RADIAL JET VELOCITY COMPONENTS AND VORTEX STRENGTH OF A JETTING JELLYFISH

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Summary A model for starting jet impulse and circulation that considers the effect of radial velocity components is investigated using the starting jet from a swimming jellyfish. The radial component of velocity plays an important role in the creation of impulse and circulation in this flow and even a simple representation of the radial velocity greatly improves the estimates of these quantities when compared to the commonly used slug model which neglects radial velocity components.

INTRODUCTION

Modeling and predicting the impulse and circulation of a vortex ring formed by a starting jet has been a common research topic for several decades with wide ranging applications. In particular, jets and vortex rings are commonly used for propulsion by creatures such as squid and jellyfish. Vortex thrusters have been developed to mimic this process and provide precise low-speed maneuvering capabilities for underwater vehicles. Such thrusters can be recessed into the vehicle body, maintaining low drag for high-speed travel while taking up very little volume within the vehicle.

Despite the large amount of previous research on the topic of starting jets and vortex generation, several issues are still not well understood. Most significantly, the commonly used slug model assumes an initially uniform axial jet velocity and zero radial velocity in the jet [1]. In fact, the radial velocity is often important and is highly dependent on nozzle shape near the orifice. In reality, non-zero radial velocity in the jet can act to significantly increase the impulse and circulation of the resulting jet.

In this presentation we discuss the presence and importance of the radial velocity components of the jet created by a jetting jellyfish. We use the results of numerical simulations of the swimming jellyfish *Sarsia tubulosa*, full details of the simulation may be found in [3].

RESULTS

At present, only preliminary results have been obtained for this study. The final presentation will included calculations of the total impulse and circulation of the resulting jet to further highlight the advantages of more accurate jet models that include the radial velocity components.

The jellyfish *S. tubulosa* uses a strong contraction to emit a jet that rolls up into a single vortex ring during each swimming stroke as shown in Fig. 1. Previously this type of jet formation has been modeled with a "slug model" where the jet is approximated as a slug of fluid with $u = \text{constant}$ and $v = 0$. In reality the v velocity is often non-negligible and this model can lead to very inaccurate predictions of the impulse and circulation of the resulting jet and vortex ring.

In this jellyfish, the shape of the sub-umbrellar cavity and the velum during contraction is responsible for generating a radial velocity component in the jet. The axial and radial velocity components at the velar opening are shown in Fig. 2. The radial velocity must necessarily be zero at $r = 0$ since the jet is axisymmetric. The axial velocity exhibits the characteristic profile seen in jets created by cavity type actuators with peaks in velocity near the extents of the jet.

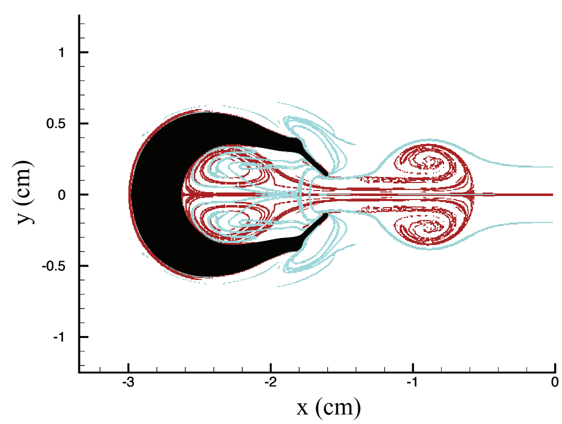

Figure 1. The vortex ejected by *S. tubulosa* as shown by Lagrangian coherent structures. Note that the cavity has an angled exit nozzle shape which creates a radial component of velocity in the jet.

Of particular note is the fact that the radial velocity component is almost $1/3$ as large as the axial component and dv/dx actually exceeds the axial velocity at some points. This is certainly not a negligible effect and must be considered when modeling such jets.

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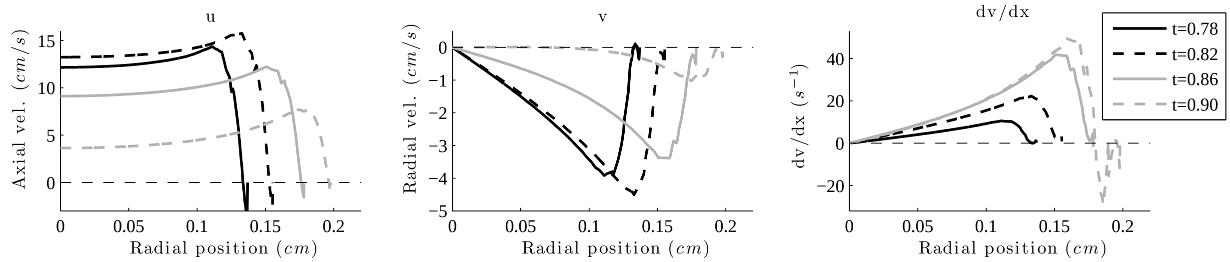

(a) **(b)** **(c)**

Figure 2. The **(a)** axial, **(b)** radial, and **(c)** dv/dx components of jet velocity at the jellyfish velar opening at four times during jet expulsion. The jellyfish contraction lasts from $t = 0.73s$ to $t = 0.95s$.

Impulse and circulation estimates

By assuming that impulse and circulation in the fluid are solely generated by the jet from the jellyfish, it is possible to estimate these properties based only on the flow behavior at the velar opening. The rate of change of circulation and hydrodynamic impulse may be computed as

$$\frac{d\Gamma}{dt} = -\frac{1}{2}{u_0}^2 - \int_0^R u\frac{dv}{dx}\bigg|_{x=x_0} dr \tag{1}$$

and

$$\frac{dI}{dt} = -\pi \int_0^R \left[2u^2 r + u\frac{dv}{dx}r^2 - v^2 r\right]_{x=x0} dr \tag{2}$$

respectively. These estimates are fully derived in an upcoming paper [2].

Many past studies have used the slug model to estimate these quantities for piston/cylinder arrangements by assuming $u = u_0$, $v = 0$, and $dv/dx = 0$ at the orifice. These assumptions lead directly to

$$\frac{d\Gamma}{dt} = -\frac{1}{2}{u_0}^2, \quad \frac{dI}{dt} = \pi {u_0}^2 R^2 \tag{3}$$

However, experiments have shown these estimates to be quite inaccurate for certain cases. It is possible to obtain much more accurate results by considering a two dimensional model which assumes v and dv/dx are linearly proportional to r with proportionality constants A_v and $A_{dv/dx}$ respectively and uses $u = u_0$ for the axial velocity. The proportionality constants may be computed with a least squares best fit from the data. Under these assumptions, Equations 1 and 2 simplify to

$$\frac{d\Gamma}{dt} = -\frac{u_0}{2}\left(u_0 + A_{dv/dx}R^2\right), \qquad \frac{dI}{dt} = -\pi\left[{u_0}^2R^2 + \frac{1}{4}(A_{dv/dx}u_0 - {A_v}^2)R^4\right]. \tag{4}$$

At the time of maximum jet velocity for the jellyfish, the actual values of $d\Gamma/dt$ and dI/dt and the two models are

	Actual value	1D Slug Model	2D Linear model
$d\Gamma/dt$	-110.6	-87.4	-108.0
dI/dt	-14.1	-13.3	-13.7

The linear model is far more accurate than the slug model in this case, especially for the circulation, highlighting the potential for improvement in models attainable from simple approximations that consider the radial component of jet velocity.

References

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