FLOQUET STABILITY ANALYSIS OF HOVERING MODEL INSECTS

Mao Sun [a)] & Jiang Hao Wu
Institute of Fluid Mechanics, Beijing University of Aeronautics and Astronautics, Beijing, China

Summary Because of the periodically varying aerodynamic and inertial forces of the flapping wings, a hovering insect is a cyclically forcing system, and generally the flight is not in a fixed-point equilibrium, but in a cyclic-motion equilibrium. Current stability theory of insect flight is based on the averaged model and treats the flight as a fixed-point equilibrium. The present study treated the flight as a cyclic-motion equilibrium and used the Floquet theory to analyze the longitudinal stability of insect hovering.

INTRODUCTION

There has been some work on flight dynamic stability of insects (e.g. [1, 2]). In these works, an averaged model borrowed directly from the literature of aircraft flight dynamics was used. Because of the periodically varying aerodynamic and inertial forces of the flapping wings, a flying insect is a cyclically forcing system, and generally it is not in a fixed-point equilibrium. But in the averaged model used in the previous studies, the wing forces were time-averaged over wingbeat cycle, the equilibrium flight was represented by a fixed-point equilibrium. In the present study, we will analyze the stability of the periodic solution that represents the "equilibrium flight", i.e. the cyclic-motion stability, using the Floquet theory. We consider the longitudinal dynamic stability of hover flight in two representative model insects, a model dronefly and a model hawkmoth; the model dronefly may represent insects with small amplitude of body oscillation and the model hawkmoth may represent insects with relatively large amplitude of body oscillation.

METHODS

The periodic solution of the equilibrium flight is obtained by numerically solving the equations of motion coupled with the Navier-Stokes; the shooting method is used to obtain the periodic solution [3]. When the equilibrium flight is disturbed, we express the state variable vector as the sum of the equilibrium-flight value and the disturbance value, and also express the aerodynamic and wing-inertial forces as the sum of the equilibrium-flight value and the disturbance value. We further use Taylor's theorem for analytical functions to express each of the disturbance values of the aerodynamic and wing-inertial forces as an infinite series about the reference condition. Substituting these expressions into the equations of motion and neglecting the second and higher order terms give:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}(t)\mathbf{x}(t) \tag{1}$$

where $\mathbf{x}(t)$ is the disturbance value of the state variable vector and $\mathbf{A}(t)$ is called as system matrix. The elements of $\mathbf{A}(t)$ are periodic functions of time with period 1 (time is non-dimensionalized by wingbeat period) and are determined by the aerodynamic derivatives and the mass of wings and body of the insect. Using the Floquet theory, the general solution of Eq. 1 can be written in the following form [4]

$$\mathbf{x}(t) = \sum_{j=1}^{4} a_j \mathbf{p}_j(t) e^{\rho_j t} \tag{2}$$

where a_j is a constant, $\mathbf{p}_j(t)$ are periodic vector functions with period 1 and ρ_j are the characteristic exponentsof the system (j=1, 2, 3 ,4). From Eq. 1, it is clear that when the real part of each of ρ_j is negative, $\mathbf{x}(t)$ will die out as time increasing and the flight is stable, otherwise the flight is unstable; that is, the flight stability is dependent on ρ_j. Aerodynamic force derivatives needed in matrix $\mathbf{A}(t)$ are computed using CFD method [5] and wing-inertial force derivatives are computed the known wing kinematic and morphological data.

For comparison, analysis using the averaged-model theory (fixed-point stability analysis) was also made. Results of both the cyclic-motion stability analysis and fixed-point stability analysis were tested by numerical simulation using complete equations of motion coupled with the Navier-Stokes equations.

RESULTS AND DISCUSSION

The characteristic exponents, ρ_j (j=1, 2, 3, 4) are shown in Table 1. For each of the two model insects, there is a pair of complex characteristic exponents ($\rho_{1,2}$) with positive real part and two negative real characteristic exponents (ρ_3 and ρ_4). For both model insects, because ρ_3 and ρ_4 are negative, the second and third terms on the right-hand-side of Eq. 21 die out as time increasing; but because the real part of $\rho_{1,2}$ is positive the first and second terms on the right-hand-side of Eq. 1 will grow with time. Thus the periodic solution, or the hover flight, is unstable.

a) Corresponding author. Email: m.sun@buaa.edu.cn.

Let us compare the results of Floquet theory (cyclic-motion stability analysis) with those of the averaged-model theory (fixed-point stability analysis). The system matrix in the averaged-model theory is a constant matrix and techniques of eigenvalue and eigenvector analysis can be used to solve the equation. The eigenvalues for the two hovering model insects are included in Table 1 (numbers in parentheses). For each of the insects, there are a pair of complex eigenvalues ($\lambda_{1,2}$) with positive real part and two negative real eigenvalues (λ_3 and λ_4). Comparing the results of the Floquet theory in the previous section with those of the averaged-model theory, the following observations can be made. For each of the two model insects, both theories predict the same unstable and stable natural modes of motion. For the model dronefly, the growth rate of the unstable mode and the decay rates of the stable modes given by the Floquet theory are very close to their counterparts of the averaged-model theory (the real part of $\rho_{1,2}$ =0.0476, ρ_3=-0.115 and ρ_4=-0.0144 in the Floquet theory; the real part of $\lambda_{1,2}$ =0.0473, λ_3=-0.114 and λ_4=-0.0147 in the averaged-model theory). But for the model hawkmoth, the growth and decay rates given by the Floquet theory are significantly different from their counterparts of the averaged-model theory: for Floquet theory, the real part of $\rho_{1,2}$=0.199, ρ_3=-0.747 and ρ_4=-0.103, whilst for averaged-model theory, the real part of $\lambda_{1,2}$=0.253, λ_3=-0.715 and λ_4=-0.094. The real part of the root of the unstable mode given by the averaged-model theory (0.253) is about 25% higher than that by the Floquet theory (0.199).

Table 1 Characteristic exponents in Floquet theory; DF, model dronefly; HM, model hawkmoth

	$\rho_{1,2}$ ($\lambda_{1,2}$)	ρ_3 (λ_3)	ρ_4 (λ_4)
Dronefly	0.0476±0.0961i (0.0473±0.0940i)	-0.115 (-0.114)	-0.0144 (-.0147)
Hawkmoth	0.199±0.562i (0.253±0.588i)	-0.747 (-0.715)	-0.103 (-0.0941)

Results of both the Floquet theory and averaged-model theory have been tested by numerical simulation using complete equations of motion coupled with the Navier-Stokes equations. For the model dronefly, both the Floquet theory and the averaged-model theory are in good agreement with the numerical simulation. For the model hawkmoth, the Floquet theory is in good agreement with the numerical simulation, whilst the averaged-model theory has pronounced differences with the simulation.

CONCLUSIONS

Because of the periodically varying aerodynamic and inertial forces of the flapping wings, a hovering insect is a cyclically forcing system, and generally the flight is not in a fixed-point equilibrium, but in a cyclic-motion equilibrium. Current stability theory of insect flight is based on the averaged model and treats the flight as a fixed-point equilibrium. In the present study, we treated the flight as a cyclic-motion equilibrium and used the Floquet theory to analyze the longitudinal stability of insect flight. Two hovering model insects were considered, a dronefly and a hawkmoth. The former has relatively high wingbeat frequency and small wing-mass to body-mass ratio, and hence very small amplitude of body oscillation; whilst the latter has relatively low wingbeat frequency and large wing-mass to body-mass ratio, and hence relatively large amplitude of body oscillation. For comparison, analysis using the averaged-model theory (fixed-point stability analysis) was also made. Results of both the cyclic-motion stability analysis and fixed-point stability analysis were tested by numerical simulation using complete equations of motion coupled with the Navier-Stokes equations. The Floquet theory (cyclic-motion stability analysis) agreed well with the simulation for both the model dronefly and model hawkmoth; but the averaged-model theory gave good results only for the dronefly. For an insect with relatively large body oscillation at wingbeat frequency, cyclic-motion stability analysis is required, and for their control analysis, existing well-developed control theories for systems of fixed-point equilibrium are no longer applicable and new methods which take the cyclic variation of the flight dynamics into account are needed.

References

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