FLOW DESTABILIZATION AND CHAOTIC MIXING IN THE CHANNEL WITH TRANSVERSELY CORRUGATED WALLS

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Summary Problem of mixing enhancement a viscous incompressible flow in a channel with wavy walls is investigated theoretically and numerically. The wall waviness is unidirectional and it is oriented spanwise, i.e., the lines of constant elevation are parallel to the driving pressure gradient. It is shown that appropriately chosen wall waviness leads to destabilization at surprisingly low Reynolds numbers. The linear stability analysis shows that the critical Reynolds number Re_{cr} can be reduced even under 60, i.e., by two orders of magnitude when compared to the Poiseuille flow between flat parallel planes. The unstable mode of disturbances has the form of a vortex array, which travels downstream. The remarkable feature is that the most destabilizing waviness does not introduce any additional flow resistance. The results of the stability analysis are consistent with the result of direct numerical simulation performed using the finite volume CFD package Fluent.

Problem of mixing enhancement plays the major role in the processes of mass and heat transfer, particularly in the laminar regime, which is typical for small-scale flows. An essential condition imposed on to the Lab-On-Chip microsystems [1] is to get chemical and/or biological components mixed together within very small distances – even lower that one millimeter. However, the achievement of highly effective mixing and transport in the micro-scale flow is far from trivial due to the lack of turbulence [2]. It is then mandatory to create an environment for chaotic advection, i.e. to obtain the flow field characterized by the presence of a sufficiently complex and time-dependent vortex structure. Such structure can be evoked by various geometric modifications (like wall waviness or the applications of surface-mounted obstacles) and/or an external forcing (e.g. oscillations of a driving pressure gradient). Unfortunately, these manipulations are often accompanied by a large increase of hydraulic resistance.

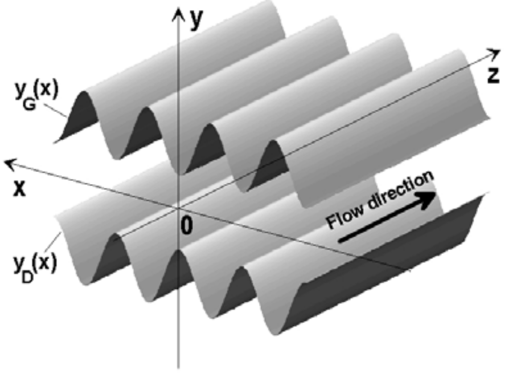

Figure 1. Channel with symmetric transversal wall waviness. The basic flow is directed along the 0z axis. The wall geometry is spanwise-periodic.

The current work describes the mixing-enhancement method based on the idea of forced chaotic convection in the channel with appropriately shaped and transversely oriented wall waviness (Fig.1). Linear stability analysis [3,4] of the simple, unidirectional flow in such domain (Fig.2) has shown that this flow can spontaneously loose stability at the Reynolds numbers as low as 60 (Fig.3), providing that the geometric period of the wall waviness is properly chosen.

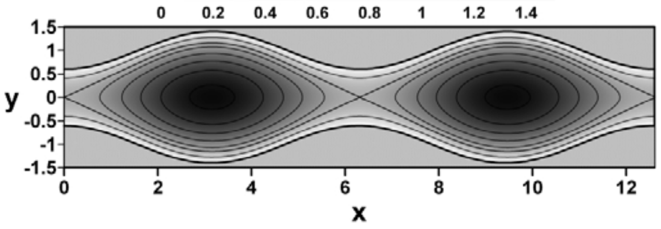

Figure 2. The velocity distribution of the basic flow. The amplitude of sinusoidal wall waviness is S=0.4 which is 20% of the average vertical wall distance.

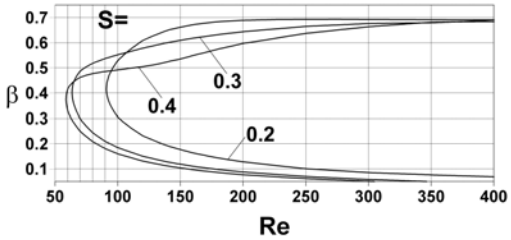

Figure 3. The lines of neutral stability in the β (streamwise wave number) and Re (the Reynolds number) plane. The curves for three different amplitudes S are shown. The geometric period of wall waviness is equal 2π. The critical Reynolds number for S=0.4 is lower than 60.

Moreover, in contrast to most alternative geometric modification leading to comparable effects, the flow resistance in the unstable range remains essentially the same as in the case of the Poiseuille flow, i.e. the flow in the channel with flat walls. The unstable mode of disturbances has the form of three-dimensional vortex array (Fig.4), which travels downstream with the speed similar to the average velocity of the fluid in the channel's center plane.

The destabilizing effect has been found rather sensitive to the spatial orientation of the wall waviness: deviation by just several degrees from exactly transversal orientation basically removes the effect. The process of the nonlinear "saturation " of the unstable mode of disturbances predicted by the linear theory has been also investigated by means of the direct numerical simulations using the finite volume CFD package Fluent (Ansys Inc.). The effect of the Reynolds number on the flow structure was analyzed for the infinite wavy channel assuming periodic boundary conditions in streamwise and spanwise directions. The computations have shown that beyond the stability threshold the flow in the wavy channel evolves into a complex, fully three-dimensional and oscillatory form. It was confirmed that unstable modes a present already for the Reynolds number equal 100. The mixing capabilities of this flow can be assessed by the numerical simulation of spreading of an initially concentrated cloud of markers; the sample trajectories of such "passive scalar" injections are depicted in (Fig.5). Divergence of the trajectories of initially close markers is evident. It seems that further increase of the flow complexity could be achieved by means of additional geometric modifications of the channel boundaries. The results already obtained as well as those of the ongoing research (concerning mostly the influence of the finite span, i.e. the presence of the side walls) will be useful in the design of the experimental setup and during further development of the prototype of a new micro-mixer.

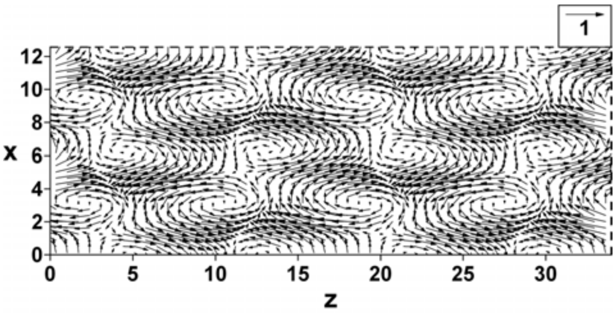

Figure 4. The velocity field of the unstable mode of disturbances, plotted in the channel's center plane y=0. The presented pattern of flow travels downstream with the velocity slightly smaller than 1.

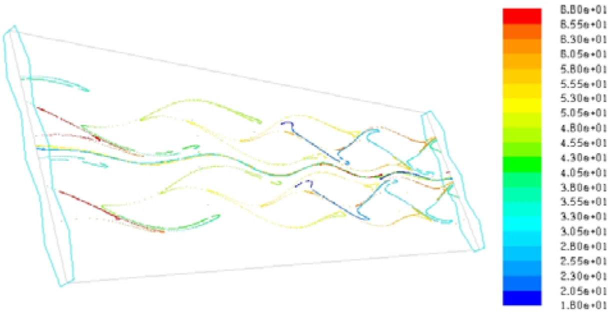

Figure 5. Numerical simulation of the markers' trajectories: particle traces colored by particle residence time.

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