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Low-Reynolds-Number Instability of the Laminar Flow in the Wavy Channel

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Enhancement of mixing in the laminar regime is of fundamental importance in numerous applications in microfluidics, biotechnology, medicine and heat transfer. Significant improvement of mass and/or heat transfer can be achieved by the chaotic mixing which might appear when the flow field contains sufficiently complex and time-dependent vortex structures. Such structures can be triggered by various geometric modifications (like wall waviness or the applications of surface-mounted obstacles), external forcing (like an oscillations of a driving pressure gradient) or the combination of both. Unfortunately, in most cases the mixing improvement is accompanied by large increase of hydraulic resistance.

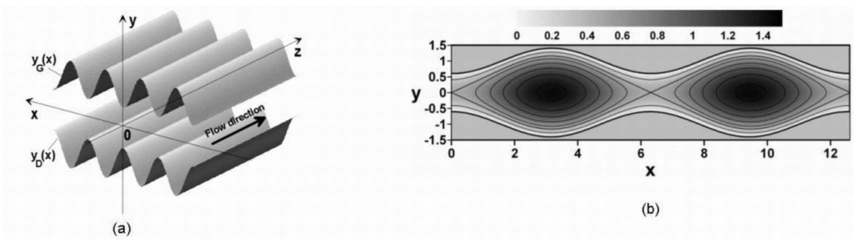

Figure 1: (a) Channel with symmetric transversal wall waviness. The basic flow is directed along the 0z axis. The wall geometry is spanwise-periodic. (b) The velocity distribution of the basic flow. The amplitude of sinusoidal wall waviness is S=0.4 which is 20% of the average vertical wall distance.

In the current study, being the extension of the results published by the 1^{st} author in [1, 2], an instability of a viscous incompressible flow in a channel with wavy walls is investigated theoretically and numerically. The wall waviness is unidirectional and - in contrast to the majority of earlier studies (like in [3, 4]) - it is oriented transversely, i.e., the lines of constant elevation are parallel to the driving pressure gradient (Fig.1a). In such configuration, the basic (undisturbed) flow is particularly simple: the velocity field contains only one nonzero streamwise component (Fig. 1b). It has been shown that appropriately chosen wall waviness leads to destabilization at surprisingly low Reynolds numbers (Fig.2a). The linear stability analysis shows that the critical Reynolds number Re_{cr} can be reduced even down to 58, i.e., by two orders of magnitude when compared to the Poiseuille flow between flat parallel planes ($\mathrm{Re}_{cr} = 5772$). The unstable mode of disturbances (which can be interpreted as the fundamental transversal Squire mode) has the form of a vortex array, which travels downstream (Fig.2b). The remarkable feature is that the most destabilizing waviness does not introduce any additional flow resistance. The results of the stability analysis are consistent with the result of direct numerical simulation performed with FLUENT package (ANSYS Inc.).

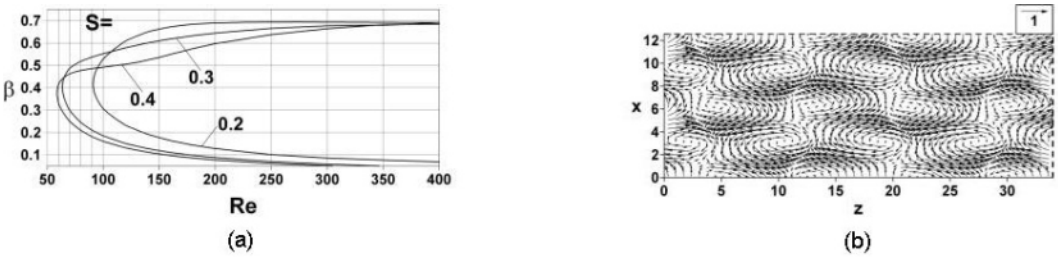

Figure 2: (a) The lines of neutral stability in the β (streamwise wave number) and Re (the Reynolds number) plane. The curves for three different amplitudes S are shown. The geometric period of wall waviness is equal 2π. The critical Reynolds number for S=0.4 is lower than 60. (b) The velocity field of the unstable mode of disturbances, plotted in the channels center plane y=0. The presented pattern of flow travels downstream with the velocity slightly smaller than 1.

In practical situations it might be difficult to maintain ideally transversal orientation of the pressure gradient with respect to the wall waviness. Therefore, the influence of small deflections from the nominal configuration on the flow stability has been investigated. Such case is much more involving - both mathematically and computationally - since the basic flow is now three-dimensional and the stability equations are more complicated. It has been shown that the increasing departure from strictly transversal orientation of the wall waviness leads to rapid reduction of the spanwise modulation of the streamwise component of the velocity of the mean flow which is accompanied by a quick growth of the critical Reynolds number (Fig.3). Still, the width of the "window" of the deflection angle where the critical Reynolds number is sufficiently small (say, lower than 100) is about 3 degrees (for each side), i.e. it is large enough to be of some practical significance.

The ongoing research includes the investigation of the effect of the finite span (the presence of the channel's sidewalls) on the flow stability as well as experimental verification of the theoretical/numerical predictions.

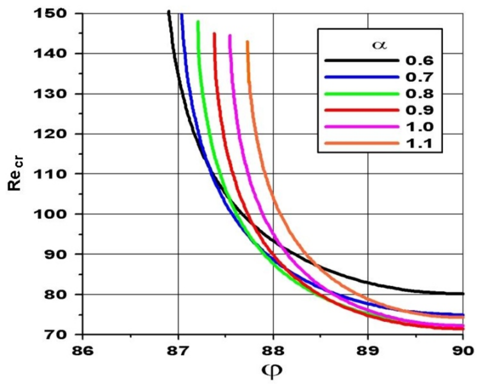

Figure 3: Departure from strictly transversal orientation of the wall waviness results in rapid increase of the critical Reynolds number; the destabilization effect dissappears when the deflection angle exceeds the value of 3 degrees. Beyond this range, the fundamental Squire mode is stable even for very large Reynolds numbers.

Acknowledgments
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