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Structural damage identification by adding virtual masses

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Abstract This paper presents a method for damage identification by adding virtual masses to the structure in order to increase its sensitivity to local damages. The main concept is based on the Virtual Distortion Method (VDM), which is a fast structural reanalysis method that employs virtual distortions or pseudo loads to simulate structural modifications. In this paper, the structure with an added virtual mass is called the virtual structure. First, the acceleration frequency response of the virtual structure is constructed numerically by the VDM using local dynamic data measured only by a single excitation sensor and a single acceleration sensor. Second, the value of the additional mass is determined via sensitivity analysis of the constructed frequency responses of the virtual structure with respect to damage parameters; only the natural frequencies with high sensitivity are selected. This process is repeated for all the considered placements of the virtual mass. At last, the selected natural frequencies of all the virtual structures are used together for damage identification of the real structure. A finite element (FE) model of a plane frame is used to introduce and verify the proposed method. The damage can be identified precisely and effectively even under simulated 5% Gaussian noise pollution.

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1 Introduction

In structural health monitoring (SHM) (Chang et al 2003), structural damage identification (Fan and Qiao 2011) plays an important role in maintaining integrity and safety of structures. In the recent decades, it has become an extensively researched field in civil engineering. Even if many effective methods for damage identification have been developed, monitoring of large and complex structures is still a challenge due to their complexity, limitation of instrumentation and response insensitivity to local damage.

The process of damage identification includes damage detection, localization and estimation of its extent. Signal processing methods, such as spectral methods (El-Shafie et al 2012), wavelet analysis (Kim and Melhem 2003) or Hilbert-Huang transform (Tang et al 2011), can detect the damage directly and quickly using the measured response without a parametric structural model. Although these methods can catch the moment when the damage occurred and its location, they usually cannot estimate the damage extent accurately. The extents of the damage are usually optimized via the finite element (FE) model using certain dynamic characteristics of the structure, such as flexibility matrix (Duan et al 2005), natural frequencies and mode shapes (Hassiotis 2000; Lin and Ewins 1990), responses in time or frequency domain (Zhang et al 2012), etc. Among dynamic structural properties, natural frequencies reflect the most basic dynamic characteristics of the structure, and they can be identified easily, accurately and reliably by several well-known modal analysis methods, such as ERA (Eigensystem

Realization Algorithm) or SSI (Stochastic Subspace Identification) method. Therefore, the methods based on modal information are widely used for damage identification. For example, Yun and Bahng (2000) monitor local stiffness modifications using natural frequencies and mode shapes. Jaishi and Ren (2007) update the parameters of the finite element model based on eigenvalue and strain energy residuals using an multiobjective optimization technique. An and Ou (2011) utilize local modes to detect local damages of a truss structure.

In practice, it is often not easy to obtain accurate higher natural frequencies from measured responses, and so only lower natural frequencies are usually estimated. For complex and large structures in civil engineering, the number of unknown damage-related parameters is large, much larger than the number of estimable natural frequencies. Therefore, the potential of damage identification methods based on natural frequencies is limited unless the information they provide can be complemented. There is a general family of approaches that aim at this problem by applying structural modifications to the monitored structure in the purpose of obtaining more information from the structures constructed this way. Nalitolela et al (1992) present a method of model updating by adding physical masses or stiffeners on the structure and utilizing modal information of the perturbed structures. This method is further improved by Nalitolela et al (1993), which adding imagined stiffness in a selected structural degree of freedom (Dof). Cha and de Pillis (2001) add known physical masses to the structure and then identify damage using the orthogonal conditions of the system eigenvalue problem. Dinh et al (2012) improve this method using the state-space transformation of the eigenvalue problem, which is applicable to non-proportionally damped structures.

Increasing the sensitivity of a structure by adding mass is a very appealing concept. However, in real application it is not always possible to fix perfectly a physical mass to the structure in a proper position and of proper weight. Therefore, this paper proposes a method that performs a virtual modification of the structure and adds a virtual mass instead of a physical mass. The general methodology of the Virtual Distortion Method (VDM) is used (Kołakowski et al 2008). The VDM belongs to fast structural reanalysis methods (Akgün et al 2001) and employs virtual distortions and pseudo loads to model various structural modifications. The response of the modified structure is constructed at a low cost using a limited set of locally measured responses of the original structure, without solving from scratch the modified equation of motion. Such a methodology is versatile and the VDM has been successfully used for modeling and/or identification of such structural characteristics as stiffness (Swiercz et al 2008), mass (Suwała and Jankowski 2012), moving mass (Zhang et al 2010; Bajer and Dyniewicz 2009), damping (Mróz et al 2009), fixed and free boundary conditions (Hou et al 2012) or prestressing (Holnicki-Szulc and Haftka 1992).

This paper employs the VDM to construct the frequency response function (FRF) of a real structure with a virtual mass added in a selected Dof (called the virtual structure). Such an approach has three main advantages: (1) the FRF of the virtual structure is constructed using only a single acceleration response and a single corresponding excitation; (2) the virtual mass can be added in an arbitrary Dof of the structure and with an arbitrary weight; (3) the sensitivity of natural frequencies to damage can be increased by adding virtual masses to the involved elements, so that the virtual structures constructed this way provide together enough dynamic information for effective and unique identification.

This paper is structured as follows. The next two sections derive the proposed approach. The process of constructing the FRF of a virtual structure is first deduced based on the VDM. Several virtual structures are then simultaneously constructed with virtual masses added in different locations. Sensitivity analysis of their natural frequencies is performed in order to select the masses, their locations and the natural frequencies with high sensitivities with respect to all the damage parameters. Finally, damage identification procedure is proposed that combines all the virtual structures. In the fourth section, a numerical example of a plane frame is utilized to illustrate and test the proposed method. Its effectiveness is demonstrated at a simulated Gaussian measurement noise level of 5% rms.

2 Virtual structure and its FRF

This section derives the FRF of a given real structure with a virtual mass added. The general case is derived and simplified to describe the case in which only a single degree of freedom (Dof) is affected, measured and excited. A simple formula is then used for approximation of the natural frequencies of the virtual structure in dependence of the added mass.

2.1 General case

Assume the acceleration response of a real structure with n_{d} Dofs is measured in n_{a} points and let the structure be excited by n_{f} loading forces. Denote by $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ respectively the mass, damping and stiffness matrix. In frequency domain, the equilibrium equation can be written as:

$$\begin{cases} \left[-\omega^2\mathbf{M}+\mathrm{j}\omega\mathbf{C}+\mathbf{K}\right]\mathbf{X}(\omega)=\mathbf{BF}(\omega), \\ \mathbf{A}(\omega)=\mathbf{D}\ddot{\mathbf{X}}(\omega), \end{cases} \tag{1}$$

where $\mathbf{B}$ is the load allocation matrix with the dimensions $n_{\mathrm{d}} \times n_{\mathrm{f}}$, $\mathbf{D}$ is the observation matrix with the dimensions

$n_a \times n_d$, the vector $\mathbf{F}(\omega)$ collects the spectra of the excitation forces and the vector $\mathbf{A}(\omega)$ contains the spectra of measured accelerations.

The frequency response function $\ddot{\mathbf{h}}(\omega)\colon \mathbf{F}(\omega) \mapsto \mathbf{A}(\omega)$ of the n_a acceleration responses of the real structure with respect to the n_f excitation forces, can be thus expressed as

$$\ddot{\mathbf{h}}(\omega) = \mathbf{D}\ddot{\mathbf{H}}(\omega)\mathbf{B}, \tag{2}$$

where $\ddot{\mathbf{H}}(\omega) = -\omega^2\mathbf{H}(\omega)$ is the acceleration frequency response matrix and $\mathbf{H}(\omega)$ is the frequency response matrix of the real structure with the dimensions $n_d \times n_d$,

$$\mathbf{H}(\omega) = \left[-\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{C} + \mathbf{K}\right]^{-1}. \tag{3}$$

Consider a modification $\Delta\mathbf{M}$ of the mass matrix. In frequency domain, the equilibrium equation of the modified structure is

$$\left[-\omega^2(\mathbf{M} + \Delta\mathbf{M}) + \mathrm{j}\omega\mathbf{C} + \mathbf{K}\right]\mathbf{X}^{\mathrm{V}}(\omega, \Delta\mathbf{M}) = \mathbf{B}\mathbf{F}(\omega), \tag{4}$$

where $\mathbf{X}^{\mathrm{V}}(\omega, \Delta\mathbf{M})$ is the response of the modified structure. The VDM moves the modification term to the right-hand side of the equation. Equation (4) is then expressed in the equivalent form:

$$\left[-\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{C} + \mathbf{K}\right]\mathbf{X}^{\mathrm{V}}(\omega, \Delta\mathbf{M}) = \mathbf{B}\mathbf{F}(\omega) + \mathbf{P}(\omega, \Delta\mathbf{M}), \tag{5}$$

which is the equilibrium equation of the real structure subjected additionally besides $\mathbf{F}(\omega)$ to the pseudo load vector $\mathbf{P}(\omega, \Delta\mathbf{M})$ that models the added mass,

$$\mathbf{P}(\omega, \Delta\mathbf{M}) = -\Delta\mathbf{M}\ddot{\mathbf{X}}^{\mathrm{V}}(\omega, \Delta\mathbf{M}), \tag{6}$$

where $\ddot{\mathbf{X}}^{\mathrm{V}}(\omega, \Delta\mathbf{M}) = -\omega^2\mathbf{X}^{\mathrm{V}}(\omega, \Delta\mathbf{M})$. Equation (5) suggests that the acceleration response of the modified structure equals the sum of original response and the response to the pseudo load,

$$\ddot{\mathbf{X}}^{\mathrm{V}}(\omega, \Delta\mathbf{M}) = \ddot{\mathbf{X}}(\omega) + \ddot{\mathbf{H}}(\omega)\mathbf{P}(\omega, \Delta\mathbf{M}). \tag{7}$$

Substitution of Eq. (7) into the right-hand side of Eq. (6) yields the following equation with the pseudo load in the role of the unknown:

$$\left[\mathbf{I} + \Delta\mathbf{M}\ddot{\mathbf{H}}(\omega)\right]\mathbf{P}(\omega, \Delta\mathbf{M}) = -\Delta\mathbf{M}\ddot{\mathbf{X}}(\omega), \tag{8}$$

where $\mathbf{I}$ is the identity matrix of the appropriate dimensions. Solving Eq. (8), the resulting pseudo load can be used in Eq. (7) to find the acceleration response of the structure with added mass.

The above formulation reveals two important features of the VDM:

1. In case of a localized mass modification, the matrix $\Delta\mathbf{M}$ is sparse and Eq. (8) simplifies to an equation of very limited dimensions in comparison to the dimensions of the full equilibrium equation. Such a feature is common to all fast reanalysis methods (Akgün et al 2001).
2. The acceleration response of the structure with added mass is computed by Eqs. (7) and (8). The response is expressed solely in terms of the modification $\Delta\mathbf{M}$ and certain basic characteristics of the real structure, $\ddot{\mathbf{H}}(\omega)$ and $\ddot{\mathbf{X}}(\omega)$. These characteristics can be computed using an updated FE model of the real structure, but they can be also measured experimentally, which makes the VDM an essentially nonparametric approach and which is unlike other reanalysis methods.

Both features become particularly clear, if a single Dof is used for excitation, measurement and modification. This is the case considered in this paper.

2.2 Case of a single affected Dof

If the observation, excitation, and an additional mass are all located only along the same ith Dof, then

$$\mathbf{D} = \mathbf{I}_i^{\mathrm{T}}, \qquad \mathbf{B} = \mathbf{I}_i, \qquad \Delta\mathbf{M} = m\mathbf{I}_i\mathbf{I}_i^{\mathrm{T}}, \tag{9}$$

where $\mathbf{I}_i$ is the n_d dimensional vector with 1 in the ith position and 0s elsewhere, and m is the additional mass that modifies the structural mass only in the ith Dof. Due to Eq. (6), the equivalent pseudo load acts only in the ith Dof and equals $-mA^{\mathrm{V}}(\omega, m)$, where $A^{\mathrm{V}}(\omega, m)$ is the acceleration of the modified structure along the same Dof. In such a setup, Eq. (7) simplifies into a scalar equation

$$A^{\mathrm{V}}(\omega, m) = A(\omega) - m\ddot{H}_{ii}(\omega)A^{\mathrm{V}}(\omega, m), \tag{10}$$

which yields the acceleration response of the modified structure,

$$A^{\mathrm{V}}(\omega, m) = \frac{A(\omega)}{1 + m\ddot{H}_{ii}(\omega)}. \tag{11}$$

Since

$$\ddot{H}_{ii}(\omega) = A(\omega)/F(\omega), \tag{12a}$$

$$\ddot{H}_{ii}^{\mathrm{V}}(\omega, m) = A^{\mathrm{V}}(\omega, m)/F(\omega), \tag{12b}$$

the acceleration FRF of the modified structure is

$$\ddot{H}_{ii}^{\mathrm{V}}(\omega, m) = \frac{\ddot{H}_{ii}(\omega)}{1 + m\ddot{H}_{ii}(\omega)} = \frac{A(\omega)}{F(\omega) + mA(\omega)}. \tag{13}$$

For each additional mass m, the acceleration frequency response $\ddot{H}_{ii}^{\mathrm{V}}(\omega, m)$ of the modified structure can be thus constructed by the Fourier transforms of the measured time-domain response $a(t)$ and structural excitation $f(t)$ of the real structure. The FRF of the modified structure is thus computed solely numerically in terms of the measured response and excitation of the real structure: there is no need to actually fix a physical additional mass to the structure, which facilitates practical implementation of the method. In applications, the weight and placement of the additional mass can be chosen flexibly according to the demand of the analysis. The modified structure with additional virtual mass modeled in this way is called the *virtual structure*.

2.3 Natural frequencies of a virtual structure

Let the virtual structure constructed by adding a virtual mass m in the ith Dof of the real structure be symbolically denoted by $G_i(m)$. By adding the virtual mass, the dynamic characteristics of the virtual structure, including its natural frequencies, are changed compared to the original real structure. The jth natural frequency $\omega_{ji}^{\mathrm{V}}(m)$ of $G_i(m)$ can be found via an analysis of the constructed acceleration FRF. Due to the measurement noise and the finite frequency resolution of the FFT, a straightforward peak picking procedure may not yield results accurate enough for the purpose of damage identification. The accuracy can be usually significantly improved by averaging out the noise with even a simple approximation of the raw values. In this paper, an approximation of the following form is used:

$$\omega_{ji}^{\mathrm{V}}(m) \approx a_0 + \mathrm{e}^{Q(m)}, \tag{14}$$

where a_0 is a constant and $Q(m)$ is a polynomial of a low order, which can be selected on a case-by-case basis in dependence on the constructed FRF nephogram and the considered range of the virtual masses m.

3 Damage identification by adding virtual masses

In practice, the virtual structure is constructed based on measurements of a damaged real structure in order to increase the sensitivity of its natural frequencies to the investigated damage parameters. Usually several virtual masses need to be separately added in different Dofs in order to cover all the parameters. The FE model of the undamaged structure is then updated, taking separately into account all the virtual masses, to fit its natural frequencies to the constructed values.

3.1 Damage model

Assume that the global structure is divided into n substructures and let the structural damage be modeled in terms of stiffness reduction ratios of the substructures. The damage extent of the lth substructure, $\mu_l \in (0,1]$, is defined as

$$\tilde{\mathbf{K}}_l = \mu_l \mathbf{K}_l, \tag{15}$$

where $\mathbf{K}_l$ is the original undamaged stiffness matrix of the lth substructure expressed in in the global Dofs and $\tilde{\mathbf{K}}_l$ is its damaged stiffness matrix. The damage of the global structure is thus quantified by the vector $\boldsymbol{\mu} = \{\mu_1, \mu_2, \ldots, \mu_n\}^{\mathrm{T}}$ of the damage extents of the substructures. The global stiffness matrix $\mathbf{K}(\boldsymbol{\mu})$ of the damaged structure is assembled as

$$\mathbf{K}(\boldsymbol{\mu}) = \sum_{l=1}^{n} \mu_l \mathbf{K}_l, \tag{16}$$

which for a vector of ones yields also the global stiffness matrix $\mathbf{K}(\boldsymbol{\mu}_0)$ of the original undamaged structure,

$$\boldsymbol{\mu}_0 = \{1, 1, \ldots, 1\}. \tag{17}$$

3.2 Sensitivity of the natural frequencies

By adding a virtual mass to the real structure, dynamic characteristics of the resulting virtual structure $G_i(m)$ are changed compared to the original real structure $G_i(0)$. In this way, virtual structure can be designed and used purposely for damage identification. Here, the virtual mass is added in the aim of increasing the relative sensitivity of natural frequencies with respect to damage parameters μ_l.

3.2.1 Structure with a known damage (or undamaged)

Assuming the damage vector $\boldsymbol{\mu}$, characteristics of the jth natural frequency $\omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)$ of the corresponding virtual structure, which is denoted symbolically by $G_i(\boldsymbol{\mu}, m)$, depend on three parameters: the order j of the natural frequency, the virtual mass m and its placement i. The virtual mass does not affect stiffness, thus the stiffness matrix of such a virtual structure is still $\mathbf{K}(\boldsymbol{\mu})$. Let R_{jil} denote the absolute sensitivity of $\omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)$ with respect to the damage extent μ_l of the lth substructure,

$$R_{jil}(\boldsymbol{\mu}, m) = \frac{\partial \omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)}{\partial \mu_l} = \frac{\left(\boldsymbol{\varphi}_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)\right)^{\mathrm{T}} \mathbf{K}_l \, \boldsymbol{\varphi}_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)}{2\omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)}, \tag{18}$$

where $\boldsymbol{\varphi}_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)$ is the jth mass-normalized global mode shape of the virtual structure $G_i(\boldsymbol{\mu}, m)$. The relative sensitivity is defined as

$$\eta_{jil}(\boldsymbol{\mu}, m) = \frac{R_{jil}(\boldsymbol{\mu}, m)}{\omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)} = \frac{\left(\boldsymbol{\varphi}_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)\right)^{\mathrm{T}} \mathbf{K}_l \, \boldsymbol{\varphi}_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)}{2\left(\omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu}, m)\right)^2}. \tag{19}$$

The problem of optimum mass placement is a combinatorial problem, which is approached here in a heuristic way based on common engineering sense: in order to increase the sensitivity to μ_l, a single virtual mass is added to the lth substructure in the approximate antinode of its lowest-order local substructural mode. Thus, the placement i in Eq. (19) is in principle a function of l.

By adjusting the value of the additional mass m, there exists an optimum value that maximizes the relative sensitivity $\eta_{jil}(\boldsymbol{\mu}, m)$. Using Eq. (19), it can be deduced that the sum of the relative sensitivities $\sum_l \eta_{jil}(\boldsymbol{\mu}, m)$ equals 1/2. The sensitivity is nonnegative, and thus $\eta_{jil}(\boldsymbol{\mu}, m) \in [0, 1/2]$. As a result, given l and the related i, a threshold value might be used to facilitate the selection process of the mass m and the order j of the natural frequency $\omega_{ji}^{\mathrm{V}}(\boldsymbol{\mu},m)$, so that it is sensitive enough to a given damage parameter μ_l. Such a procedure can yield a limited number n_{f} of natural frequencies and virtual structures that are highly sensitive to all the considered damage parameters. The selected frequencies are denoted by $\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p)$, where $p=1,2,\ldots,n_{\mathrm{f}}$.

3.2.2 Real structure with an unknown damage

The relative sensitivity $\eta_{jil}(\boldsymbol{\mu},m)$ can be computed using Eq. (18) only if the corresponding mode shape $\boldsymbol{\varphi}_{ji}^{\mathrm{V}}(\boldsymbol{\mu},m)$ is known. Thus, the selection process described above can be performed only for a structure with a known damage vector $\boldsymbol{\mu}$, while the real damaged structure has an unknown $\boldsymbol{\mu}$ that needs to be identified. The above sensitivity analysis is thus usually performed based on the FE model of the undamaged structure defined by Eq. (17). The results obtained this way provide only a baseline for the selection of the virtual structures and their natural frequencies. Once selected, the corresponding natural frequencies $\omega_{j_p i_p}^{\mathrm{V}}(m_p)$ can be constructed for the real damaged structure based on the experimentally measured responses and excitations using the approach described in Section 2 and used as a known dynamic information for updating the vector $\boldsymbol{\mu}$ through fitting of the theoretical values $\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p)$, see Section 3.3.

The baseline results of the selection process are obtained by analyzing the undamaged structure. Given the real structure, which is damaged in an unknown way, the process can be supported with heuristic information provided by the constructed acceleration FRF. Consider the formula for the relative sensitivity Eq. (19) and the approximate formula for the acceleration FRF of a virtual structure at its jth natural frequency,

$$\ddot{H}_{ii}^{\mathrm{V}}(\omega_{ji}^{\mathrm{V}}(m),m) \approx \frac{\mathrm{j}\left(\boldsymbol{\varphi}_{ji}^{\mathrm{V}}(m)\right)^{\mathrm{T}} \mathbf{I}_i \mathbf{I}_i^{\mathrm{T}} \boldsymbol{\varphi}_{ji}^{\mathrm{V}}(m)}{2\zeta_j}, \tag{20}$$

which is obtained for $\omega=\omega_j=\omega_{ji}^{\mathrm{V}}(m)$ and $\boldsymbol{\phi}_j=\boldsymbol{\varphi}_{ij}^{\mathrm{V}}(m)$ from the general exact formula,

$$\ddot{H}_{ii}(\omega) = \sum_{j=1}^{n} \frac{(\mathrm{j}\omega)^2 \phi_{ji}\phi_{ji}}{\omega_j^2-\omega^2+2\mathrm{j}\zeta_j\omega_j\omega}, \tag{21}$$

where ω_j is the jth natural frequency, ζ_j is the damping ratio of the jth structural mode and the vector $\boldsymbol{\phi}_j=\left\{\phi_{j1},\ldots,\phi_{jn_{\mathrm{d}}}\right\}$ is its shape. The formal similarity of Eqs. (19) and (20), together with the fact that i is the antinode of the first distortion eigenvector of $\mathbf{K}_l$, suggests that pronounced peaks of the relative sensitivity might be correlated with pronounced peaks of the constructed acceleration FRF.

3.3 Damage identification

Basically, the Gauss–Newton optimization procedure is employed to fit in relative terms the selected natural frequencies computed using the FE model of the structure to their counterparts constructed using the measurements of the real damaged structure. The following objective function is used:

$$F(\boldsymbol{\mu}) = \frac{1}{2}\sum_{p=1}^{n_{\mathrm{f}}} \left(\Delta\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p)\right)^2, \tag{22}$$

where

$$\Delta\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p) = \frac{\omega_{j_p i_p}^{\mathrm{V}}(m_p)-\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p)}{\omega_{j_p i_p}^{\mathrm{V}}(m_p)} \tag{23}$$

is the relative discrepancy between $\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p)$, which is the natural frequency computed using the damage vector $\boldsymbol{\mu}$ and the FE model of the structure, and $\omega_{j_p i_p}^{\mathrm{V}}(m_p)$, which is the corresponding natural frequency constructed using measurements of the real damaged structure. In the vector notation, the objective function and its gradient can be stated as

$$F(\boldsymbol{\mu}) = \frac{1}{2}\|\Delta\boldsymbol{\omega}^{\mathrm{V}}(\boldsymbol{\mu})\|^2, \tag{24a}$$

$$\nabla F(\boldsymbol{\mu}) = -\mathbf{J}^{\mathrm{T}}(\boldsymbol{\mu})\,\Delta\boldsymbol{\omega}^{\mathrm{V}}(\boldsymbol{\mu}), \tag{24b}$$

where the vector $\Delta\boldsymbol{\omega}^{\mathrm{V}}(\boldsymbol{\mu})$ collects together all the n_{f} relative discrepancies $\Delta\omega_{j_p i_p}^{\mathrm{V}}(\boldsymbol{\mu},m_p)$ and $\mathbf{J}(\boldsymbol{\mu})$ is the Jacobi matrix of $\Delta\boldsymbol{\omega}^{\mathrm{V}}(\boldsymbol{\mu})$ with respect to $\boldsymbol{\mu}$, which can be computed using Eq. (18). If the Gauss approximation to the Hessian is used,

$$\nabla^2 F(\boldsymbol{\mu}) \approx \mathbf{J}^{\mathrm{T}}(\boldsymbol{\mu})\,\mathbf{J}(\boldsymbol{\mu}), \tag{25}$$

then the Newton update rule yields

$$\boldsymbol{\mu}_{k+1} = \boldsymbol{\mu}_k + \mathbf{J}^{\star}(\boldsymbol{\mu})\,\Delta\boldsymbol{\omega}^{\mathrm{V}}(\boldsymbol{\mu}), \tag{26}$$

where $\mathbf{J}^{\star}(\boldsymbol{\mu})$ is the pseudo-inverse of the Jacobi matrix. The starting point for optimization can be selected to be $\boldsymbol{\mu}_0$ defined in Eq. (17), which corresponds to the undamaged state of the structure.

4 Numerical verification

First, a simple two-story plane frame model is used to verify that virtual masses can be used to increase the sensitivities of natural frequencies to local damages, as well as to illustrate the application principles of the proposed approach. Second, a four-story frame is employed to test the damage identification procedure.

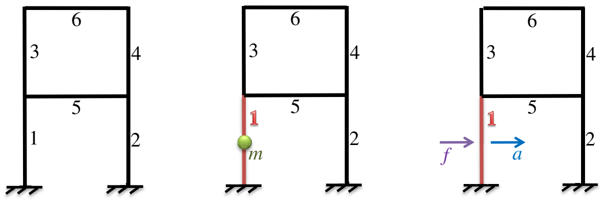

Fig. 1 Two-story plane frame: (*left*) the original structure; (*middle*) a virtual mass added in the middle of the first substructure; (*right*) testing excitation and measurement

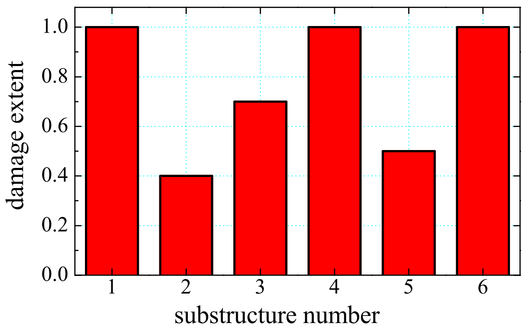

Fig. 2 Two-story plane frame: damage extents of the substructures

Table 1 Two-story plane frame: the first five natural frequencies of the undamaged original structure and the damaged structure

order	1	2	3	4	5
undamaged [Hz]	5.90	19.48	42.84	61.06	67.07
damaged [Hz]	4.78	16.92	37.60	50.87	54.84

4.1 Two-story plane frame

4.1.1 The structure and the damage

A two-story plane frame model is shown in Fig. 1 (left). The height of each floor and the width of the span are 0.6 m. The modulus of elasticity is 2.1 GPa, and the density is 7850 kg/m^3. The cross-sections of all the elements are the same, 3×10^{-4} m^2, and their second moment of area is 9×10^{-10} m^4. Each pillar and each beam is further divided into 4 finite elements.

The frame consists of 4 pillars and 2 beams, which are the 6 substructures to be identified in this example. The substructures are numbered as shown in Fig. 1. The 2nd, 3rd and 5th substructures are assumed to be damaged with the damage extents shown in Fig. 2. The first five natural frequencies of the undamaged original structure and the damaged structure are listed in Table 1.

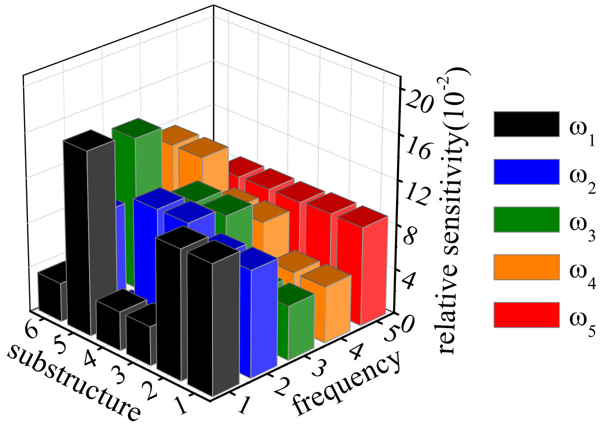

Fig. 3 Two-story plane frame: the relative sensitivity matrix of the natural frequencies for the original undamaged structure

4.1.2 Sensitivity analysis and the FRF of the undamaged structure

The model of the undamaged structure is used to illustrate the sensitivity analysis and for verification of the process of constructing the frequency responses of the original structure with a virtual mass.

Structural sensitivity analysis. Using Eq. (19) and the undamaged original FE model, the relative sensitivity is computed for the first five natural frequencies with respect to the six substructural damage extents, see Fig. 3. The maximum value is less than 0.14, and moreover, due to the structural symmetry, the patterns for substructures 1 and 2 as well as 3 and 4 are exactly the same. As a result, the first five natural frequencies of the original structure are as a whole not sensitive enough to be used for damage identification.

An additional mass is added to the structure to increase the sensitivity with respect to damage parameters. Assume that mass m is added in the middle of substructure 1 of the undamaged structure, as shown in Fig. 1 (middle). The modified structure is symbolically denoted by $G_1(\boldsymbol{\mu}_0, m)$. Given the additional mass m, the jth natural frequency $\omega_{j1}^{\mathrm{V}}(\boldsymbol{\mu}_0, m)$ and the jth mode shape $\boldsymbol{\varphi}_{j1}^{\mathrm{V}}(\boldsymbol{\mu}_0, m)$ can be computed using the FE model of the undamaged structure. Equation (19) can be then directly used to compute the relative sensitivity $\eta_{j11}(\boldsymbol{\mu}_0, m)$ of the jth natural frequency with respect to the damage extent μ_1 of the first substructure. Fig. 4 plots the relative sensitivities of the first five natural frequencies in dependence on the additional mass m. The figure clearly illustrates that the additional mass can considerably affect the relative sensitivities (increase or decrease by almost an order of magnitude). In order to improve the accuracy of identification, natural frequencies with high relative sensitivity can be selected. If 0.2 is used as the threshold level, then the 2nd, 3rd and 4th natural frequencies can be picked. However, the

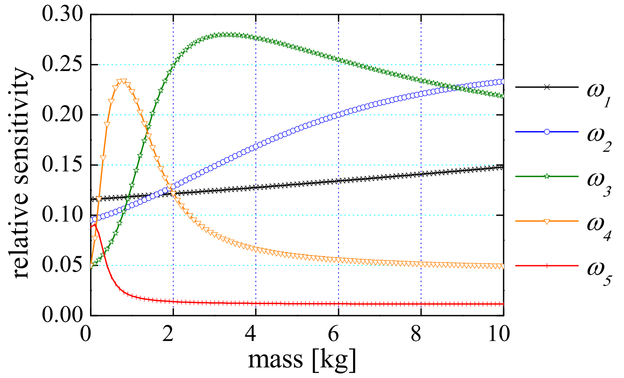

Fig. 4 Two-story plane frame: the relative sensitivities $\eta_{j11}(\boldsymbol{\mu}_0, m)$ with respect to the damage of the first substructure in dependence on the mass m

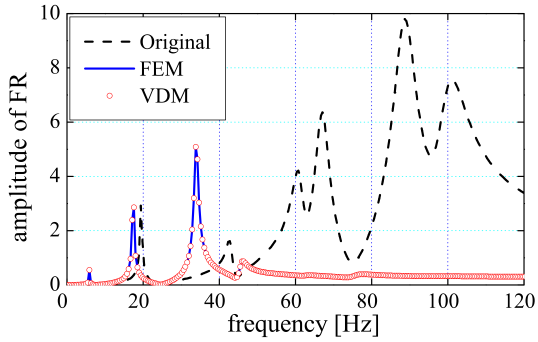

Fig. 6 Two-story plane frame: comparison of the computed and constructed frequency responses

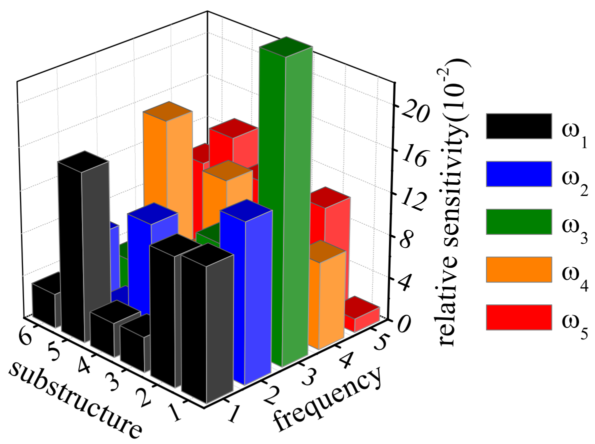

Fig. 5 Two-story plane frame: the relative sensitivity matrix of the natural frequencies for the structure $G_1(\mu_0, 3\text{ kg})$

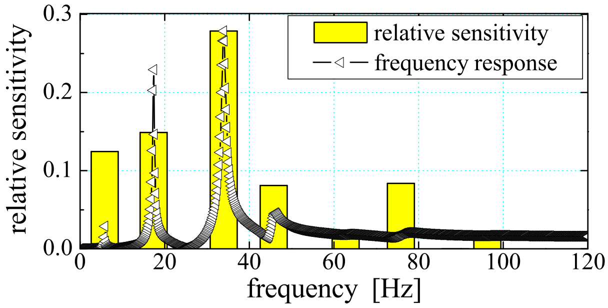

Fig. 7 Two-story plane frame: the constructed frequency response and the relative sensitivities

sensitivity of the 4th natural frequency changes rapidly with the additional mass, so that it might not be stable also for a structure with a different, unknown damage $\boldsymbol{\mu}$. Fig. 5 shows the relative sensitivity matrix for the undamaged structure with a mass of 3 kg added to the first substructure, which contains a higher sensitivity with respect to substructure 1.

The sensitivity analysis is computed based on the FE model of undamaged structure, which is different from the real damaged structure. As discussed in Section 3.2.2, the relationship of the relative sensitivities and the additional mass in Fig. 4 can provide a reference for selection of the order j of the natural frequency and the additional mass m, but the final decision should be taken after considering the constructed frequency response.

Construction of the frequency response. Figure 6 compares three curves of the acceleration frequency response of the middle of substructure 1 along the horizontal direction to the excitation applied in the same Dof, see Fig. 1 (right). The responses labeled “Original” and “FEM” are computed by Eq. (21) based on the FE models of the original undamaged structure and the modified structure $G_1(\boldsymbol{\mu}_0, 3\text{ kg})$, respectively; the frequency response labeled “VDM” is computed by Eq. (13) using the frequency response of the original undamaged structure (the curve labeled “Original” in Fig. 6). The curve labeled “VDM” closely follows the curve labeled “FEM”, which confirms that the frequency response of the virtual structure can be precisely constructed using the proposed VDM-based approach.

Section 3.2.2 suggests a relation between pronounced peaks of the constructed FRF and pronounced peaks of the relative sensitivities. Thus, Fig. 7 compares the (rescaled for convenience) constructed frequency response of the considered modified structure $G_1(\boldsymbol{\mu}_0, 3\text{ kg})$ with the relative sensitivities $\eta_{j11}(\boldsymbol{\mu}_0, 3\text{ kg})$ of the corresponding natural frequencies with respect to the damage of substructure 1. In general, the plot confirms the suggested relation between the high amplitudes of the frequency response and the high relative sensitivities of the corresponding natural frequencies. The rule is rather heuristic than absolute, but it can be used as a source of additional information to facilitate the process of selecting natural frequencies and additional masses in damaged structures.

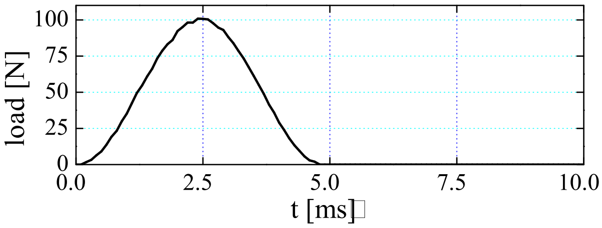

Fig. 8 Two-story plane frame: the simulated measured excitation

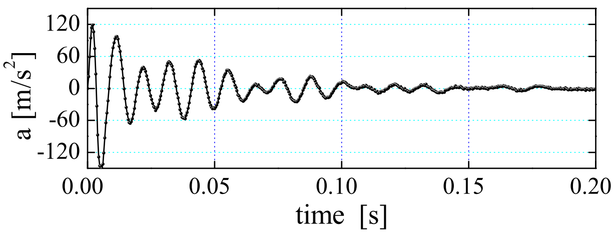

Fig. 9 Two-story plane frame: the response to the simulated measured acceleration

4.1.3 Damage identification

This section illustrates the damage identification procedure using the proposed method. First, the frequency response of the virtual structures are constructed using the (simulated) measured response of damaged structure to hammer excitation. Then, the natural frequencies with high sensitivity to damage parameters are selected through the sensitivity analysis and used for damage identification.

Excitation and response. A simulated hammer excitation is applied in the middle of substructure 1 of the damaged structure, see Fig. 1 (right). The sampling frequency is 10 kHz, and the time interval of 2 s is considered. The simulated excitation lasts 5 ms and models an impact by a modal hammer, see Fig. 8. In order to simulate a real application, 5% Gaussian white noise is added to both excitation and the response. The excitation without noise is applied to the structure, and the structural responses are computed via the FE model of damaged structure. The simulated measured acceleration response of the middle substructure along the horizontal direction is shown in Fig. 9.

Construction of the FRF. To construct the frequency response of the virtual structure with additional virtual masses, the Fourier Transform with exponential window is performed on the noisy simulated measured excitation and responses. The result is used in Eq. (13) to obtain the frequency response of the virtual damaged structure $G_1(\boldsymbol{\mu}, m)$ for the virtual mass $m \in [0, 10]$. Fig. 10 shows the nephogram of the constructed frequency response with respect to the virtual mass. The brightness reflects the amplitude of the frequency response, so that the brightest points correspond to its peaks, that is to the natural frequencies of the virtual structure. As the mass increases, the natural frequencies decrease. The constructed nephogram around the 3rd natural frequency (from 24 Hz to 40 Hz) is selected from Fig. 8 and shown zoomed in Fig. 11 (top). The corresponding theoretical nephogram is computed using the FE model of the virtual damaged structure and shown in Fig. 11 (bottom). By comparison of both figures, the following can be noted:

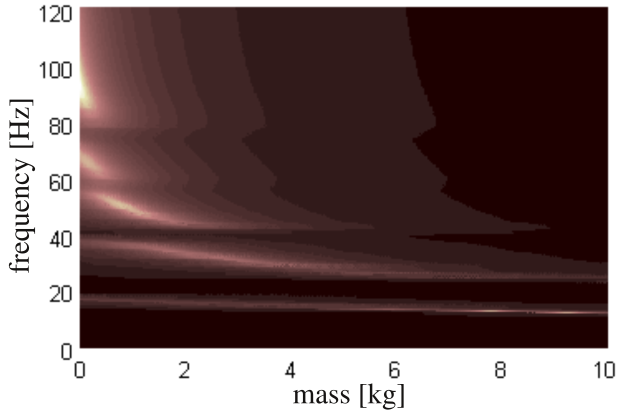

Fig. 10 Two-story plane frame: nephogram of the constructed frequency response of the damaged structure $G_1(\boldsymbol{\mu}, m)$ in dependence on the frequency and virtual mass m

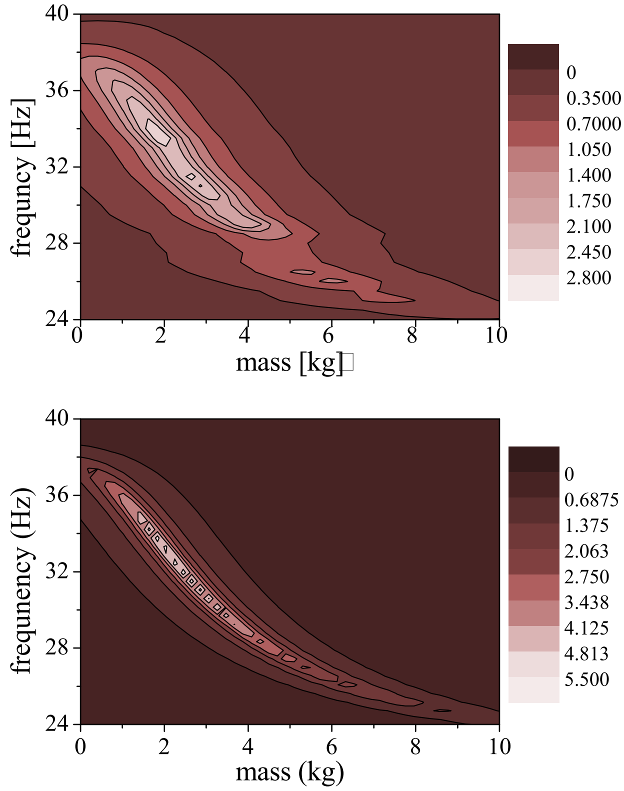

Fig. 11 Two-story plane frame: nephogram of the frequency response, zoomed to show the 3rd natural frequency: (*top*) constructed from noisy simulated measurements; (*bottom*) exact theoretical

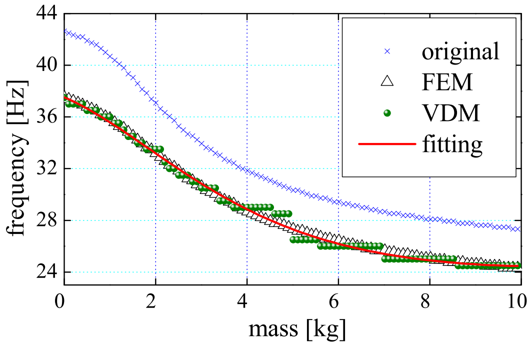

Fig. 12 Two-story plane frame: the 3rd natural frequency in dependence on the virtual mass

1. Even if the amplitudes of the natural frequencies are not same, the natural frequencies corresponding to the amplitude peaks are almost the same, see Fig. 12, which plots the 3rd natural frequency of the virtual structure as a function of the additional mass. The curve "VDM" is obtained by direct peak-picking using the constructed frequency response in Figure 11 (top), while the curve "FEM" plots the accurate values computed using the FE model of the virtual damaged structure. The curve "Original" is computed using the undamaged FE model.
2. For small virtual masses, the discrepancy between the curves "VDM" and "FEM" are negligible. However, the error increases with the mass and can affect the identification results, if the "VDM" curve is directly used for damage monitoring.
3. The curve "FEM" confirms that the relation between the natural frequency and the additional mass is smooth, and that the natural frequency decreases with the mass increasing in an exponential-like decay.

To reduce the influence of errors, an exponential-like approximation is used to fit the relation between the constructed frequency and the additional virtual masses, see Eq. (14). In the considered example, a third-order polynomial is used in the exponent. The approximation is shown in Fig. 12 (curve "fitting") and seems to effectively smooth out the errors of direct peak-picking. If a narrower range of virtual masses is considered, a polynomial of a lower order can be often used in the exponent.

Selection of natural frequencies. The considered excitation is applied in the middle of all the six substructures, and the corresponding acceleration responses are simulated. The nephograms of the corresponding virtual structures are constructed by Eq. (13) with respect to the virtual masses added separately in the middle of the six substructures along the same direction as the excitations, see Fig. 13. Via the analysis of these six nephograms and the sensitivity analysis, eight natural frequencies are selected to be used for damage identification, see Table 2. For example, Case 1 refers to the 2nd natural frequency of the virtual structure with a 3 kg virtual mass added to substructure 1. Fig. 14 plots the relative sensitivities of the selected natural frequencies in dependence on the respectively placed virtual masses.

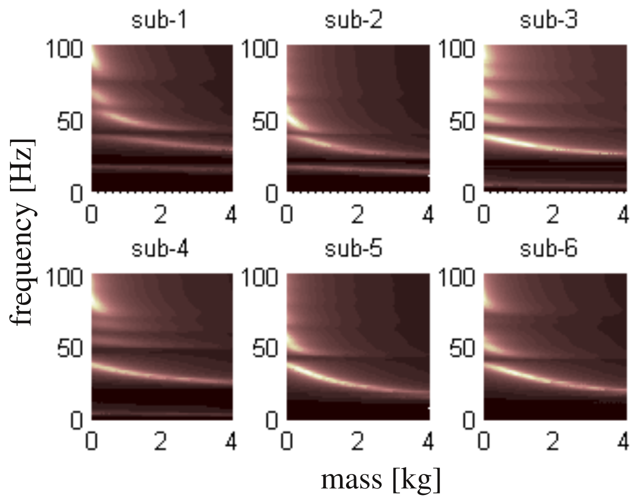

Fig. 13 Two-story plane frame: nephograms of the constructed frequency responses of the six virtual structures

Table 2 Two-story plane frame: the eight natural frequencies selected for damage identification

Case	1	2	3	4	5	6	7	8
Substructure	1	1	2	2	3	4	5	6
Order of natural frequency	2	3	2	3	3	3	3	3
Mass [kg]	3	3	3	3	4	4	3	3

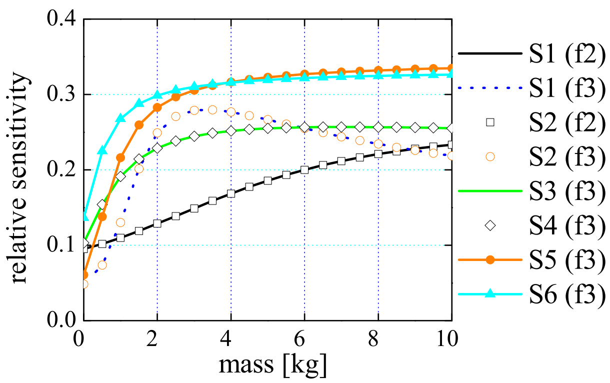

Fig. 14 Two-story plane frame: the relative sensitivities of the eight selected natural frequencies with respect to the respectively placed virtual masses

The eight selected natural frequencies are computed by fitting the relation between the picked natural frequency and the virtual mass, see the column "VDM, approximated" in Table 3. They are close to the accurate theoretical values computed using the FE models of the damaged virtual structures and the exact damage extents ("Damaged virtual structure"). For comparison, the column "Undamaged virtual struc-

Table 3 Two-story plane frame: comparison of the selected natural frequencies

Case	Undamaged virtual structure	Damaged virtual structure	VDM, approximated	Identified virtual structure
1	17.37	15.43	15.34	15.46
2	33.93	30.61	30.85	30.62
3	17.37	14.00	14.12	14.05
4	33.93	25.46	25.59	25.54
5	28.68	25.62	25.83	25.75
6	28.68	25.74	25.94	25.72
7	25.92	20.23	20.02	19.96
8	23.13	22.21	22.25	22.19

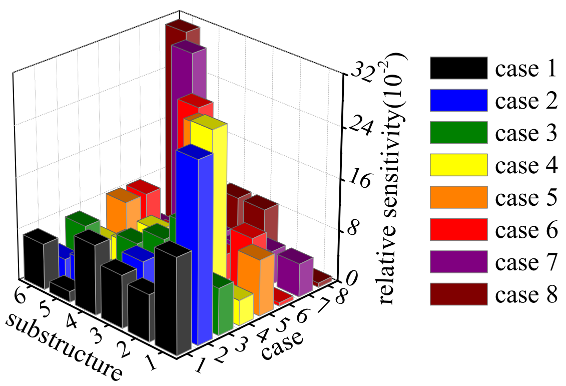

Fig. 15 Two-story plane frame: the relative sensitivity matrix of the eight selected natural frequencies

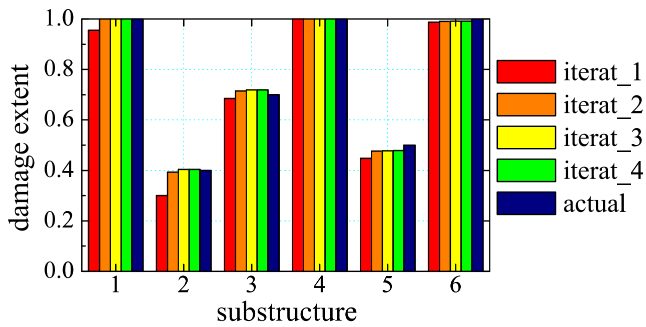

Fig. 16 Two-story plane frame: damage extents in successive iterations

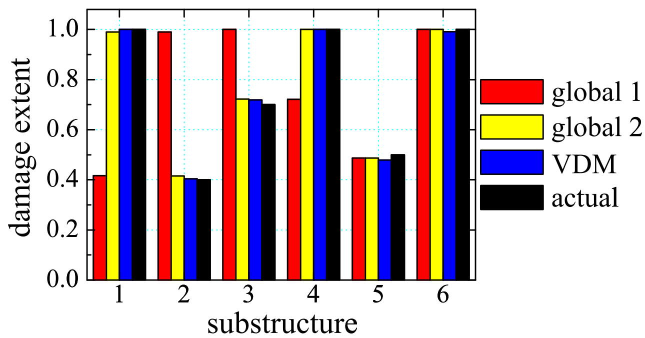

Fig. 17 Two-story plane frame: comparison of the identification results of a standard approach ("global 1" and "global 2") and the proposed approach ("VDM") with the actual damages ("actual")

ture" lists the same natural frequencies in the corresponding undamaged virtual structures.

Damage identification Damage is identified by updating the vector $\boldsymbol{\mu}$ to fit the computed natural frequencies to the approximated ones, see Eq. (22). The update rule Eq. (26) uses the Jacobi matrix $\mathbf{J}(\boldsymbol{\mu})$, which contains the relative sensitivities of the selected natural frequencies of the virtual structures. They are computed using the FE model and shown in Fig. 15. Compared to Fig. 3, it is a diagonally dominant matrix with a small condition number of 3.61 only. Therefore, by utilizing the sensitivities of properly selected virtual structures, the optimization quickly converges in only 4 iterations. Fig. 16 attests that the damage extents are identified precisely. The corresponding fitted values of the natural frequencies (computed using the FE models of the virtual structures with the identified damage extents) are listed in the column "Identified virtual structure" of Table 3.

For comparison purposes, a standard optimization is also performed using natural frequencies of the global structure (with no virtual masses) and the standard Matlab implementation of a genetic algorithm (GA). During the optimization, the first 8 natural frequencies (with 2% errors) of the global structure are employed. Two different global minima are identified in several optimization runs and denoted by "global 1" and "global 2", see Fig. 17. The result "global 2" is precise, while "global 1" is wrong. Therefore, standard natural frequencies cannot guarantee an accurate identification, especially in the considered case of a symmetrical structure. Here, although the result "global 2" is precise, the optimization efficiency of GA is not optimistic, especially for a large complex structure with a large number of damage extents. Comparatively, the proposed method of additional virtual masses required only four iterations, provided accurate results and utilized natural frequencies of a significantly lower order. To further test the approach, a more complex structure of a four-story frame model is considered in the following section.

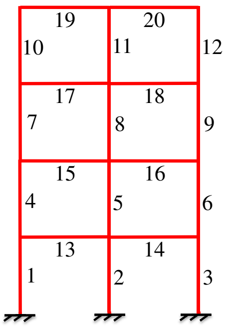

Fig. 18 A four-story plane frame model

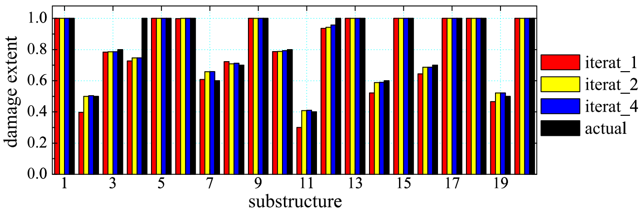

Fig. 19 Four-story plane frame: identified damage extents

4.2 Four-story frame model

Fig. 18 shows a four-story frame model, which consists of 12 columns and 8 beams. It is divided into 20 substructures, and each member is a substructure. Their assumed damages extents are shown in Fig. 19 ("actual"). Excitations are separately applied in the middle positions of all the substructures and are measured together with the corresponding simulated accelerations, both contaminated with 5% Gaussian white noise. Then the corresponding 20 nephograms of the constructed frequency responses are computed. Via the nephograms and the sensitivity analysis, the additional virtual masses and natural frequencies are determined and listed in Table 4 in the columns "mass" and "VDM, approximated". The constructed values are very close to those computed from the damaged FE model ("Damaged virtual structure"). The damage extents are identified quickly and accurately in only 4 iterations, see Figure 19. The natural frequencies of the updated virtual structures are shown in Table 4 in the column "Identified virtual structure". The accuracy of the estimated natural frequencies and damage extents confirms the effectiveness of the proposed method.

5 Conclusion

This paper proposes an effective method for structural damage identification by constructing virtual structures with additional virtual masses. The virtual structures are constructed in order to increase their sensitivity with respect to the considered damage parameters. A plane frame model is employed to test the method. The most important features can be summarized as follows:

1. The virtual structures are united together and all their natural frequencies with high sensitivity are collected into a single sensitivity matrix which is a well-conditioned diagonally dominant matrix. As a result, the optimization is quickly convergent.
2. Damage identification is performed using only lower-order natural frequencies, which can be reliably obtained from measurements.
3. The mass is added to the structure virtually (numerically): the practical difficulties of adding a real mass to the structure are thus avoided.
4. Damage identification of global structure requires only two sensors: one excitation sensor and one acceleration sensor, which are sequentially used for constructing all the considered virtual structures one by one. Therefore, the experimental costs can be significantly reduced.

In the proposed method, the number of identified natural frequencies used for damage identification is in direct proportion to the number of measurements. No matter how large the structure is, if the testing time is long enough, a virtual mass can be added to each member and the collected modal information will be enough for damage identification. However, if the number of the unknown parameters is very large, it might not be easy to use the method due to practical limitations of a real application. Testing all the members in a large structure is rarely feasible: it is very time-consuming and some members might not be physically accessible for measurement.

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Table 4 Four-story plane frame: comparison of the virtual natural frequencies

Substructure	Order	mass [kg]	Damaged virtual structure	VDM, approximated	Identified virtual structure
1	5	3	31.90	32.25	31.89
2	5	3	26.11	26.22	26.15
3	5	3	29.42	29.25	29.20
4	5	3	29.18	29.25	29.15
5	5	3	28.76	29.01	28.76
6	5	3	28.93	28.97	28.85
7	5	3	25.55	25.85	25.78
8	5	3	26.53	26.80	26.69
9	5	3	29.09	29.22	29.02
10	5	3	24.91	25.05	24.97
11	5	2.5	22.84	23.13	23.05
12	5	2.5	28.97	28.87	28.72
13	5	2.5	27.84	28.00	27.82
14	5	3	22.21	22.11	22.08
15	5	3	25.57	25.54	25.50
16	5	3	23.51	23.44	23.39
17	5	3	24.67	25.12	24.73
18	5	3	25.48	25.68	25.46
19	5	1.5	24.19	24.53	24.47
20	5	2.5	24.40	24.60	24.38

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