A NUMERICAL STUDY OF SWIMMING MICRO-ORGANISMS INSIDE A CAPILLARY TUBE

Lailai Zhu*, Eric Lauga** & Luca Brandt*[a)]

*Linné Flow Center, Department of Mechanics, KTH, Royal Institute of Technology, Stockholm, 10044, Sweden

**Department of Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, 92093-0411, USA. 10044, Sweden

Summary Numerical simulations based on boundary element method are performed to investigate the locomotion of natural micro-organisms and artificial swimmers inside straight and bent capillary tubes. Kinematics and energetics of the micro locomotion are studied for different levels of confinement. Trajectories of micro-organisms with varying gaits are analyzed with a focus on the locomotory stability: the squirmer model is used to model ciliated micro-organisms moving by tangential surface deformation. Helical trajectories are reproduced, which agrees well with previous experimental observations.

INTRODUCTION

Micro-organisms exhibit interesting locomotive behaviors due to complex surrounding environments. In addition to biological reactions, swimming hydrodynamics play an important role to successfully explain the accumulation of swimming *Eschenchia coli* confined near flat solid boundaries and the corresponding circular trajectories [1], to cite one example. Cell locomotion through higher degree of geometrical confinement like a capillary tube is relevant when considering restricted motions of spermatozoa *Trypanosoma* inside channel-like paths and bacteria through microporous environments. In this work, we numerically study swimming microorganisms inside a tube with boundary element method (BEM), where the squirmer [2] is used as our model micro-swimmer. We calculate swimming speed and power consumption of squirmers confined in a capillary tube. We also show trajectories of a cell with different initial positions and orientations, exploring the relation between swimming gaits and locomotory stability. Cell locomotion in bent tubes is also simulated, giving some hints for the design and operation of artificial micro-swimmers in complicated confined geometries.

Squirmer model

Tangential axisymmetric velocity $u_s(\theta)$ is imposed on a spherical squirmer to represent the metachronal waves generated by the beating cilia, $u_s(\theta) = B_1 \sin\theta + (B_2/2)\sin 2\theta$, where $\theta = \arccos(\mathbf{e}\cdot\mathbf{r}/r)$, $\mathbf{e}$ is the swimming direction and $\mathbf{r}$ is the position vector of the evaluated point relative to the sphere center. Note that here we consider steady boundary condition to model fast beating cilia. $\alpha = B_2/B_1$ and we vary α to achieve different swimming gaits. $\alpha = 0$ corresponds to a neutral squirmer, positive and negative α stand for a puller and pusher respectively. The swimming speed of a squirmer in the free space is $2/3B_1$.

NUMERICAL METHODS

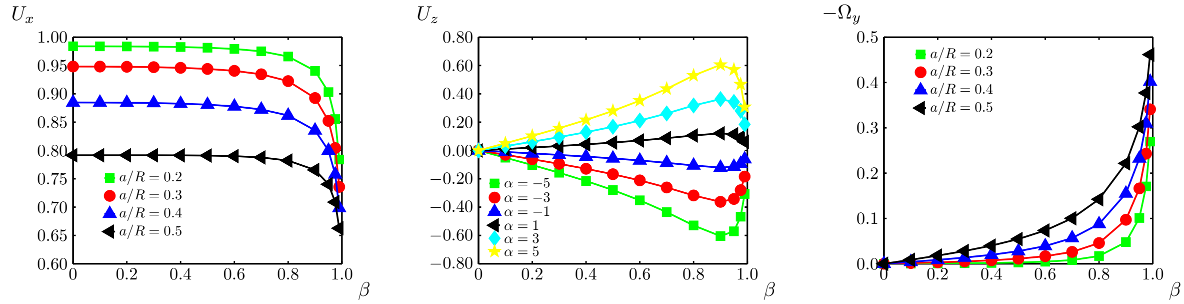
We use constant-force boundary element method where the integration and force discretization are 'decoupled' as introduced by D. J. Smith [3]. Analytical integration is performed for singular elements using plane polar coordinates with Gauss-Legendre quadrature. Mesh subdivision is implemented to handle nearly singular integration as the swimming cells are getting close to solid boundaries. The code is well validated for a translating sphere in the free space, near a plane wall and inside a small tube.

RESULTS AND DISCUSSION

Instantaneous kinematics and energetics

We numerically calculate the swimming speed and power consumption for a squirmer exploiting pure tangential surface deformation, thus we fix $B1$ to 1 and vary the swimming gait α. Different values of a/R and β are chosen to study the effect of confinement and eccentricity on locomotion (a cell radius, R pipe radius, β non-dimensional off-axis distance). In Fig. 1, we report the instantaneous swimming velocity of a squirmer inside a tube with the confinement $a/R = 0.3$. The cell is located at $(0, 0, -\beta\,(R - a))$ with its orientation parallel to the tube axis. We show that the confinement decreases the swimming speed of the cell inside the tube. Puller are repelled by the nearest wall while pushers are attracted. Moreover, squirmers tend to rotate away from the nearest wall due to the cell-wall hydrodynamic interaction. We also find that tube confinement increases the power consumption of swimming cells, probably resulting from higher hydrodynamic resistance.

[a)] Corresponding author. E-mail: luca@mech.kth.se.

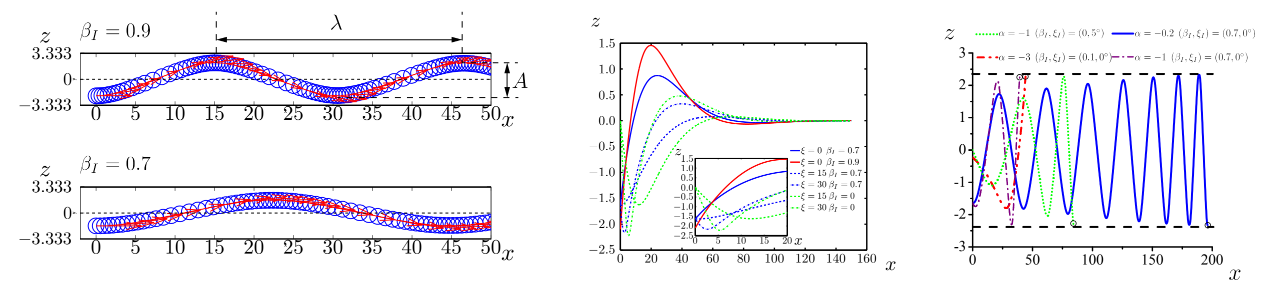


(a) Swimming velocity in the axial direction U_x (independent of α).

(b) Swimming velocity in the transverse direction U_z (dependent on α).

(c) Rotational velocity in the reference system $-\Omega_y$ (independent of α).

Figure 1. Swimming velocity of a squirmer located at $(0, 0, -\beta (R - a))$ inside the tube with its orientation parallel to the tube axis.



(a) Typical wave-like motions of a neutral squirmer. Blue circles stand for the cell and red arrows point out the cell orientation.

(b) Trajectories of a $\alpha = 3$ puller with different initial positions and orientations.

(c) Trajectories of a pusher with different α, initial positions and orientations.

Figure 2. Trajectories of neutral squirmers, pullers and pushers inside a capillary tube with confinement $a/R = 0.3$.

Locomotory trajectories and the dynamic stability

We study the trajectories of a neutral squirmer, puller and pusher inside the capillary tube, when the cell motion is confined in a 2D plane. Different cell positions and orientations are investigated. We show that for all the three gaits, the cell execute wave-like motions inside the tube. For a neutral squirmer, the wavelength and wave magnitude stay constant with time (see Fig. 2(a)), indicating such a trajectory is marginally stable. For a puller, we observe increasing wavelength and decreasing wave magnitude (Fig. 2(b)), thus the trajectories are passively asymptotical stable, whereas the opposite trend is detected for a pusher (Fig. 2(c)), implying that the motion of the pusher inside the tube is unstable.

3D trajectories and motion inside a bent tube

We also study 3D trajectories of squirmers and we find that a neutral squirmer follows helical trajectories, which agrees with the experimental observations of swimming *Paramecium* inside a capillary tube [4]. We show that such a helical motion is a combination of wave motions developing in the axial and azimuthal direction. Cells bounce back from the tube wall purely from hydrodynamic interactions. Different initial cell positions and orientations correspond to varying trajectories and the locomotion is hindered as the helical trajectory shifts towards the wall. Moreover, the puller is asymptotically stable as in the 2D case. In the final contribution, we will present simulations of a squirmer swimming inside a bent tube; artificial swimmers propeled by a rotating flagella is also being investigated.

References

[1] Lauga E., DiLuzio W. R., Whitesides G. M. and Stone H. A.: Swimming in Circles: Motion of Bacteria near Solid Boundaries. *Biophys. J.* **90**:400-412, 2006

[2] Blake J. R.: A spherical envelope approach to ciliary propulsion. *J. Fluid Mech.* **46**:199-208, 1971

[3] Smith D. J.: A boundary element regularized Stokeslet method applied to cilia- and flagella-driven flow. *Proc. R. Soc. A* **465**:3605-3626, 2009

[4] Jana S., UM S. H., and Jung S.:Paramecium swimming in capillary tube. *arXiv* **1012.4812v1**, 2010