Clusters of particles falling in a viscous fluid with periodic boundary conditions

M. L. Ekiel-jeżewska\textsuperscript{a)}
Institute of Fundamental Technological Research, Polish Academy of Sciences, Świętokrzyska 21, 00-049 Warsaw, Poland

B. U. Felderhoff\textsuperscript{b)}
Institut für Theoretische Physik A, Rheinisch-Westfälische Technische Hochschule Aachen, Templergraben 55, 52056 Aachen, Germany

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Exemplary dynamics of three point particles falling under gravity in Stokes flow with periodic boundary conditions is presented in a movie. Stable and unstable solutions of the equations of Stokesian dynamics are explicitly shown for two initial configurations: equilateral triangles with side lengths close to a critical size. © 2006 American Institute of Physics.

\[ \frac{dR_j}{dt} = -K \sum_{k \neq j} T_{jk}(R_j - R_k) \cdot e_z, \quad (j = 1, \ldots, N). \]  

One or two point particles in a periodic cell always fall steadily with no change of relative position. Three point particles in general undergo irregular relative motion. But if they initially form an equilateral horizontal triangle with a base parallel to an axis of the cubic lattice, then later the triangle changes into isosceles with the parallel base. Each particle oscillates with the same period $T$ while settling.\textsuperscript{7} For small side length $d$, the oscillations are neutrally stable, with the same time-averaged settling velocities of all three particles. A critical size $d_0 = 0.4242\ldots$ exists with an infinite period of oscillations. For larger $d$, three oscillating particles split into a pair and a singlet, which settle with two different time-averaged velocities and their motion is unstable, after one or several periods becomes irregular.\textsuperscript{7}

These two types of periodic motions are animated in Fig. 1; four periodic cells are visible and the particle recirculating trajectories are also shown. The initial sizes $d = 0.42$ and $d = 0.43$ are close to the critical $d_0$ from both sides. The frame of reference is moving with the time-averaged velocity of the base (dark gray and black; red and blue online) particles, to trace their pure oscillatory motion. Senses of the motion along the two corresponding trajectories, with $d = 0.42$ and $d = 0.43$, are the same for the apex (light gray; green online) particle, and opposite for each of the base (dark gray and black; red or blue) particles. The difference may be understood taking into account that for $d = 0.43$, a base particle comes close to, and “teams up” with, the other base particle from a neighboring periodic cell (rather than from the same periodic cell as for $d = 0.42$). In both cases, when the two coupled particles come closest to each other, vertical velocities of their oscillatory motion are oriented along gravity and are of the largest value. For $d = 0.43$, the apex particle slowly drags the base ones up, and they seem to form a single cluster, until the twins suddenly turn around, leave their companion alone, and join another apex partner.

\textsuperscript{a)}Electronic mail: mekiel@ippt.gov.pl

\textsuperscript{b)}Electronic mail: ufelder@physik.rwth-aachen.de

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Significant anisotropy of velocity fluctuations (averaged over particles and over the period) is shown in Fig. 2. The horizontal fluctuations $\sigma_{\parallel}$ dominate the vertical ones $\sigma_{\perp}$, if clusters are smaller than the critical size, and the opposite otherwise. This difference is related to a change from one to two settling velocities, and from oblate to prolate shapes of the particle recirculating trajectories (see Fig. 1 and Ref. 7).

III. CONCLUSIONS

In summary, two sediment systems have been shown. In the first one recirculating particles form clusters, which settle with the same time-averaged velocities. This example illustrates that regions of different density, and different mean velocity, are not necessary for the existence of “swirls.” The second system consists of recirculating unstable columns passing each other with different time-averaged velocities. This example shows that not only fibers, but also sediment-spheres may form “streamers” for certain many-particle distributions.