UNSTEADY BOUNDARY LAYERS ON FLAPPING WINGS

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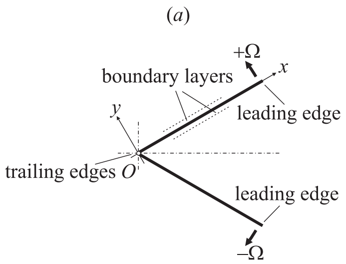
Summary
Flow near the trailing edges of insect wings during the high Reynolds number clap-fling-sweep process is related to unsteady stagnation-point flow. Similarity solutions to this problem are considered. A wide range of possible behaviour is revealed, depending on whether the flow in the far field is accelerating or decelerating, and depending on the flow direction.

INTRODUCTION

Unsteady fluid-body interactions play a central role in the flapping flight dynamics of insects and birds. This is particularly evident in the clap-fling-sweep flight mechanism discovered by Weis-Fogh [1], and analysed theoretically by Lighthill [2] who introduced the idealised potential-flow model shown in figure 1(*a*): two flat plates rotating about a common hinge point O with angular velocities $\pm\Omega$. In a real fluid, viscous effects are important. A typical vorticity field, computed by solving the two-dimensional Navier–Stokes equations with a spectral method and a volume penalisation technique [3], is displayed in figure 1(*b*): vortices are shed from the leading edges [4], and the flow near the trailing edges is also dominated by viscous stresses [5]. When the Reynolds number is large, boundary layers form on (and may separate from) the wing surfaces. These boundary layers, for which the external flow is linear in distance from the hinge point, are closely related to unsteady stagnation-point boundary layers. This motivates the following treatment.



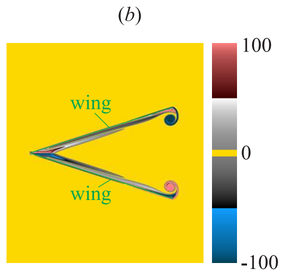


Figure 1. Sketch of a two-dimensional section of the wings (*a*) and a snapshot of the vorticity field computed by solving the two-dimensional Navier–Stokes equations (*b*).

SIMILARITY SOLUTIONS

Let us assume that the flow far from the boundary is the plane irrotational strain with streamfunction $\Psi(x,y,t) = \pm A(t)xy$, where the flow strength $A(t) > 0$ is obtained from the asymptotic expansion of Lighthill's complex potential [2] near O, and the plus or minus sign is chosen according as the flow is towards or away from the boundary (*i.e.*, 'forward' or 'rear' stagnation-point flow, respectively). The streamfunction $\psi(x,y,t)$ of the flow then satisfies the vorticity equation

$$\frac{\partial(\nabla^2\psi)}{\partial t} - \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)} = \nu\nabla^4\psi\,, \tag{1}$$

where ν is the kinematic viscosity of the fluid, and the boundary conditions $\psi = \partial\psi/\partial y = 0$ on $y = 0$, $\psi \sim \Psi$ as $y \to \infty$. We consider similarity solutions of (1) of the form $\psi(x,y,t) = (\nu A(t))^{1/2} x f(\eta)$, where $\eta = y(A(t)/\nu)^{1/2}$. Substitution in (1) and integration once gives an ordinary differential equation

$$f''' + ff'' - f'^2 + 1 = \kappa\left(\frac{\eta}{2}f'' + f' \mp 1\right) \;\text{ with } f(0)=0,\; f'(0)=0,\; f'(\infty)=\pm 1\,, \tag{2}$$

where the upper or lower sign corresponds to the forward or rear stagnation-point flow, respectively, and $\kappa = \dot{A}/A^2$, a dimensionless measure of the unsteadiness of the flow. Clearly, (2) requires that κ be constant in time, and then $A(t) = (\kappa(t_0 - t))^{-1}$, where t_0 is constant, such that $A(0) = (\kappa t_0)^{-1} > 0$, and we suppose that $\kappa \neq 0$. Since by assumption $A(t) > 0$, and the initial conditions are specified at $t = 0$, there are two distinct possibilities (for both types of the stagnation points): accelerating flow with $\kappa > 0$ and $0 \le t < t_0$, or decelerating flow with $\kappa < 0$ and $t_0 < 0 \le t < \infty$. For a given value of κ, boundary-layer velocity profiles are obtained by numerical integration of (2).

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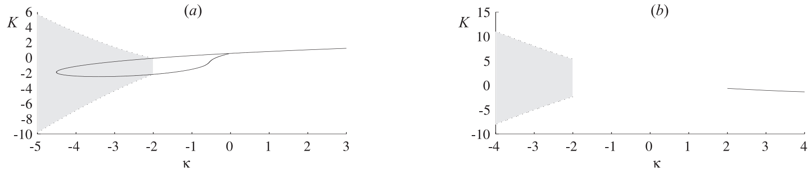

Figure 2. Velocity gradient at the wall, $K = f''(0)$, in flows towards (*a*) and away (*b*) from a stagnation point.

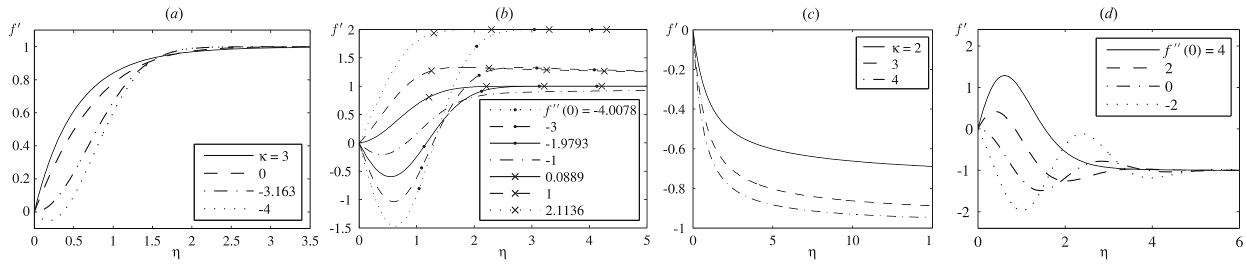

Figure 3. Velocity profiles for flows towards (*a,b*) and away (*c,d*) from a stagnation point.

The unsteady forward-stagnation-point flow was first studied by Yang [6] who computed solutions for a limited range of κ $(-3 < \kappa < +1)$, and who inferred by extrapolation that the wall stress (proportional to $f''(0)$) vanishes at $\kappa \approx -3.175$. The question of what happens for $\kappa < -3.175$ was left open. We have computed solutions by integrating outwards from $\eta = 0$ to a suitably large value of η with three initial conditions ($f(0) = 0$, $f'(0) = 0$, $f''(0) = K$), and then choosing K in such a way as to ensure that $f'(\eta) \sim 1$ for large η. Figure 2(*a*) shows the values of K thus obtained as a function of κ. For κ positive, K is single-valued, *i.e.*, the velocity profile is unique for accelerating flow. For κ negative (decelerating flow), the situation is more complex, as might be expected. The solution bifurcates into an 'upper branch' and a 'lower branch'; these may be followed continuously down to $\kappa \approx -4.6$, where they join. Typical upper branch profiles are shown in figure 3(*a*). We notice that the upper-branch profiles $f'(\eta)$ are positive if $K > 0$, and that the wall stress vanishes at $\kappa \approx -3.163$. For $\kappa < -2$, there is in addition a one-parameter family of solutions (of which the above upper and lower branch solutions are particular members), as indicated by the shaded area in figure 2(*a*). For example, for the particular case $\kappa = -3$, any value of K in the open interval $]-4.0078, 2.1136[$ gives a solution for which $f' \to 1$ as $\eta \to \infty$. Figure 3(*b*) shows seven different possible profiles $f'(\eta)$ for this value of κ, including the two 'limiting' solutions (for $K = -4.0078$ and 2.1136) and the two that correspond to points on the solid curve within the shaded region in figure 2(*a*) ($K = -1.9793$ and 0.0889). These two are distinguished by exponentially rapid convergence as $\eta \to \infty$, whereas solutions corresponding to all other values of K in the above open interval converge much slower.

The boundary-layer problem for the rear-stagnation-point flow has no steady-state solution ($\kappa = 0$). However, unsteady solutions do exist when $|\kappa|$ is large enough. Figure 2(*b*) shows the values of K consistent with the outer boundary condition $f'(\infty) = -1$. Now there are two distinct regimes. (i) For any $\kappa \geq 2$, there is a unique profile $f'(\eta)$. Figure 3(*c*) shows some of these. The convergence is very slow compared with that for accelerating flow towards a stagnation point. In the limiting case $\kappa = 2$, f' approaches -1 like $f' \sim 2/\ln C\eta - 1$ as $\eta \to \infty$, where $C \approx 140$. (ii) For any $\kappa \leq -2$ there is a one-parameter family of solutions. Such pairs (κ, K) lie in the shaded area in figure 2(*b*). Some typical velocity profiles $f'(\eta)$ for $\kappa = -3$ are shown in figure 3(*d*) and all of them have inflection points.

CONCLUSIONS AND PERSPECTIVES

Similarity solutions have been obtained for unsteady stagnation-point flows. They also describe the flow near the trailing edges of flapping wings during clap-fling-sweep. Such solutions for the rear-stagnation-point flow exist if the unsteadiness parameter is large enough, $|\kappa| \geq 2$. It is planned to compare these results with numerical simulations of the two-dimensional Navier–Stokes equations.

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