Clusters of particles falling in a viscous fluid with periodic boundary conditions

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Exemplary dynamics of three point particles falling under gravity in Stokes flow with periodic boundary conditions is presented in a movie. Stable and unstable solutions of the equations of Stokesian dynamics are explicitly shown for two initial configurations: equilateral triangles with side lengths close to a critical size. © *2006 American Institute of Physics*.
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I. INTRODUCTION

Sedimentation of non-Brownian suspensions has been extensively studied recently.[1] Velocity distribution of the suspended particles has been measured experimentally. For sphere suspensions, swirl-like structures have been observed.[2] For a dilute fiber suspension, large-scale fast streamers have been detected.[3] In both types of systems, fluctuations of the particle velocities are large and anisotropic. There is no theoretical explanation of these phenomena. However, it is clear that they should result from an interplay between hydrodynamic interactions of individual close particles, and collective long-distance many-particle effects. It is known that in infinite viscous fluid, clustering of particles is often accompanied by their recirculation.[4,5] In this work, a movie is shown which illustrates how collective effects can generate recirculation, oscillations, and clustering of particles.

II. THE MODEL

A simple model of a sedimenting suspension is used: a viscous fluid with periodic boundary conditions, and a small cluster of $j=1,\ldots,N$ point forces in each cubic cell. Each cell has a unit length $L=1$, and is labeled by $\boldsymbol{n}=(k,l,m)$, with the integers k,l,m. The fluid velocity and pressure satisfy the Stokes equations. The corresponding periodic Green function is the Hasimoto tensor,

$$\mathsf{T}_H(\boldsymbol{r}) = \frac{1}{4\pi^2\eta}\sum_{\boldsymbol{n}}{}' \frac{\mathbf{1}-\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}}{|\boldsymbol{n}|^2}\exp[2\pi i\boldsymbol{n}\cdot\boldsymbol{r}], \quad (1)$$

where $\hat{\boldsymbol{n}}=\boldsymbol{n}/|\boldsymbol{n}|$ and the prime on the summation sign indicates that the term $\boldsymbol{n}=\mathbf{0}$ is omitted. $\mathsf{T}_H(\boldsymbol{r})$ is calculated numerically, using the efficient scheme from Ref. 6. The point forces, located at $\boldsymbol{R}_j+\boldsymbol{n}$, are antiparallel to the z axis, $\boldsymbol{K}=-K\boldsymbol{e}_z$. Their dynamics reads as

$$\frac{d\boldsymbol{R}_j}{dt} = -K\sum_{k\neq j}^{N}\mathsf{T}_H(\boldsymbol{R}_j-\boldsymbol{R}_k)\cdot\boldsymbol{e}_z, \quad (j=1,\ldots,N). \quad (2)$$

One or two point particles in a periodic cell always fall steadily with no change of relative position. Three point particles in general undergo irregular relative motion. But if they initially form an equilateral horizontal triangle with a base parallel to an axis of the cubic lattice, then later the triangle changes into isosceles with the parallel base. Each particle oscillates with the same period T while settling.[7] For small side length d, the oscillations are neutrally stable, with the same time-averaged settling velocities of all three particles. A critical size $d_0=0.4242\ldots$ exists with an infinite period of oscillations. For larger d, three oscillating particles split into a pair and a singlet, which settle with two different time-averaged velocities and their motion is unstable, after one or several periods becomes irregular.[7]

These two types of periodic motions are animated in Fig. 1; four periodic cells are visible and the particle recirculating trajectories are also shown. The initial sizes $d=0.42$ and $d=0.43$ are close to the critical d_0 from both sides. The frame of reference is moving with the time-averaged velocity of the base (dark gray and black; red and blue online) particles, to trace their pure oscillatory motion. Senses of the motion along the two corresponding trajectories, with $d=0.42$ and $d=0.43$, are the same for the apex (light gray; green online) particle, and opposite for each of the base (dark gray and black; red or blue) particles. The difference may be understood taking into account that for $d=0.43$, a base particle comes close to, and "teams up" with, the other base particle from a *neighboring* periodic cell (rather than from *the same* periodic cell as for $d=0.42$). In both cases, when the two coupled particles come closest to each other, vertical velocities of their oscillatory motion are oriented along gravity and are of the largest value. For $d=0.43$, the apex particle slowly drags the base ones up, and they seem to form a single cluster, until the twins suddenly turn around, leave their companion alone, and join another apex partner.

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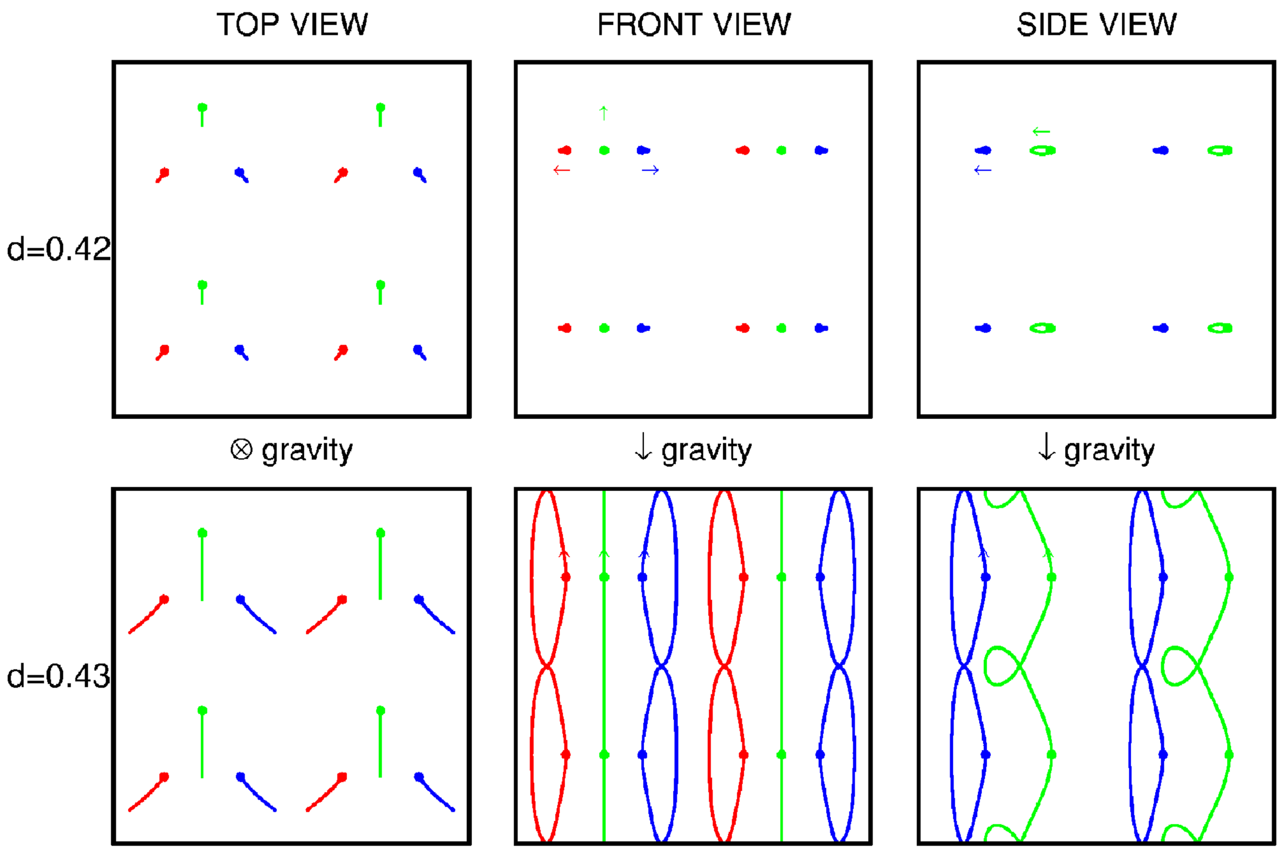

FIG. 1. (Color online) Motion of three point particles in periodic cubic boundary conditions, initially at a horizontal equilateral triangle of side length *d*, with a base parallel to an axis of the periodic cell. The frame is moving with the time-averaged velocity of the base (dark gray and black; red and blue online) particles (enhanced online).

Significant anisotropy of velocity fluctuations (averaged over particles and over the period) is shown in Fig. 2. The horizontal fluctuations $\sigma_\perp$ dominate the vertical ones $\sigma_{||}$, if clusters are smaller than the critical size, and the opposite otherwise. This difference is related to a change from one to two settling velocities, and from oblate to prolate shapes of the particle recirculating trajectories (see Fig. 1 and Ref. 7).

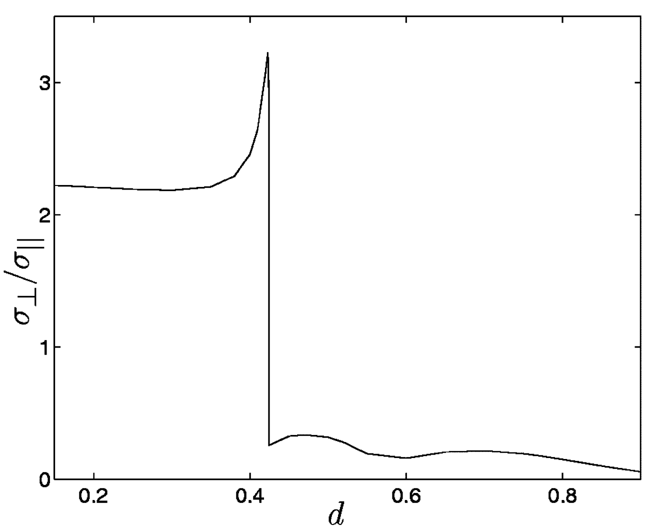

FIG. 2. Ratio of horizontal and vertical velocity fluctuations $\sigma_\perp / \sigma_{||}$ versus initial side length *d*.

III. CONCLUSIONS

In summary, two sediment systems have been shown. In the first one recirculating particles form clusters, which settle with *the same* time-averaged velocities. This example illustrates that regions of different density, and different mean velocity, are not necessary for the existence of “swirls.” The second system consists of recirculating unstable columns passing each other with *different* time-averaged velocities. This example shows that not only fibers, but also sediment-spheres may form “streamers” for certain many-particle distributions.

[1]S. Ramaswamy, “Issues in the statistical mechanics of steady sedimentation,” Adv. Phys. **50**, 297 (2001).
[2]P. N. Segrè, E. Herbolzheimer, and P. M. Chaikin, “Long-range correlations in sedimentation,” Phys. Rev. Lett. **79**, 2574 (1997).
[3]B. Metzger, É. Guazzelli, and J. Butler, “Large-scale streamers in the sedimentation of a dilute fiber suspension,” Phys. Rev. Lett. **95**, 164506 (2005).
[4]L. M. Hocking, “The behaviour of clusters of spheres falling in a viscous fluid. Part 2. Slow motion theory,” J. Fluid Mech. **20**, 129 (1964).
[5]J. M. Nitsche and G. K. Batchelor, “Break-up of a falling drop containing dispersed particles,” J. Fluid Mech. **340**, 161 (1997).
[6]B. Cichocki and B. U. Felderhof, “Periodic fundamental solution of the linear Navier-Stokes equations,” Physica A **159**, 19 (1989).
[7]M. L. Ekiel-Jeżewska and B. U. Felderhof, “Periodic sedimentation of three particles in periodic boundary conditions,” Phys. Fluids **17**, 093102 (2005).