

Nonparametric Identification of Added Masses in Frequency Domain

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Abstract

This paper presents a frequency-domain, nonparametric method for identification of added masses, and reports on its experimental verification. The identification is directly based on experimentally collected characteristics of the unmodified structure, so that no parametric numerical model of the monitored structure is required. Consequently, there is no need for the initial stage of model updating. This is a continuation of and an improvement over a previous research that resulted in a time-domain identification method, which was tested to be accurate but significantly time-consuming. For the experimental verification, a 4 m long 3D truss structure with 26 nodes and 70 elements is used. A total of 12 modification cases is tested: in each of 3 selected nodes, 4 additional masses are separately added and successfully identified.

I. INTRODUCTION

In recent decades, Structure Health Monitoring (SHM) has become an important and widely researched field with a number of dedicated international journals and specialized international conferences. All monitoring methods can be roughly divided into two general categories: local and global approaches. Local approaches are used for precise local identification of small defects in a narrow inspection zone and are usually based on manual ultrasonic testing; they are outside the scope of this paper. Global approaches are used for identification of significant defects in a larger inspection zone (often the entire monitored structure). Most of the global methods, including mass identification methods, can be classified into two general groups:

- 1) *Model-based methods* rely on a parametric numerical model of the monitored structure that is most often a finite element (FE) model [1], [2], [3], [4]. The identification is stated in the form of a minimization problem of the discrepancy between the measured response of the modified/damaged structure and the computed response of the modeled structure. Selected parameters of the model, which are assumed to describe the structural modification being identified, are used as the optimization variables.
- 2) *Pattern recognition methods* rely on a database of numerical fingerprints of low dimension that are extracted from several responses of the modified/damaged structure [5], [6]. The responses used to form the database should be collected either by simulations or by experimental measurements of the structure with introduced modification scenarios, which are to be identified later. The fingerprints should discriminate well between the scenarios. Given the database and the measured response of the involved structure, the actual modification is identified using the fingerprints only.

Methods of both groups have their strengths and weaknesses. In particular, in case of many structures, it may not be possible to actually introduce the modifications in order to perform the measurements and build the fingerprint database. Similarly, an accurate numerical model of a complex real-world structure may be hard to obtain and update.

Therefore, the authors have developed and investigated a nonparametric monitoring method [7], [8]. The method is directly based on experimentally collected characteristics of the unmodified structure, which constitute its nonparametric model. As a result, no parametric numerical model of the monitored structure (such as a finite element model) is required, and consequently, there is no need for the difficult initial stage of model updating that is typical in other parametric model-based approaches. On the other hand, the method is based on the equation of motion and allows parametrized mass modifications to be identified, which makes the method different from typical pattern recognition methods. However, even though the approach allows parametrized modifications to be identified, the response of the modified structure is modeled in an essentially local and nonparametric or data-driven way. Moreover, no topological information about the global structure is required, besides potential locations of the modifications given in the terms of the set of the related degrees of freedom (DOFs). The experimentally obtained structural characteristics are limited to these DOFs, so that the nonparametric model is in fact also reduced with respect to them. A general motivation behind the research is the need for a practical identification technique that could be used in black-box type global monitoring systems for identification of structural modifications, external dynamic loads and various damages of real-world, large and complex structures.

This paper presents a frequency-domain version of the method for identification of added masses, and reports on its experimental verification. It is a continuation of and an improvement over a previous research that resulted in a time-domain identification method, which was tested to be accurate but significantly time-consuming [7], [9]. The frequency-domain methodology has been briefly outlined in its initial stages and tested numerically in [10]. The paper is structured as follows. First, the general methodology is discussed, including the direct problem of nonparametric modeling of the response of the

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modified structure (Section II), the inverse problem of identification of structural modifications along with the optimization approach (Section III), and then an experimental verification is reported (Section IV). For the verification, a 4 m long 3D truss structure with 26 nodes and 70 elements is used. A total of 12 modification cases is tested: in each of 3 selected nodes, 4 additional masses are separately added and identified.

II. DIRECT PROBLEM

This section introduces the methodology used for modeling the response of the modified structure. Based on the numerical conditioning of the direct problem and on spectral content of the experimentally collected data, a spectral reliability index is defined to assess the reliability of the modeled response.

The direct problem is understood here as a problem of computing the response $\mathbf{u}(\omega)$ of the modified structure in frequency domain to a known testing excitation $\mathbf{f}(\omega)$, given

- Mass modification, which is expressed parametrically in terms of the modification $\Delta\mathbf{M}$ to the unknown structural mass matrix \mathbf{M} .
- Certain characteristics of the original unmodified structure, which are purely experimental in nature and consist of
 - the response $\mathbf{u}^L(\omega)$ of the original structure to the same testing excitation $\mathbf{f}(\omega)$, and
 - the matrix $\mathbf{B}(\omega)$ of impulse response functions of the unmodified structure, which are applied and/or measured in the degrees of freedom (DOFs) of the testing excitation, potential mass modifications and sensors. The matrix $\mathbf{B}(\omega)$ constitutes a nonparametric model of the unmodified structure, which is reduced with respect to the involved DOFs.

A. Virtual distortion method

In agreement with the methodology of the virtual distortion method (VDM), which is a method for fast reanalysis of structures [11], [12], [13], [14], [15], [16] and other systems of various nature [17], [18], the effects of the modifications are equivalently modeled with certain pseudo loads $\mathbf{p}(\omega)$ that act in the unmodified structure. The response $\mathbf{u}(\omega)$ of the modified structure to the excitation $\mathbf{f}(\omega)$ is thus expressed as a sum of

- 1) the response $\mathbf{u}^L(\omega)$ of the unmodified structure to $\mathbf{f}(\omega)$, and
- 2) the effect $\mathbf{u}^R(\omega)$ of the pseudo loads on the response, which is expressed in the form of their convolution $\mathbf{B}(\omega)\mathbf{p}(\omega)$ with the (frequency-domain) impulse responses included in the matrix $\mathbf{B}(\omega)$.

The response of the modified structure is found in two steps: first the pseudo loads are computed and then the corresponding response. The pseudo loads are coupled to the response, hence they are given in an implicit form of a certain integral equation, whose solution in time domain is computationally a very demanding task, which is one of the main shortcomings of the approach proposed in [7].

B. Pseudo loads

Let $\mathbf{p}(\omega)$ be a vector of pseudo loads that excite the original unmodified structure. Denote by $\mathbf{u}^R(\omega)$ the vector of the corresponding structural response. The unmodified structure is assumed to satisfy the standard form of the quasi-static equation of motion,

$$[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}] \mathbf{u}^R(\omega) = \mathbf{p}(\omega), \quad (1)$$

where \mathbf{M} , \mathbf{C} and \mathbf{K} denote respectively the mass, damping and stiffness matrices. Since the structure is linear, the response $\mathbf{u}(\omega)$ can be expressed in the form of a convolution of the excitation $\mathbf{p}(\omega)$ with the matrix $\mathbf{B}(\omega)$ of structural impulse responses,

$$\mathbf{u}^R(\omega) = \mathbf{B}(\omega)\mathbf{p}(\omega). \quad (2)$$

C. Governing equation

Let $\mathbf{f}(\omega)$ be an external testing excitation, and denote respectively by $\mathbf{u}^L(\omega)$ and $\mathbf{u}(\omega)$ the responses of the unmodified and modified structures. If the effect of the considered mass modifications $\Delta\mathbf{M}$ on structural damping and stiffness is neglected, the corresponding equations of motion can be stated as

$$[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}] \mathbf{u}^L(\omega) = \mathbf{f}(\omega), \quad (3)$$

$$[-\omega^2\mathbf{M} - \omega^2\Delta\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}] \mathbf{u}(\omega) = \mathbf{f}(\omega). \quad (4)$$

Notice that (4) can be rearranged into the equation of motion of the unmodified structure,

$$[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}] \mathbf{u}(\omega) = \mathbf{f}(\omega) + \mathbf{p}(\omega), \quad (5)$$

where the modifications are modeled by the pseudo load $\mathbf{p}(\omega)$,

$$\mathbf{p}(\omega) = \omega^2\Delta\mathbf{M}\mathbf{u}(\omega). \quad (6)$$

Equation (5) confirms that the response $\mathbf{u}(\omega)$ depends on the pseudo load $\mathbf{p}(\omega)$, so that (6) states the pseudo load in an implicit way. The structure is linear, thus according to 5, the response $\mathbf{u}(\omega)$ is the sum of the responses of the unmodified structure to $\mathbf{f}(\omega)$ and $\mathbf{p}(\omega)$, which are respectively $\mathbf{u}^L(\omega)$ and $\mathbf{u}^R(\omega)$, see (3) and (1),

$$\mathbf{u}(\omega) = \mathbf{u}^L(\omega) + \mathbf{u}^R(\omega). \quad (7)$$

If (7) is substituted into (6) and then (2) is used, the following equation is obtained:

$$[\mathbf{I} - \omega^2 \Delta \mathbf{M} \mathbf{B}(\omega)] \mathbf{p}(\omega) = \omega^2 \Delta \mathbf{M} \mathbf{u}^L(\omega), \quad (8)$$

where \mathbf{I} is an identity matrix of proper dimensions. Equation (8) is a frequency-domain representation of a system of Volterra integral equations of the second kind with the pseudo load vector $\mathbf{p}(\omega)$ as the unknown. An important result of the Riesz theory [19] states that it is well-posed, provided $\mathbf{M} + \Delta \mathbf{M}$ is nonsingular. Therefore, (8) has a unique solution, if the mass modification upholds the positive definiteness of the mass matrix $\mathbf{M} + \Delta \mathbf{M}$.

D. Nonparametrically modeled response

For a testing excitation $\mathbf{f}(\omega)$, the pseudo loads equivalent to given modification $\Delta \mathbf{M}$ are found by solving the system (8). According to (7) and (2), the response of the modified structure is the following sum of the response of the unmodified structure and the effects of the pseudo loads:

$$\mathbf{u}(\omega) = \mathbf{u}^L(\omega) + \mathbf{B}(\omega) \mathbf{p}(\omega). \quad (9)$$

Besides $\Delta \mathbf{M}$, the computations require only selected nonparametric characteristics of the unmodified structure, which can be measured experimentally:

- 1) the response $\mathbf{u}^L(\omega)$ to the testing excitation $\mathbf{f}(\omega)$,
- 2) the impulse response matrix $\mathbf{B}(\omega)$.

As a result, there is no need to build and update a parametric numerical model of neither the unmodified nor the modified structure. According to (6), the pseudo loads are nonzero only in the DOFs related to the modifications, which allows the number of necessary measurements to be significantly restricted and effectively reduces the nonparametric model.

E. Spectral reliability index

The crucial step in the procedure outlined above is the solution of (8), which in time-domain analysis takes the form of a system of Volterra integral equations of the second kind. In time domain, such a system, after time discretization, constitutes a large, dense and block-Toeplitz structured linear system. Numerical solution of such a system, even with the several effective numerical techniques proposed in [7] (including block embedment in a larger circulant matrix, computing exact matrix-vector products by the fast Fourier transform and an iterative solution through the conjugate gradient least squares algorithm), is computationally very expensive and time-consuming. On the other hand, since in practice the discretized version of (8) is always significantly ill-conditioned, an advantage of the time-domain solution is that it allows a fine control to be exerted over the level of numerical regularization through the regularizing properties of the CGLS algorithm [20].

The approach in this paper is formulated in frequency domain. It converts the time-domain system into the small and discrete system (8), which is stated in frequency domain and constitutes an independent system for each frequency line ω . It is a huge computational advantage over the time-domain approach, which computes the solution for all frequencies at once in a large convolution-type equation. Nevertheless, even if the Riesz theory implies that (8) is well-posed, such a system is usually significantly ill-conditioned. This is exemplified in a high conditioning coefficient of the system matrix $\mathbf{I} - \omega^2 \Delta \mathbf{M} \mathbf{B}(\omega)$ for certain frequency lines ω , which in time-domain formulation is controlled by the regularizing properties of the CGLS algorithm. In frequency domain, naive direct solutions of (8) for all ω and an application of the inverse fast Fourier transform to compute the time-domain response will rarely yield a numerically meaningful result. It suggests that the frequency-domain response of the modified structure can be reliably computed only for a certain subset of all possible frequency lines ω . The reliability level of the computed solution depends on at least two following factors:

- 1) Numerical conditioning of (8): reliable computations are possible only if the system is not excessively ill-conditioned.
- 2) Reliability of the nonparametric structural model at the considered frequency line ω : meaningful computations are possible only if ω is well-represented in all the quasi-impulsive excitations used to measure the impulse response matrix $\mathbf{B}(\omega)$.

This paper quantifies the reliability level at a given frequency line ω by means of the following reliability index:

$$w(\omega, \Delta \mathbf{M}) := \begin{cases} \kappa^{-1} (\mathbf{I} - \omega^2 \Delta \mathbf{M} \mathbf{B}(\omega)) & \text{if } \omega \leq \omega_{\max}, \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

where $\kappa(\mathbf{A})$ denotes the standard conditioning coefficient of the matrix \mathbf{A} , defined as the ratio of its largest and smallest singular values,

$$\kappa(\mathbf{A}) := \frac{\sigma_{\max}(\mathbf{A})}{\sigma_{\min}(\mathbf{A})}, \quad (11)$$

and ω_{\max} is the maximum frequency reliably represented in all the quasi-impulsive excitations used to measure the impulse response matrix $\mathbf{B}(\omega)$.

III. INVERSE PROBLEM

The inverse problem is the problem of identification of mass modifications $\Delta\mathbf{M}$, based on the

- 1) measured response of the modified structure $\mathbf{u}^M(\omega)$ to the testing excitation $\mathbf{f}(\omega)$,
- 2) measured response of the original unmodified structure $\mathbf{u}^L(\omega)$ to the testing excitation $\mathbf{f}(\omega)$, and
- 3) nonparametric model of the original unmodified structure, given in the form of the frequency-domain matrix $\mathbf{B}(\omega)$ of experimentally measured structural impulse responses.

A significant discrepancy between the measured response and the reference response suggests that a modification has occurred. The modification can be then identified by minimizing the discrepancy between the measured response and the response modeled nonparametrically in the direct problem.

In the following, it is assumed that the unknown mass modifications are defined in terms of a vector

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix} \quad (12)$$

of mass modification parameters, which will usually be the added masses. As a result, identification of the modifications $\Delta\mathbf{M}(\boldsymbol{\mu})$ amounts to identification of the equivalent vector $\boldsymbol{\mu}$ of mass modification parameters.

The vague notion of discrepancy is formalized here in the form of an objective function, and the identification problem is formulated in terms of an optimization problem of the objective function. In [7], the standard ℓ^2 norm is used to compare the involved responses in time domain. Here, the following frequency-domain objective function is used:

$$F(\boldsymbol{\mu}) := \frac{1}{2} \int_0^{\omega_{\max}} w(\omega, \boldsymbol{\mu}) \|\mathbf{d}(\omega)\|^2 d\omega, \quad (13)$$

where $\boldsymbol{\mu}$ is the unknown vector of modification parameters that needs to be identified, $w(\omega, \boldsymbol{\mu})$ is the spectral reliability index,

$$w(\omega, \boldsymbol{\mu}) := w(\omega, \Delta\mathbf{M}(\boldsymbol{\mu})), \quad (14)$$

and $\mathbf{d}(\omega)$ is the difference between the measured response $\mathbf{u}^M(\omega)$ and the nonparametrically modeled response $\mathbf{u}(\omega)$,

$$\mathbf{d}(\omega) := \mathbf{u}^M(\omega) - \mathbf{u}(\omega). \quad (15)$$

In practice, to facilitate a comparison between objective functions computed in different identification cases, it might be useful to normalize (13) with respect to the measured response by dividing F by the w -weighted norm of $\mathbf{u}^M(\omega)$,

$$F_{\text{norm}}(\boldsymbol{\mu}) := \frac{1}{2} \frac{\int_0^{\omega_{\max}} w(\omega, \boldsymbol{\mu}) \|\mathbf{d}(\omega)\|^2 d\omega}{\int_0^{\omega_{\max}} w(\omega, \boldsymbol{\mu}) \|\mathbf{u}^M(\omega)\|^2 d\omega}. \quad (16)$$

Identification of mass modifications amounts here to minimization of the objective function (13) or (16) with respect to the modification parameters $\boldsymbol{\mu}$. The simplest approach is to use a zero-order optimization method. A more advanced method might use gradient-based or even Hessian-based optimization algorithms, and such an approach would require a dedicated sensitivity analysis method to be developed. Previous experiences with the time-domain version of the method [7] suggest that the adjoint variable method might be most effective in such an application.

IV. EXPERIMENTAL VERIFICATION

This section presents selected experimental results of identification of an added nodal mass in a 3D truss structure, and compares them to the corresponding results obtained using the time-domain formulation [7].

A. The structure

The experimental verification uses a 3D truss structure with 26 nodes and 70 elements, see Fig. 1. It is constructed using a commercially available system of spherical joints and connecting tubes. The structure is 4 m long, and its elements are circular steel tubes with the radius of 22 mm, the thickness of 1 mm and the lengths of 0.500 m or 0.707 m. The mass of each joint is 0.23 kg, and the total mass of the structure is approximately 32 kg. The two right-hand side nodes in the scheme are free to move in the longitudinal direction only, while the two opposite left-hand side nodes have restrained all their DOFs and are thus turned into fixed supports.

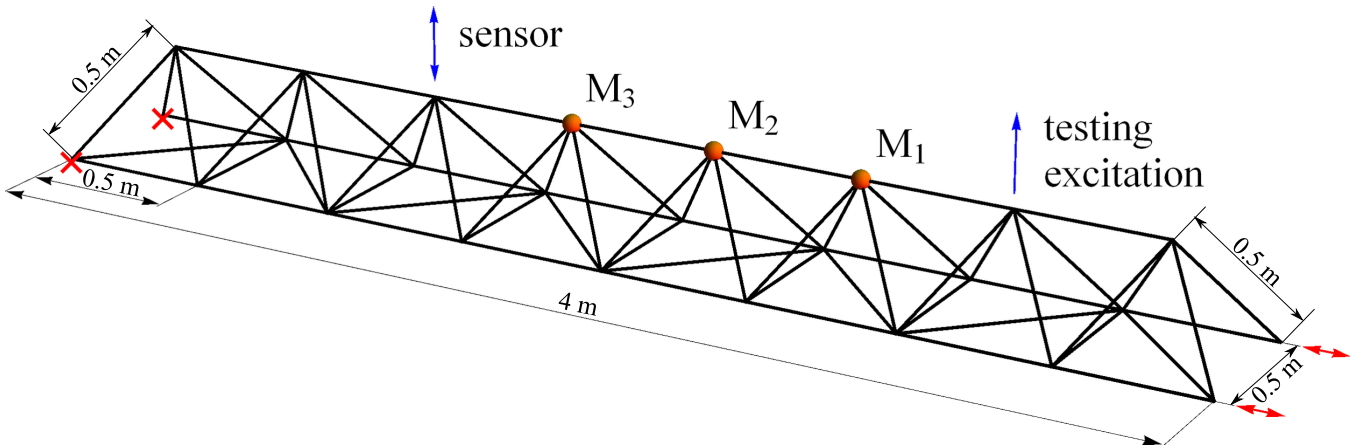
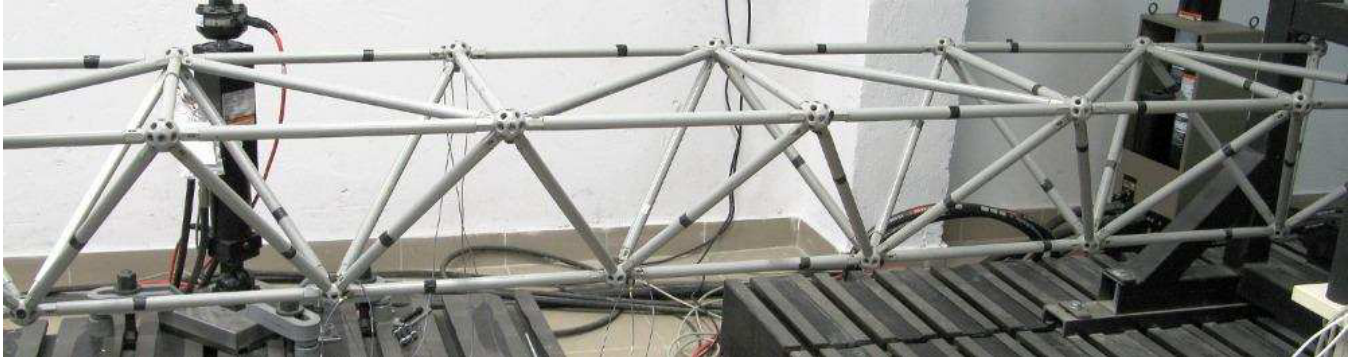


Fig. 1. The 3D truss structure used in experimental verification: (top) a fragment of the real structure; (bottom) scheme

B. Excitations and measurements

Fig. 1 shows the location of the testing excitation $f(t)$ and of the single sensor intended for identification and used to measure the response $\mathbf{u}^L(\omega)$ of the original structure and the response $\mathbf{u}^M(\omega)$ of the modified structure. A modal hammer is used to generate the testing excitation, as well as all the quasi impulsive excitations used in measurements of the necessary impulse responses. All the sensors are accelerometers; the signals from the accelerometers and from the modal hammer are collected by a Brüel & Kjær data acquisition system PULSE at 65.5 kHz and transferred to a desktop PC for further analysis. For each response, time-domain measurements are recorded independently four times and then averaged in order to diminish the adverse effects of the measurement noise. A total of 16 000 time steps is recorded for each response, which corresponds approximately to the time interval of 245 ms or 7.7 periods of the first natural vibration (31.5 Hz); in frequency-domain terms, it corresponds to approx. 4.1 Hz per frequency line. Fig. 2 plots the power spectra of all the impact hammer excitations used in the experiment. Based on the plots, the upper limit for the frequency integration ω_{\max} has been set at the safe level of 100 spectral lines, which approximately corresponds to 400 Hz.

C. Mass modifications

A total of 12 mass modification cases is tested. They are implemented experimentally by attaching concentrated masses at one of the nodes marked M_1 , M_2 and M_3 in Fig. 1(bottom). It is assumed that the location of the modifications is known in advance and need not be identified. Four different masses of 1.36 kg, 2.86 kg, 3.86 kg and 5.36 kg are used in each of the three nodes, which amounts to a total of 12 cases. Such modifications considerably alter the local structural dynamics in their neighborhood, since they range from 100% to almost 400% of the structural mass related to the modified node (joint mass plus half of the masses of the six neighboring elements, which is 1.36 kg altogether). On the other hand, in comparison to the total mass of the unmodified structure (32 kg), the relative mass modifications are much smaller and range from 4.25% to 16.75%. As an example, Fig. 3 plots the spectral reliability indices computed for the frequency range up to $2\omega_{\max}$ for a single mass of 3 kg added separately in on of the three considered nodes M_1 , M_2 and M_3 . The results confirm that the selected value of ω_{\max} is safe: up to ω_{\max} the reliability index remains at relatively high level with first significant drops well beyond ω_{\max} .

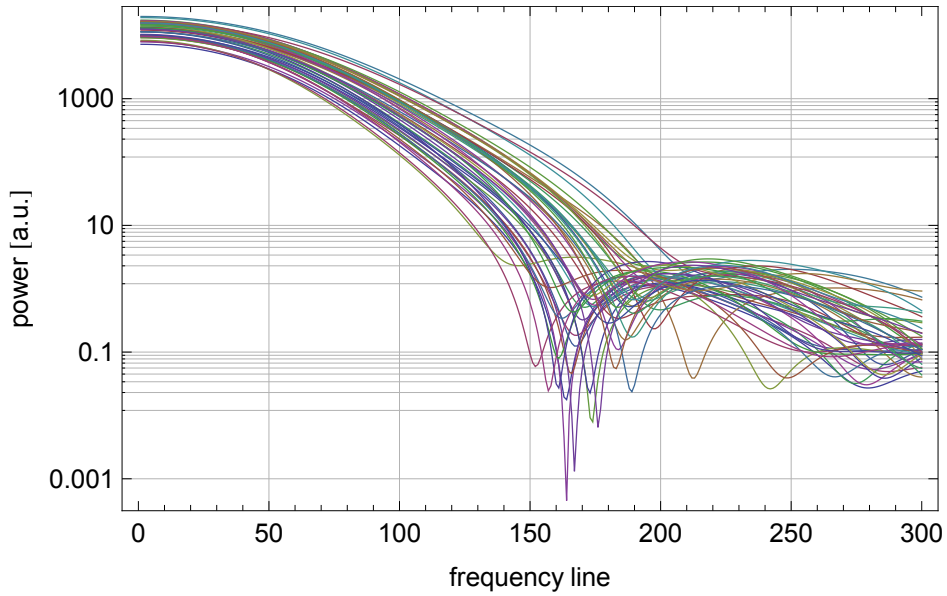


Fig. 2. Power spectra of all the impact hammer excitations used in the experiment

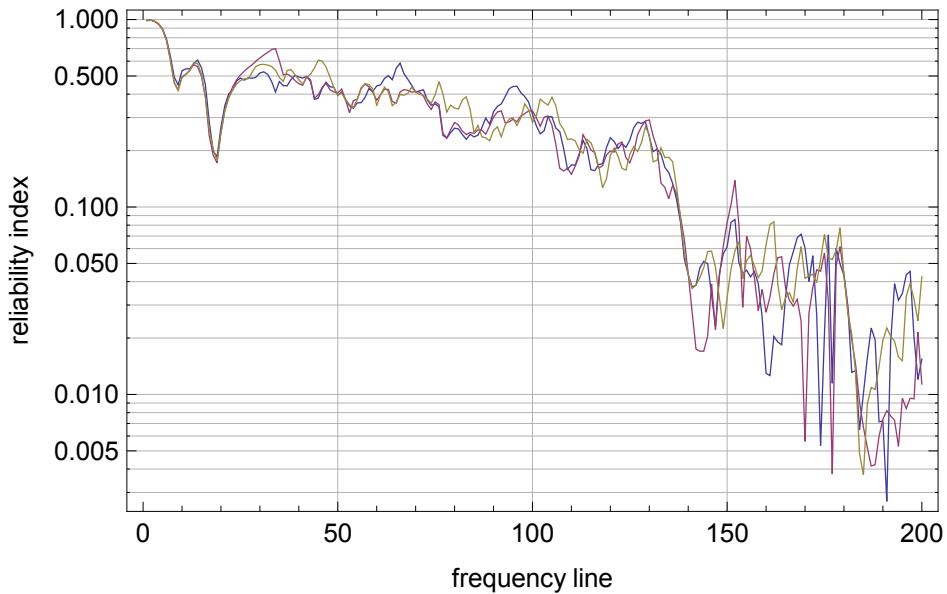


Fig. 3. Spectral reliability index computed for the frequency range 0 to $2\omega_{\max}$ for a single mass of 3 kg added in M_1 (blue), M_2 (purple) or M_3 (yellow)

D. Identification results

The results of identification depend on the decay rate of the exponential window used in computations of the fast Fourier transform (FFT), which is quantified in the terms of the window attenuation r at the end of the recorded time interval (e.g., if $\log r = 2$, the window drops to 0.01 at the end of the time interval). In order to test stability of the results, all the identifications are performed repeatedly using the attenuation rate varying from 1.0 up to 8.0. The results are plotted in Fig. 4: the top figure plots the mass identification results, while the bottom figure plots the relative accuracies in all 12 tested cases (thin lines) and the average relative accuracy (thick black line). It seems that there is a large window for the FFT window attenuation, in which the average relative accuracy of the identification remains at a relatively low level (below 2.5%). However, the worst-case accuracy is the best at approximately 1.8 window attenuation and then significantly increases. The second worst-case accuracy remains at a fairly subdued level in a large interval of 2.6 to 4.0 window attenuation. It suggests that a relatively large window of attenuation values can be used. In further computations, a compromise value of 2 is used, which means that the used FFT window drops to approximately 1% of the initial value at the end of the considered time interval. Such a value is typical and widely used in practice.

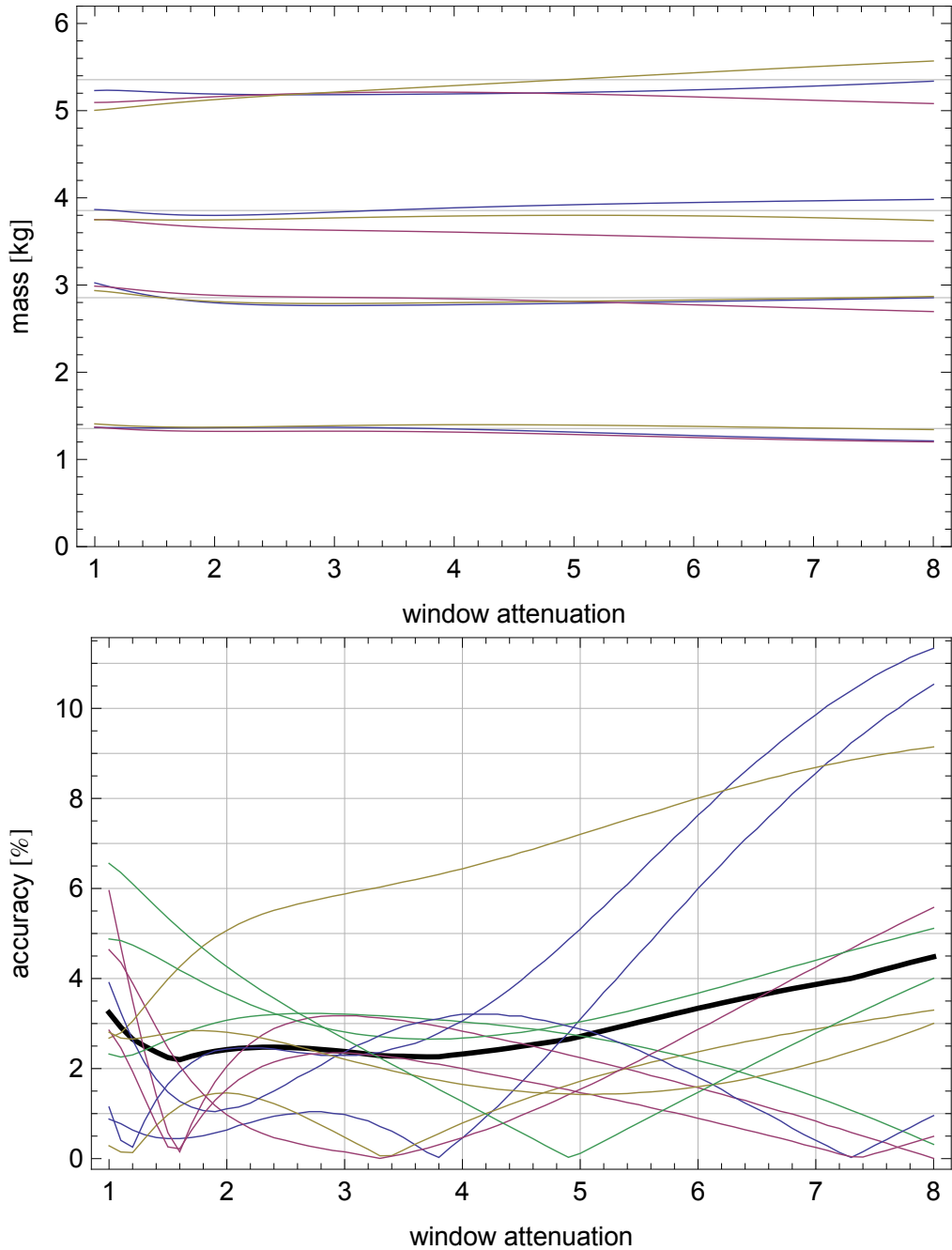


Fig. 4. Identification results in dependence on the attenuation of the FFT window: (top) identified masses (the dotted gridlines mark the actual modifications); (bottom) identification relative accuracies (thin lines) and the average relative accuracy (thick black line)

At the selected FFT window attenuation, the results of identification seem to slightly underestimate the actual modifications; the relative identification errors range between -5% and $+1\%$ with the average relative error of -2% . Fig. 5 (top) plots in the logarithmic scale the normalized objective functions F_{norm} computed for all the four masses and the three considered nodes; the functions are unimodal and their minima are clearly distinguishable. For comparison, Fig. 5 (bottom) plots the corresponding normalized objective functions obtained in [7] using the computationally extremely expensive time-domain approach. General characteristics of both families of the objective functions are very similar, even if the fit seems to be slightly better in the frequency-domain approach.

V. CONCLUSIONS AND FURTHER WORK

This paper presents and tests experimentally a frequency-domain version of a nonparametric approach to identification of structural mass modifications. The approach is based on the virtual distortion method (VDM) and requires neither parametric numerical model of the monitored structure nor any topological information, besides the locations of the potential

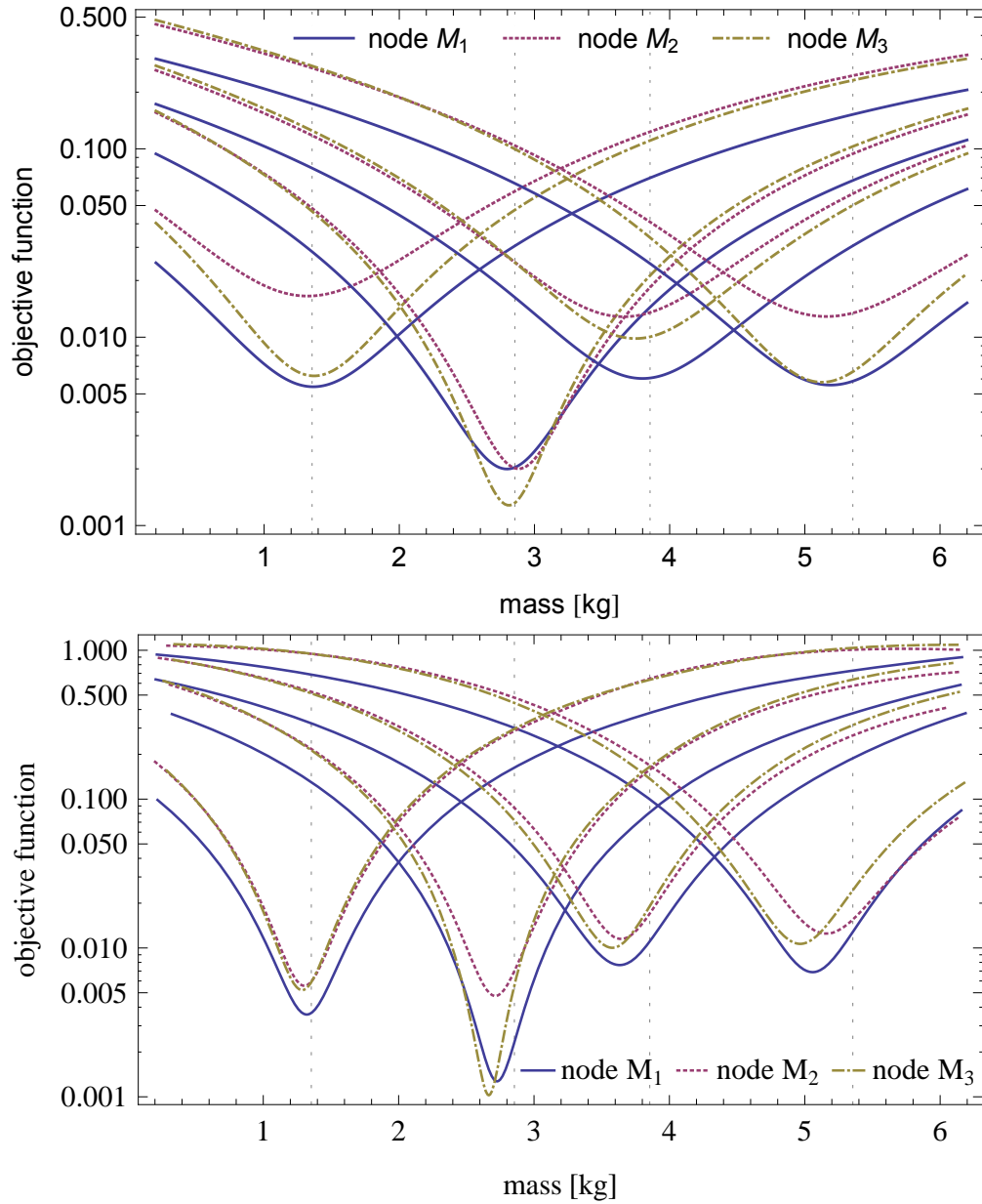


Fig. 5. Normalized objective functions: (top) frequency-domain approach; (bottom) time-domain approach [7]. Vertical gridlines mark the actual values of the modifications being identified

modifications. Experimentally measured local impulse-responses are directly used to model the response of the modified structure in an essentially nonparametric way. A 3D truss structure is used in the experimental validation. A total of 12 cases of single nodal mass modification are identified using a single impact test excitation and a single test sensor. The average relative accuracy of identification is less than 2.5% and noticeably better than the average accuracy of 5% obtained at much higher computational cost in the time-domain version of the approach [7].

The accuracy depends on the definition of the objective function, as well as on the characteristics, number and placement of the test excitations and test sensors, and on the measurement error level and structural topology. All these factors are reflected in the collected experimental impulse responses, and development of methods for assessing the ultimate accuracy based on the experimental data seems to be an interesting research problem. The methodology has several promising further research directions, including nonparametric modeling of bridge-like structures for the purpose of retrofitting them with semi-active dampers with the objective of reduction of vibration from traffic-induced moving loads [21], [22], [23], [24], nonparametric modeling for identification of impact-type loads [25] or in semi-active absorption of such loads of various origins [26], [27], [28], [29], [30], [31].

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