SHAPE AND MOTION OPTIMIZATION OF 3D SELF-PROPELLED SWIMMERS

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Summary We present an optimization framework for shapes and motions of self-propelled swimmers. The present study combines 3D remeshed vortex methods with Lagrangian level sets, a penalization technique and a projection approach for the flow simulations, along with an evolution strategy for the optimization. We apply our algorithm to study the escape response of larval fish, and to investigate the relationship between a swimmer's morphology and fast or efficient anguilliform swimming. The results of the simulations are compared with experiments in order to examine the perceived optimality of biological swimmers.

MOTIVATION

Fish hydromechanics are widely studied to extract design principles which could improve the design of man-made vehicles. These studies are often limited to one or a few swimmers with fixed shape and motion for which swimming characteristics and the corresponding hydrodynamics are analyzed. We apply instead a reverse-engineering approach to investigate and understand the optimality of biological swimmers. We achieve this by combining a flexible three-dimensional incompressible viscous flow solver with an evolution-based optimization technique. Given an objective, this approach allows us to find the corresponding optimal swimmers. The results give unprecedented insights in the kinematics and hydrodynamical features of self-propelled swimmers.

EQUATIONS AND NUMERICAL METHOD

We consider a deforming and self-propelling body in a viscous incompressible flow determined by solving the incompressible Navier-Stokes equations. The action of the body on the fluid is realized through the no-slip boundary condition at the interface, and the feedback from the fluid to the body is in turn described by Newton's equation of motion.

The flow is solved via a remeshed vortex method, characterized by a Lagrangian particle advection, followed by particle remeshing and the use of an FFT-based Poisson solver [9]. The remeshed vortex method is coupled with Brinkman penalization to enforce the no-slip boundary condition at the body interface, a projection approach handles the feedback from the flow to the body while the body surface is tracked implicitly by a level set [2]. This algorithm has been extended to non-divergence free deformations for single and multiple bodies [3].

The Navier-Stokes equations are cast into velocity ($\mathbf{u}$) - vorticity ($\boldsymbol{\omega} = \nabla \times \mathbf{u}$) formulation and appended with the penalization term:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \nabla \cdot (\mathbf{u} : \boldsymbol{\omega}) = (\boldsymbol{\omega} \cdot \nabla)\, \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \lambda \nabla \times \chi_s (\mathbf{u}_s - \mathbf{u}) \tag{1}$$

where $\nabla \cdot (\mathbf{u} : \boldsymbol{\omega})$ is the vector with components $\partial/\partial x_i (u_i \omega_j)$, χ_s the characteristic function [3] describing the body shape, $\lambda \gg 1$ the penalization factor and $\mathbf{u}_s$ is the local body velocity of the swimmer. The translational ($\mathbf{u}_T$) and rotational ($\mathbf{u}_R$) components of $\mathbf{u}_s = \mathbf{u}_T + \mathbf{u}_R + \mathbf{u}_{DEF}$ are recovered through a projection approach, while the deformation velocity $\mathbf{u}_{DEF}$ is specific to the shape under study. The representation of the swimmer as a level-set function and the flexibility of the penalization approach allow an easy extension to multiple swimmers, as shown in Fig. 1 (middle).

The optimization is performed using the Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) in its rank-μ and weighted recombination form [6]. The CMA-ES is an iterative algorithm that operates by sampling, at each generation, p candidate parameter vectors from a multivariate Gaussian distribution. Its mean, overall standard deviation and covariance matrix are adapted based on past successful parameter vectors, ranked according to their cost function values.

A successful application of this framework confirmed the optimality of C-start as escape response for larval zebrafish [5] (Fig. 1, left). This study also served as validation of our methodology, showing good agreement with experimental results. Given the strong localization of the generated flow structures, we are extending our framework with wavelet-based multiresolution techniques [7] to fully take advantage of the computational resources (Fig.1, right). Our wavelet-based multiresolution approach for flow-structure interaction in a viscous environment has been implemented and validated in the 2D case. It allowed for the investigation of the influence of Reynolds number on the dynamics of arrangements of submerged bodies [4]. These simulations showed how the Reynolds number is responsible for sharp transitions in qualitative system behavior and highlighted the limits of inviscid approaches.

SHAPE OPTIMIZATION FOR STEADILY SWIMMING LARVAL FISH

In addition to optimizing swimming motions for a given shape [8, 5], we also optimized swimmer shapes for a given motion pattern. The motion is inspired by anguilliform swimmers as described in [1] and used in [8], and is based on

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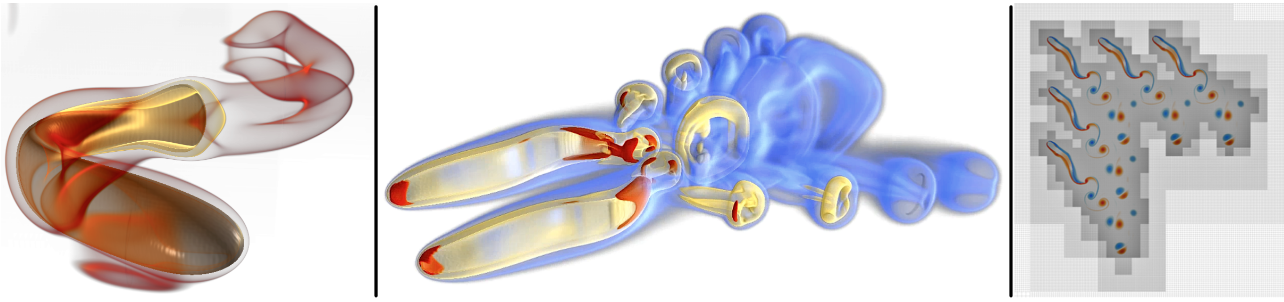

Figure 1. Snapshots of a zebrafish larva performing a C-start (left), two swimmers swimming out-of-phase in 3D (middle) and a 2D multi-resolution simulation of a small school of five fish (right)

a wave traveling along the midline of the swimmer. The three-dimensional shapes consist of an elliptical cross-section at each axial location, with semi-axes given by a width and a height curve defined along the midline of the swimmer. We parametrize each of these curves by a uniform cubic B-spline. During the optimization, the vertical coordinates of the control points are optimized to maximize speed or efficiency of the swimmer at the moderate Reynolds number[b)] of 550. The results (Fig. 2) indicate that for the considered flow regime, from a purely hydrodynamical point of view a fast swimmer needs to be thin with a pronounced tail and a bump in the mid-length of the shape, while an efficient swimmer combines a blunt head with a slender tail.

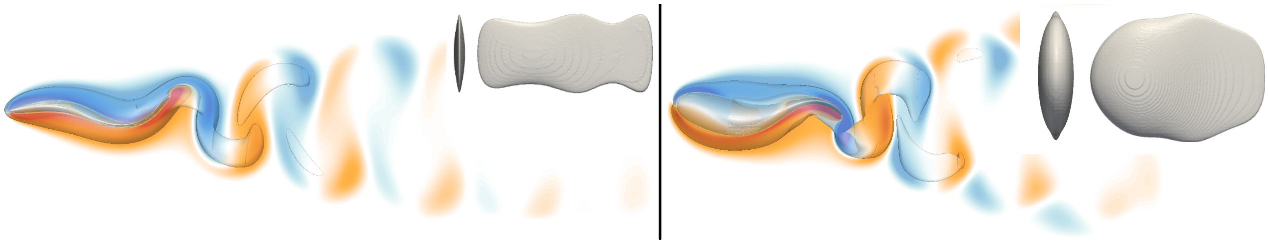

Figure 2. Vorticity fields in the mid-planes of the domain, colored by the out-of-plane vorticity component. Fastest swimmer (left) and most efficient swimmer (right). The insets show a front and side view of the geometry.

CONCLUSIONS

In this work, we showed how the combination of a three-dimensional fluid solver and an evolutionary optimization strategy provides a powerful method to investigate biolocomotion. Our reverse-engineering approach reveals optimal features in the shape and motion of biological swimmers. Future work will focus on increasing the Reynolds number, increasing the number of swimmers and add online-learning to our framework to simulate and understand the dynamics and hydrodynamical aspects of fish schooling.

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[b)] defined as $Re = (L^2/T)/\nu$, with L the fish length, T the period of the traveling wave and ν the kinematic viscosity