mathcal{A}_{12}\mathcal{A}_{21})\left[\frac{1}{2}(A_{11} + A_{22})\right]. \end{aligned} \tag{7}$$

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## SURFACE SKIN FRICTION AND SURFACE VORTICITY FIELDS

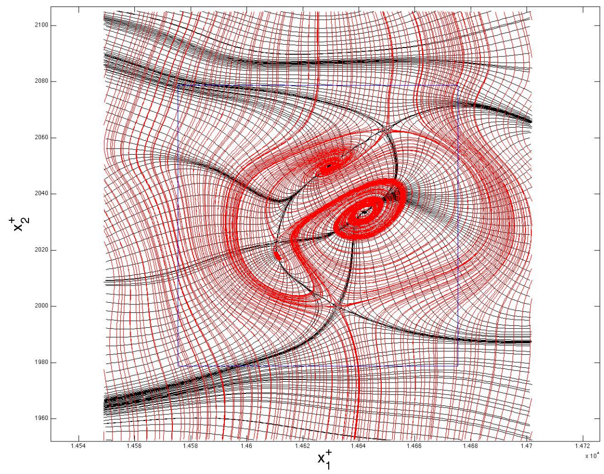

**Figure 1.** Surface skin friction lines (black) and surface vorticity lines (red) at the wall using data from DNS of channel flow, see [5] for details of computations. The size region shown if for a small region of the entire channel wall in the computational domain, which is $23,474 \times 8,803$ in viscous units. A measure of the three-dimensional structures is given by the blue square is which is $100 \times 100$ viscous wall units.

The no slip tensor and its invariants can be used to investigate the topology of skin friction fields and the surface vorticity field at the wall in any wall-bounded flows, especially using data from Direct Numerical Simulations (DNS) of wall bounded flows. Wall shear stress or skin friction is defined as the wall normal derivative of the velocity vector at the wall. In simulations of wall bounded flow over a smooth wall, the first plane away from the wall is usually parallel to the wall. Hence the direction of the near-wall velocity vector when projected on to the plane is in the same direction as the wall shear stress vector if the first plane is sufficiently close to the wall. Thus skin friction lines (also known as limiting streamlines at the no slip wall) can be obtained by integrating the streamwise velocity $u_1$ and spanwise velocity $u_2$ in the first plane from the wall. It is generally assumed that this first plane is very close to the wall. Figure 1 shows skin friction lines (black) and surface vorticity lines (red) in a streamwise-spanwise planer surface at the no slip wall using data from the DNS of channel flow made available by [5]. The Reynolds number is $Re_\tau = hu_\tau/\nu = 934$ where $h$ is the channel half height. Similar critical points have also been found on the surface using data from the DNS of a pipe flow by [9] and also from the recent spatial computation of a zero pressure gradient turbulent boundary layer by [4]. The topology of the critical points at the wall confirms the conjecture that the surface of the no slip surface in the DNS of wall bounded flow can be mapped on to the surface of a toroid. The Poincaré-Hopf index theorem or Poincaré-Bendixson theorem, see [6], [7] or [8], states that the topology of a smooth vector field on a toroidal surface is such that the number of nodes minus the number of saddles is equal to zero, i.e.

$$\sum(Nodes) - \sum(Saddles) = 0. \quad (8)$$

## CONCLUSION

The discovery of critical points in the instantaneous velocity field at the wall is important since it is generally assumed that near the wall the flow is two-dimensional and devoid of any features. The invariants of the no slip tensor provide an analytical framework and methodology to study turbulent structures near the wall. In an incompressible flow, vorticity is only generated at the boundary, see [7] and it is conjectured that these critical points must be associated with the generation and transport of vorticity and, therefore, turbulence 'at the wall'. However, it has yet to be established conclusively that critical points exist in *all* DNS of wall bounded flows and if they are *necessary* for the generation of 'turbulence' at the wall.

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