PAIRWISE INTERACTIONS BETWEEN SQUIRMING MICRO-ORGANISMS

Richard Clarke[*a)] & Matthew Finn[**]
[*]*Department of Engineering Science, University of Auckland, Auckland, New Zealand*
[**]*School of Mathematical Sciences, University of Adelaide, Adelaide, Australia*

Summary Squirmers have been shown in the past to be a useful model for understanding the non-trivial dynamics exhibited by ciliated swimming micro-organisms, such as *Opalina* or *Volvox* [1]. We use this framework to consider the pairwise interaction of such micro-organisms, and show that in two-dimensions the interaction is non-negligible even at asymptotically-large distances between the swimmers. Furthermore, we demonstrate that the nature of this long-range interaction has a limiting form which can be derived asymptotically.

INTRODUCTION

There has been much recent interest in the use of certain prototype microscopic swimmers, called Squirmers, to theoretically explain the involved behaviour observed when micro-organisms such as *Opalina* or *Volvox* swim near neighbours [1], or surfaces [2]. Squirmer models aim to emulate, in the most mathematically-tractable way, the means by which such micro-organisms propel themselves through fluid via a syncronised beating of their surface coating of cilia. Indeed it has been shown that the simple dynamical systems derived using two-dimensional Squirmers can successfully reproduce the complex near-wall behaviour seen in more advanced simulations and experiments [2]. Such a simple theoretical framework is especially appealing in light of our long-term ambition to incorporate pairwise interactions into a kinetic equation for semi-concentrate suspensions of squirmers, in an approach akin to that used to derive the Boltzmann Equation in Kinetic Gas Theory. Hence we derive a simple dynamical system that encapsulates the interaction, and discuss an interesting and unexpected phenomenon whereby interactions at asymptotically large-distances remain significant. Moreover, we derive a simple asymptotic form for this long-range interaction, and show that it depends solely on the initial relative orientation of the swimmers.

FORMULATION

Consider two swimmers whose locations we specify in the complex plane as $s_1(= x_1 + iy_1)$ and s_2. There is a distinguished direction for each swimmer (which, when isolated, coincides with their swimming direction) which we denote as θ_1 and θ_2 (see Fig. 1a). We choose to work in a frame of reference in which Swimmer 1 is stationary at the origin, and orientated along the x-axis. The interaction can then be parameterised by just two quantities: the (complex-valued) relative displacement $s = (s_2 - s_1)\exp(-i\theta_1)$ and $q = \exp(i\theta)$ where $\theta = \theta_2 - \theta_1$ is the relative orientation.

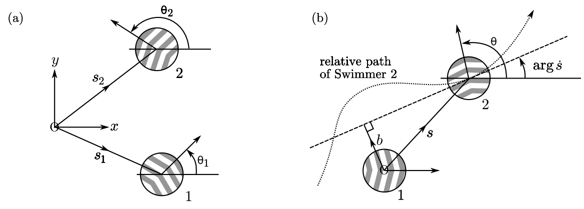

Figure 1. The pairwise interaction of two microscopic swimmers, as seen in the laboratory frame (left) and the frame of reference of Swimmer 1 (right).

[a)] Corresponding author. E-mail: rj.clarke@auckland.ac.nz.

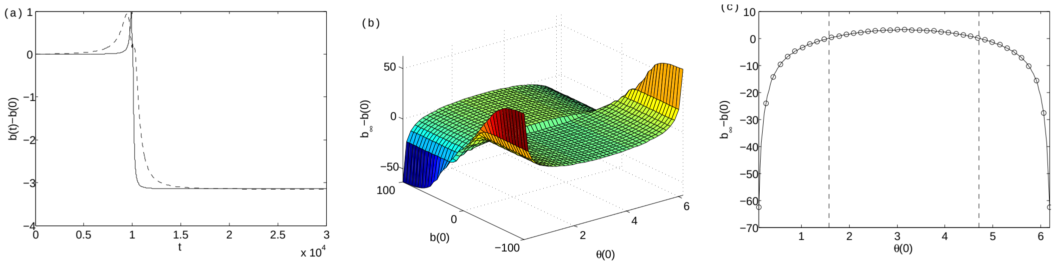

Figure 2. (a) Shift in Impact Parameter $\Delta b(t) = b(t) - b(0)$ for $b(0) = 100$ (full line) and $b(0) = 500$ (dashed line), with $\theta(0) = \pi/3$ in both cases. (b) Limiting shift in Impact Parameter $\Delta b(t \to \infty)$ against $b(0)$ and $\theta(0)$. (c) $\Delta b(t \to)$ when $b(0) = +10$ as a function of $\theta(0)$. Full line denotes solution to full dynamical system, whilst the asymptotic limit (1) is shown by markers. Dashed lines mark Δb passing through zero at $\theta = \pi/2,\ 3\pi/2$.

In addition to its own self-generated motion, each swimmer is advected and rotated by the flow generated by its neighbour. The following dynamical system then describes the interaction:

$$\dot{s} = w(s) + q - qw(-s\bar{q}) - 1 - \tfrac{1}{2}\mathrm{i}s\omega(-s\bar{q}), \quad \dot{q} = \tfrac{1}{2}\mathrm{i}q\left(\omega(s) - \omega(-s\bar{q})\right),$$

with w, ω the complex-velocity and vorticity, respectively, of the flows generated by the swimmers [1].

RESULTS

We concentrate on the Impact Parameter, $b(t)$, defined as the distance of closest approach between the swimmers were the instantaneous velocities to be maintained (see Fig. 1b). Obviously if there is no interaction between the swimmers, b should remain constant. Given that the flows decay with distance from the swimmers, this might be expected to be the case for sufficiently distant swimmers. However, as illustrated in Fig. 2(a) which plots the shift in Impact Parameter $\Delta b(t) = b(t) - b(0)$ for two different long-range interactions, this is not the case. Instead, the shift tends to the same limiting value irrespective of the (large) initial value of b. This trend is further evident in the exploration of parameter space (Fig. 2(b)), where it can be seen that the limit $\Delta b(t \to \infty)$ is reached for relatively small values of $b(0)$, and depends upon $\theta(0)$. In fact we are able to deduce asymptotically that for these interactions (see Fig. 2(c)):

$$\Delta b \to \pm 2\pi \Re\left(q(0)/|1 - q(0)|\right) \text{ as } b(0) \to \mp\infty. \tag{1}$$

CONCLUSIONS

We show that the interaction between two prototype swimming micro-organisms can be captured by a simple dynamical system which depends solely upon the relative displacement and orientations between the swimmers. Moreover, we demonstrate that interactions between swimmers do not decay away at increasingly large distances (in spite of the decaying flow), but rather tend towards some limiting form which can be predicted by asymptotic analysis. It is envisaged such effects could have important implications for the collective dynamics of such swimmers, an issue which we intend to pursue further.

References

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