# TOPOLOGY OPTIMIZATION OF THE THREE-DIMENSIONAL FISH TAIL

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*Summary* The topology optimization of the caudal fin of the 3D self-propelled swimming fish is investigated by combining unsteady computational fluid dynamics with moving boundary and topology optimization algorithms. The optimal tail, whose topology is different from that of the natural fish tail, make the fish achieve higher swimming efficiency, faster swimming speed and better maneuverability. The mechanism of high performance swimming of the optimal fish is analyzed with the vortex dynamics theory.

## NUMERICAL SIMULATION ALGORITHM

### Computational fluid dynamics numerical algorithm with the moving boundary

The finite volume method is used to solve incompressible unsteady Navier-Stokes equations. The technique of adaptive mesh refinement is used to increase computational savings. The 3D self-propelled fish swimming is a moving boundary problem with complex geometry changing constantly. In this paper, moving boundaries are handled with ghost-cell immersed boundary method(IBM). The method of adaptive multi-grids is used and the mixed adaptive refinement criteria are both vorticity and $\nabla T$, where T is a tracer of volume of fluid(VOF). The detailed numerical algorithm is referred to Ref. [1]. The dynamics equations for the self-propelled fish are

$$m\frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = \vec{F}, \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \vec{M}, \tag{1}$$

$$\vec{F} = -\int_{\partial B} (-p\vec{n} + \mu\vec{\omega} \times \vec{n})dS, \tag{2}$$

$$\vec{F} = -\int_{\partial B} \rho\vec{x} \times (1/2\vec{\sigma}_p + \vec{\sigma}_{vis})dS \tag{3}$$

where $m$ is the mass of the bionic fish; $\vec{u}$ is the velocity vector; $\vec{F}$ is the hydrodynamic force; $\vec{M}$ is the moment and $\vec{L}$ is the moment of momentum; $\vec{\sigma}_p = \vec{n} \times \vec{\nabla} p/\rho$,$\vec{\sigma}_{vis} = \nu(\vec{n} \times \vec{\nabla}) \times \vec{\omega}$. Eq.(3) is a force expression of BVF while Eq.(2) is a familiar force formula directly derived from the momentum balance[2].

The body shape of RoboTuna is used in the study of the self-propelled swimming of fish. The initial condition is $u = v = w = 0$. The computational region has six boundaries. All of them are set to be no slip boundary conditions, i.e.,$u_b = v_b = w_b = 0$. The velocity on the fish body boundary is $\vec{V_b} = \vec{V_0} + \vec{V_r} + \vec{V_f}$, where velocity $\vec{V_0}$, $\vec{V_r}$ and $\vec{V_f}$ arose from hydrodynamic force, rotation and flapping respectively.

### Algorithm of the topology optimization of the caudal fin

The topology optimization functional of the caudal fin of the three-dimensional bionic fish is the function of the swimming efficiency, the swimming speed and the amplitude of the head of fish. The objective of the optimization study is to find the optimal geometry of the caudal fin, which can make fish swim more efficiency, faster and flexible. The coefficient of water permeability of regions in the tail are the optimization parameters. The region that is favorable to the swimming performance, is impermeable. Otherwise it is permeable and is removed from the tail. With the immersed boundary method, the coefficients of water permeability of all regions are applied to the 3D bionic fish body in the process of topology optimization. Take the triangular caudal fin as the initial shape, as shown in Figure 1(a), caculate the average swimming efficiency, average swimming speed and the swing amplitude of the head of fish within one swimming period; Then doing the topology optimization to adjust the coefficient of water permeability; Apply the optimized parameters to the CFD, until the realization of optimal functional.

## RESULTS OF THE TOPOLOGY OPTIMIZATION

### The comparison of the swimming performances before and after the topology optimization

The topology optimizations of the caudal fin of the 3D bionic fish are investigated at four different types of initial swimming speeds $u = -0.1$, $u = -0.2$, $u = -0.3$, $u = -0.35$, respectively. Thus the various of moving modes, containing the low speed swimming, the moderate speed and the high speed swimming, are considered in the present study. The optimal caudal fin of most of swimming modes, as shown in Figure 1(b), whose topology is different from that of the natural fish tail, is a hollow tail, and the geometry of the region near the leading edge of the caudal fin is still similar to the lunate tail. In conclusion, the 3D topology optimized bionic fish achieves high swimming capabilities in a variety

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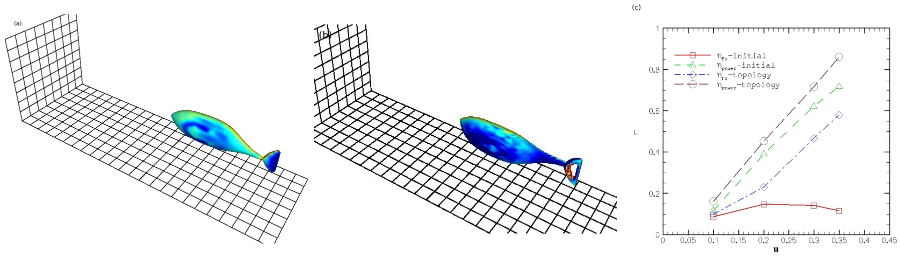

**Figure 1.** Comparison between the initial bionic fish and the 3D topology optimized bionic fish

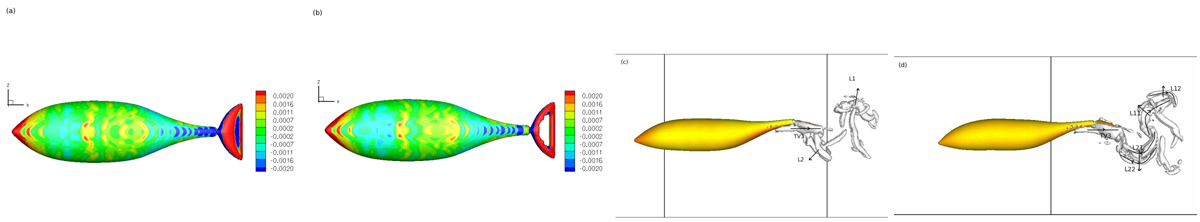

**Figure 2.** The analysis of vorticity dynamics of self-propelled swimming of the 3D bionic fish.

of swimming modes, as shown in Figure 1(c). Especially the swimming efficiency and the swimming speed of the 3D topology optimized bionic fish increased significantly, compared with the initial bionic fish with the triangular caudal fin. All indexes of the swimming performance of the 3D topology optimized bionic fish improve obviously with the increase of the swimming speed. The initial bionic fish is not suitable for moderate and high-speed swimming.

**The BVF analysis of self-propelled swimming of the 3D bionic fish**

Since the BVF in the centre region of the tail is opposite to the BVF in the region near the leading edge and the trailing edge, as shown in Figure 2(a), it leads to the decrease of the total vorticity generating on the tail. This is detrimental to swimming propulsion. After the topology optimization, the 3D topology optimized bionic fish just eliminates the central regions of the caudal fin of the initial bionic fish, as shown in in Figure 2(b). So the total strength of vorticity on the body of the 3D topology optimized bionic fish is much higher than the initial bionic fish. Due to the special shape of the caudal fin, the 3D topology optimized bionic fish also has stronger vorticity output than the fish with the lunate tail. The swimming performances of the 3D topology optimized bionic fish make great improvements.

**Comparison between the topology optimized bionic fish and the fish with the lunate tail**

In many studies, it is found that the fishes with the lunate tail have the highest swimming efficiency among most fishes in the natural world. The comparative analyses of the swimming performances of the fish with the lunate tail and the 3D topology optimized bionic fish at the same initial swimming speed are performed. The 3D topology optimized bionic fish not only achieves higher swimming efficiency and speed, but also swims more flexible. Compared Figure 2(c) with Figure 2(d), the strength of the vortex induced by the 3D topology optimized bionic fish is much higher than that of the fish with the lunate tail. Due to the distinctive feature of the caudal fin of the 3D topology optimized bionic fish, it forms the nested double vortex rings attached to the caudal fin. For the 3D topology optimized bionic fish, vortex rings shed successively in a swing cycle interconnect, but a set of vortex contrails connect two adjacent vortex loops in the wake of the fish with the lunate tail. Thus The jet induced by the double vortex rings in the wake of 3D bionic fish with topology optimized caudal fin is much stronger than that of lunate caudal fin.

## CONCLUSIONS

The 3D optimal caudal fin of all kinds of swimming modes is basically the same. The topology optimization caudal fin whose topology is different from that of the natural fish tail, is a hollow tail. The 3D topology optimized bionic fish has high swimming performances in a variety of swimming modes. The area of the topology optimized tail is larger than the lunate tail, the hollow structural characteristic reduces the adverse effect on propulsion at the high swimming speed.

## References


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