# THREE-DIMENSIONAL SIMULATION OF A RED BLOOD CELL IN SHEAR FLOW

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Summary We present an improved penalty immersed boundary method in conjunction with the subdivision finite element method for 3D simulation of the deformation of a biconcave circular disk as a model of the red blood cell in a linear shear flow. In our simulation, the swinging motion is observed due to the shape memory effect. By decreasing the dimensionless shear rate or increasing the reduced bending modulus, the swinging motion is transited into the tumbling motion.

## INTRODUCTION

Deformation of a liquid-filled capsule enclosed by an elastic membrane due to external shear flow is an important problem in fundamental research as well as in bioengineering applications. An example of widespread concern is the red blood cell (RBC) with its membrane composed of a lipid bilayer and a cytoskeleton, which behaves like a two-dimensional flexible but nearly area-conserved material [1]. The deformability of the RBC has a significant effect on the physiological function of the RBC and the rheology of the whole blood [2]. Thus understanding of the dynamics of the RBC is indispensable in cellular biology and hemodynamics, and may help us diagnose blood diseases.

Experimental observations on the RBC in shear flow have revealed rich dynamics of the system, e.g. the swing motion and the tumbling motion [3, 4]. Theoretical studies have also been performed to qualitatively describe the tumbling-to-swinging mode transition observed in experiments using simplified models [5]. However, to fully resolve the dynamics of capsules with complex geometries and finite deformations where theoretical prediction is difficult if not impossible, numerical simulations have to be performed.

## NUMERICAL METHOD

To solve the fluid-flexible body coupled system, we adopt the improved penalty immersed boundary (pIB) method proposed in our previous study [6]. The immersed boundary is assumed to be composed of two parts: massive material points and massless material points, which are assumed to be linked closely by a stiff spring with damping. The massive material points are subjected to the elastic force of solid deformation but do not interact with the fluid directly, while the massless material points interact with the fluid by moving with the local fluid velocity. In this way, we can make use of the governing equations for fluid and solid motions, which are linked through the Lagrangian and Eulerian momentum forcings [6].

In the framework of the pIB method, the flow solver and the solid solver can be developed separately by different methods. In the present study, the fractional step method is adopted to solve the incompressible fluid motion on a staggered Cartesian grid, while the finite element (FE) method is developed to simulate the membrane deformation using an unstructured triangular mesh. Since the bending force term of the RBC membrane takes the fourth-order partial differential form, the discretized surface is required to be $C^1$ continuous in order to ensure the convergence of the finite-element solutions. A method to solve the $C^1$ continuity difficulty has been found by using the subdivision surface, which represents the limit of a recursive iteration of surface node positions. The corresponding formulation for geometric interpolation of the subdivision surface can be used as the shape functions in the FE method for plate and shell computations [7]. In the present study, we apply the subdivision FE method in conjunction with the pIB method for 3D simulation of the fluid-membrane interaction. Details of the problem formulation and the computational procedure can be found in [8].

## RESULTS AND DISCUSSION

In this section, we study the deformation of the RBC in a linear shear flow by using the proposed method. The fluid domain size is 10×10×10 scaled by the equivalent volumetric radius $a_{eq}$ in the *x*-, *y*- and *z*-directions, with the origin located at the center of the domain, which coincides with the center of the RBC. Dirichlet conditions ($u = \dot{\gamma} y,\ v = w = 0$) are applied at the top and bottom boundaries, while periodic conditions are adopted in the *x*- and *z*-directions. The geometry of the RBC is taken to be a biconcave circular disk as shown in Fig.1 [9]. According to the grid convergence test, we use a resolution of 8192 elements for the RBC mesh and 192×192×192 grids for the fluid domain. Moreover, the computational time step is $\Delta t = 0.0001$ and the Reynolds number based on $a_{eq}$ is set as Re=0.025.

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Figure 2 shows time histories of the inclination angle $\theta$ of the RBC with the Skalak membrane [10], which conserves the surface area by giving a large material constant. The initial inclination angle is set to be $\pi/4$. To investigate the effects of the elastic properties of the membrane, different dimensionless shear rates $\eta$ and reduced bending moduli $\zeta$ are tested in our simulations. For the case without bending rigidity $\zeta$=0 (see Fig.2(a)), we can observe the swinging motion at $\eta$=1.2 and 0.1, and the oscillation amplitude of $\theta$ is increased as $\eta$ decreases. At $\eta$=0.025, $\theta$ is jumped from $-0.5\pi$ to $0.5\pi$ at each rotating period, indicating the tumbling motion. Similarly, the transition from the swinging motion to the tumbling motion can also be induced by increasing the bending rigidity, which makes the RBC behave more like a rigid body. For a given dimensionless shear rate $\eta$=0.1 (see Fig.2(b)), the swinging motion occurs at $\zeta$=0 while the tumbling motion is formed at $\zeta$=0.09. Figures 3 and 4 depict snapshots of the swinging motion and the tumbling motion of the RBC of $\eta$=0.1 but different $\zeta$, i.e. $\zeta$=0 and 0.09 respectively. At $\zeta$=0 (see Fig.3), buckling occurs obviously on the capsule surface induced by the compressive stresses. As expected, wrinkles are suppressed and the tumbling motion is resulted by including the bending rigidity, as seen in Fig.4 for $\zeta$=0.09.

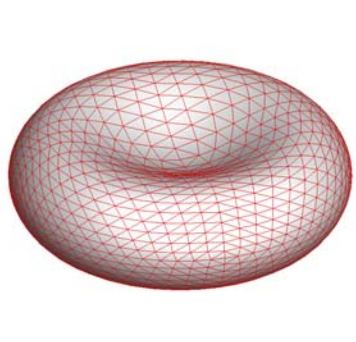

Fig.1 Surface mesh for RBC

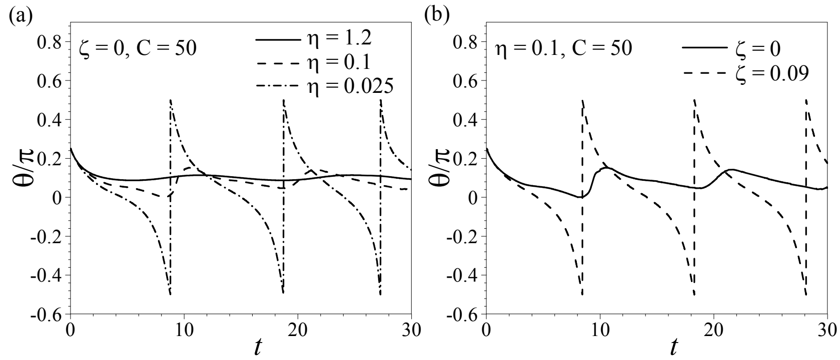


Fig.2 Time histories of the inclination angle: (a) ζ=0; (b) η=0.1.

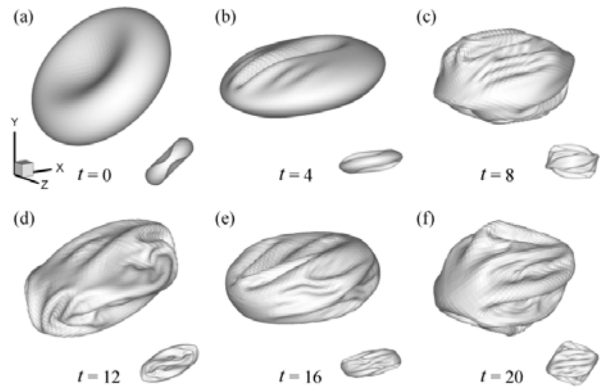


Fig.3 Instantaneous shapes of RBC with η=0.1 and ζ=0

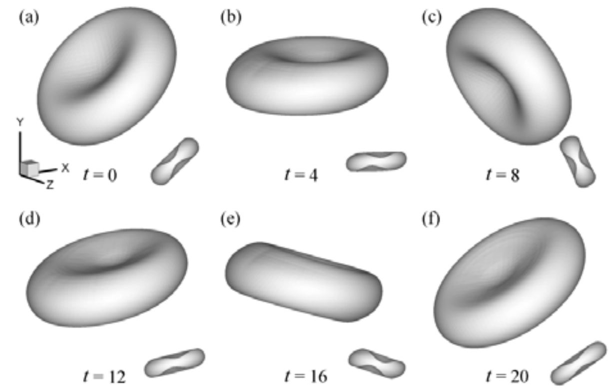


Fig.4 Instantaneous shapes of RBC with η=0.1 and ζ=0.09

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