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Model-free identification of added mass

Grzegorz Suwała, Łukasz Jankowski

Institute of Fundamental Technological Research, ul. Swietokrzyska 21, 00-049 Warsaw, Poland
e-mail: gsuwala@ippt.gov.pl, ljank@ippt.gov.pl

1. Abstract
This paper presents and experimentally validates a model-free methodology for off-line identification of modifications of structural mass. The proposed approach makes use of the Virtual Distortion Method (VDM) and is based entirely on experimentally measured data of the original unmodified structure, which is a significant advantage: no numerical modeling of the structure and tedious updating of the model are necessary. The mass modification is modeled using equivalent virtual distortion forces and experimentally obtained local impulse-responses of the unmodified structure. The identification amounts to solving an optimization problem of minimizing the mean-square distance between measured and modeled responses of the modified structure; a quick first- and second-order sensitivity analysis using the adjoint variable method is proposed. The method is validated experimentally using a 4-meter-long 70-element truss structure.
2. Keywords: mass identification, model-free SHM, virtual distortion method (VDM), adjoint variable method

3. Introduction

The long-term motivation behind this research is a need for an experimentally robust analysis technique for mass and damage monitoring that could be used in black-box type monitoring systems. In general, identification (or quantification) methods used in global structural health monitoring (SHM), including mass identification methods, can be classified into two broad groups [1]:

1. *Pattern recognition methods*, which use a database of numerical fingerprints of low dimension that are extracted from several responses [4] and sometimes encoded in the form of a trained neural network [5]. The database has to be previously constructed by simulations or by experimental measurements of the intact structure and of the several damage or modification scenarios, which are to be identified later. Given the database and a measured response of the affected structure, the actual scenario is identified using the fingerprints only, without the insight into their actual mechanical meaning, so that neither a numerical model of the structure nor a simulation of the actual scenario are necessary.

2. *Model-based methods* require a pre-calibrated numerical model of the structure [1,2,3,13,14]; the identification amounts to iterative modification of parameters of the model, simulation of the corresponding response and its comparison with an experimentally measured response of the modified or damaged structure.

The approach proposed in this paper uses the virtual distortion method (VDM) [1,6] in order to address the two fundamental deficiencies of the global identification methods: the need for numerical modeling of the involved structure or for its repetitive actual modifications. To a certain degree, the proposed method can be called a model-based approach, since it makes use of certain characteristics of the unmodified structure in order to compute the response of the modified structure. However, instead of building and updating the numerical model, the characteristics is directly provided by easy-to-measure impulse-responses of the unmodified structure, which are limited to the potential modification points. Thus, similar as in the model-based approach, the proposed method is directly based on mechanical principles and computes full responses of the modified structure, but only experimentally measured data is used for that purpose, so that no numerical model of the structure is required. Moreover, there is even no need for any topological information, besides the locations of the potential modifications. Given the test excitation, the response of the structure with modified mass is modeled using equivalent virtual distortion forces and the experimentally measured impulse-responses. The distortion forces are given in the form of the unique solution to a certain integral equation, which is solved numerically by discretization that transforms it

into a large and extremely ill-conditioned linear system with Toeplitz blocks. Its efficient and accurate solution is made possible by employing a quick iterative algorithm with regularizing properties, storing the system in computer memory in a reduced form and an FFT-based quick matrix-vector multiplication. Given the virtual forces, their convolution with the impulse-responses equals to the influence of the added mass on the structural response.
The identification of the added mass amounts to solving an inverse problem, which is formulated here as an optimization problem of minimizing the mean square distance between measured and modeled structural responses. The proposed approach makes possible fast and exact first- and second-order sensitivity analyses by the adjoint variable method [8,9,10], so that quickly convergent Newton optimization algorithms can be directly used.
The paper is structured as follows: the next (fourth) section describes the direct problem of modeling the response of a modified structure, the fifth section states the inverse problem of mass identification, including the first- and second-order sensitivity analyses. In practical implementation both the direct and the inverse problems are discrete and have to be solved numerically; this is discussed in the sixth section. The seventh section presents the experimental results.

4. Direct problem

4.1. Response as a convolution

Let $\mathbf{p}(t)$ be any excitation vector of the original structure (*unmodified structure*) and denote by $\mathbf{u}^{\mathrm{R}}(t)$ the vector of the corresponding response. It is assumed that the structure is linear and that its equation of motion can be stated as

$$\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{R}}(t) + \mathbf{K}\mathbf{u}^{\mathrm{R}}(t) = \mathbf{p}(t), \tag{1}$$

where the original mass and stiffness matrices have been denoted by $\mathbf{M}$ and $\mathbf{K}$, respectively. Later in the paper, the excitation $\mathbf{p}(t)$ stands for the modification-modeling force distortions, thus the superscript R (residual) in the symbol $\mathbf{u}^{\mathrm{R}}(t)$ that denotes the response.
Since Eq.(1) is linear, the response can by expressed in the form of a convolution of the excitation $\mathbf{p}(t)$ with the impulse-response of the structure $\mathbf{B}(t)$,

$$\mathbf{u}^{\mathrm{R}}(t) = \int_0^t \mathbf{B}(t-\tau)\mathbf{p}(\tau)\, d\tau, \tag{2a}$$

$$\ddot{\mathbf{u}}^{\mathrm{R}}(t) = \mathbf{M}^{-1}\mathbf{p}(t) + \int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}(\tau)\, d\tau. \tag{2b}$$

4.2. Force distortions

Let $\mathbf{f}(t)$ denote the *test excitation*, that is the excitation used for testing for the mass modification, and let $\mathbf{u}^{\mathrm{L}}(t)$ denote the corresponding reference response of the *unmodified structure*. The equation of motion takes the form of

$$\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{K}\mathbf{u}^{\mathrm{L}}(t) = \mathbf{f}(t). \tag{3}$$

The same test excitation $\mathbf{f}$ applied to the *modified structure* results in the response $\mathbf{u}(t)$, which is described by the following equation of motion:

$$(\mathbf{M} + \Delta\mathbf{M})\,\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{4}$$

where $\Delta\mathbf{M}$ stands for the mass modification. The formula Eq.(4) can be rearranged to obtain the equation of motion of the unmodified structure

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{p}(t), \tag{5}$$

where the mass modification is modeled by the vector $\mathbf{p}(t)$ of respective virtual force distortions (that is, pseudo forces or residual forces)

$$\mathbf{p}(t) = -\Delta\mathbf{M}\ddot{\mathbf{u}}(t). \tag{6}$$

According to Eqs.(1), (3) and (5), the response of the modified structure $\mathbf{u}(t)$ to the test excitation is the sum of the responses of the unmodified structure to the test excitation $\mathbf{u}^{\mathrm{L}}(t)$ and to the force distortions $\mathbf{u}^{\mathrm{R}}(t)$,

$$\mathbf{u}(t) = \mathbf{u}^{\mathrm{L}}(t) + \mathbf{u}^{\mathrm{R}}(t). \tag{7}$$

Formula Eq.(7), together with Eqs.(6) and (2b) yield

$$\mathbf{p}(t) = -\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t) - \Delta\mathbf{M}\left[\mathbf{M}^{-1}\mathbf{p}(t) + \int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}(\tau)\,d\tau\right],$$

which stated in the standard form, is a system of Volterra integral equations of the second kind with weakly singular kernels [7]:

$$\left[\mathbf{I} + \Delta\mathbf{M}\mathbf{M}^{-1}\right]\mathbf{p}(t) + \Delta\mathbf{M}\int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}(\tau)\,d\tau = -\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t), \tag{8}$$

where the unknowns are the force distortions $\mathbf{p}(t)$. This system may be (and usually indeed is) ill-conditioned. However, it follows from the Riesz theory [7] that the system Eq.(8) is well-posed and so has always a unique solution, provided the matrix $\mathbf{I} + \Delta\mathbf{M}\mathbf{M}^{-1}$ is nonsingular.
According to Eq.(6), the force distortions vanish in the degrees of freedom (DOFs) that are unrelated to the assumed mass modification $\Delta\mathbf{M}$. As the result, the system Eq.(8) can be in practice restricted to the DOFs that are related to the modifications (that is, the DOFs corresponding to the non-vanishing rows of $\Delta\mathbf{M}$).
Note that, in order to solve Eq.(8), besides the assumed mass modification $\Delta\mathbf{M}$, three other components are necessary:

1. the matrix $\mathbf{B}(t)$ of the impulse-responses of the unmodified structure. Its size is restricted to the DOFs related to the locations of mass modifications, which makes $\mathbf{B}(t)$ relatively easy to be experimentally measured.
2. the reference response $\mathbf{u}^{\mathrm{L}}(t)$ of the unmodified structure to the test excitation $\mathbf{f}(t)$. Again, the response has to be known only in the limited number of the DOFs that are related to the locations of mass modifications, which makes $\mathbf{u}^{\mathrm{L}}(t)$ relatively easy to be experimentally measured.
3. a respective submatrix of the mass matrix $\mathbf{M}$. This is the only component that is not straightforwardly measurable, hence seems to contradict the statement of a model-free formulation. However, in any real-world implementation all measurements are discretized, and in the discretized version of the problem (Section 6), the mass matrix is integrated into the measured impulse-response.

4.3. The response

For a given mass modification $\Delta\mathbf{M}$ and the corresponding force distortions $\mathbf{p}(t)$ obtained by solving the system Eq.(8), the response of the modified structure can be computed by Eqs.(7) and (2a) as a sum of the response of the unaffected structure and the cumulative effects of the forces distortions that model the modifications,

$$\mathbf{u}(t) = \mathbf{u}^{\mathrm{L}}(t) + \int_0^t \mathbf{B}(t-\tau)\mathbf{p}(\tau)\,d\tau. \tag{9}$$

5. Inverse problem

The inverse problem is stated in the standard way as a problem of minimization of the discrepancy between the measured and the modeled responses of the modified structure. Quick and exact first- and second-order sensitivity analyses based on the method of the adjoint variable [8,9,10] are proposed here, which allows quickly converging exact Newton optimization methods to be implemented.

5.1. Objective function

Given the test excitation, the identification of the unknown mass modification is based on the comparison between the measured response of the modified structure and its modeled response. The following objective function is used:

$$F(\Delta\mathbf{M}) = \frac{1}{2}\int_0^T \|\mathbf{u}^{\mathrm{M}}(t) - \mathbf{u}(t)\|^2\,dt, \tag{10}$$

where $\mathbf{u}^{\mathrm{M}}(t)$ is the measured response of the modified structure to the test excitation $\mathbf{f}(t)$. The identification amounts to the minimization of the function F.

5.2. First-order sensitivity analysis

A direct differentiation of Eq.(10) with respect to the ith parameter of mass modification (its precise meaning is left to the user) yields a formula that contains the corresponding derivatives of the response,

$$F_i(\Delta\mathbf{M}) = -\int_0^T \left[\mathbf{u}^{\mathrm{M}}(t) - \mathbf{u}(t)\right]^{\mathrm{T}} \mathbf{u}_i(t)\, dt. \tag{11}$$

The direct method of sensitivity analysis (direct differentiation method) computes these derivatives by differentiating Eqs.(9) and (8) and solving the latter equation several times, separately for each i. A much quicker way (one order of magnitude) can be provided by the method of adjoint variable: the differentiated equation Eq.(8) is scalarly multiplied by a vector $\boldsymbol{\lambda}(t)$ of the adjoint variables and integrated with respect to time, the result is added to Eq.(11) and the adjoint variable is assigned the value that eliminates the derivatives of the response $\mathbf{u}_i(t)$. In this way the derivative $F_i(\Delta\mathbf{M})$ can be computed as

$$F_i(\Delta\mathbf{M}) = \int_0^T \boldsymbol{\lambda}^{\mathrm{T}}(t)\Delta\mathbf{M}_i\ddot{\mathbf{u}}(t)\, dt, \tag{12}$$

where the adjoint variable $\boldsymbol{\lambda}(t)$ turns out to be independent of i and can be computed at the cost of only single solution of the integral adjoint equation

$$\left[\mathbf{I} + \Delta\mathbf{M}\mathbf{M}^{-1}\right]^{\mathrm{T}} \boldsymbol{\lambda}(t) + \int_t^T \ddot{\mathbf{B}}^{\mathrm{T}}(\tau - t)\Delta\mathbf{M}\boldsymbol{\lambda}(\tau)\, d\tau = \int_t^T \mathbf{B}^{\mathrm{T}}(\tau - t)\left[\mathbf{u}^{\mathrm{M}}(t) - \mathbf{u}(t)\right]\, d\tau. \tag{13}$$

5.3. Second-order sensitivity analysis

The direct method of computing the Hessian would require a double differentiation of Eqs.(10), (9) and (8) and solving the latter separately for each element of the Hessian. The direct-adjoint method, which seems to be the quickest from the family of second-order adjoint variable methods [10], can reduce the numerical costs by one order of magnitude. In the same way as in the first-order analysis, the adjoint variable is used to eliminate the second derivatives of the response from the double-differentiated Eq.(10). Additionally, it is assumed here that the modification parameters are simply the added masses, which yields $\Delta\mathbf{M}_{ij} = \mathbf{0}$ and a considerably simpler formula for the element $F_{ij}(\Delta\mathbf{M})$ of the Hessian:

$$\begin{aligned} F_{ij}(\Delta\mathbf{M}) &= \int_0^T \mathbf{u}_i(t)\mathbf{u}_j(t)\, dt \\ &\quad + \int_0^T \boldsymbol{\lambda}^{\mathrm{T}}(t)\left[\Delta\mathbf{M}_i \int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}_j(\tau)\, d\tau + \Delta\mathbf{M}_j \int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}_i(\tau)\, d\tau\right] dt, \end{aligned}$$

where $\boldsymbol{\lambda}(t)$ denotes the adjoint variable defined by Eq.(13). The derivatives of the response $\mathbf{u}_i(t)$ and of the force distortion $\mathbf{p}_i(t)$ have to be obtained using the direct differentiation method by solving differentiated Eq.(8) separately for each modification parameter i,

$$\left[\mathbf{I} + \Delta\mathbf{M}\mathbf{M}^{-1}\right]\mathbf{p}_i(t) + \Delta\mathbf{M}\int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}_i(\tau)\, d\tau = -\Delta\mathbf{M}\ddot{\mathbf{u}}(t),$$

and by a substitution of the result into differentiated Eq.(9).

Note that, if the second-order analysis is performed, then as a low-cost by-product, the first derivatives of the objective function can be directly computed using Eq.(11) and compared to Eq.(12) for verification purposes.

6. Discretization and numerical solution

6.1. Discretization of the direct problem

The proposed approach is intended for practical implementation. In practice all data are experimentally measured. Hence, instead of being continuous, all required responses $\mathbf{u}^{\mathrm{L}}$, $\ddot{\mathbf{u}}^{\mathrm{L}}$, $\mathbf{u}^{\mathrm{M}}$, $\mathbf{B}$ and $\ddot{\mathbf{B}}$ are vectors that are sampled in discrete time points every Δt. In the model-free approach, the discretization turns out to be an advantage, since it allows the unknown mass matrix $\mathbf{M}$, which originates from the feed-through term $\mathbf{M}^{-1}\mathbf{p}(t)$ in Eq.(2b), to be eliminated from the integral equations. If the measured discrete impulse-response $\ddot{\mathbf{D}}(t)$ is used instead of the continuous $\ddot{\mathbf{B}}(t)$, the term becomes integrated into $\ddot{\mathbf{D}}(0)$,

$$\ddot{\mathbf{D}}(t) = \begin{cases} \mathbf{M}^{-1} + \ddot{\mathbf{B}}(0)\Delta t & \text{if } t = 0, \\ \ddot{\mathbf{B}}(t)\Delta t & \text{otherwise,} \end{cases}$$

so that

$$\ddot{\mathbf{u}}^{\mathrm{R}}(t) = \sum_{\tau=0}^{t} \left[\delta_{t\tau}\mathbf{M}^{-1} + \ddot{\mathbf{B}}(t-\tau)\Delta t\right] \mathbf{p}(\tau) = \sum_{\tau=0}^{t} \ddot{\mathbf{D}}(t-\tau)\mathbf{p}(\tau).$$

Therefore, the continuous integral equation Eq.(8) is transformed into the following discrete linear system without the explicit dependence on the unknown mass matrix $\mathbf{M}$:

$$\sum_{\tau=0}^{t} \left[\Delta\mathbf{M}\ddot{\mathbf{D}}(t-\tau) + \delta_{\tau t}\right] \mathbf{p}(\tau) = -\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t),$$

which can be stated in the form of a single large linear system

$$\mathbf{A}\mathbf{p} = -\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}, \tag{14}$$

where the vectors $\mathbf{p}$ and $\ddot{\mathbf{u}}^{\mathrm{L}}$ collect the dicretized force distortions and responses, respectively. The system matrix $\mathbf{A}$ is a $3n \times 3n$ block matrix with $T \times T$ lower triangular Teoplitz blocks, where T is the number of the considered time steps and n is the number of the nodes related to mass modifications. The structure of a typical system matrix is illustrated in Figure 4.

Given, by solving Eq.(14), the discretized virtual forces $\mathbf{p}$, the modeled discretized response $\mathbf{u}$ of the modified structure can be computed as

$$\mathbf{u} = \mathbf{u}^{\mathrm{L}} + \mathbf{D}\mathbf{p},$$

where the matrix $\mathbf{D}$ is a block matrix with Toeplitz blocks and is a discrete counterpart of the convolution operator in Eq.(9).

6.2. Discretization of the inverse problem and sensitivity analysis

Identification of mass modifications amounts to the minimization of the objective function

$$F(\Delta\mathbf{M}) = \frac{1}{2}\mathbf{d}^{\mathrm{T}}\mathbf{d}, \qquad \text{where} \qquad \mathbf{d} = \mathbf{u}^{\mathrm{M}} - \mathbf{u}.$$

The derivative F_i of the objective function can be computed by a discretization of Eqs.(12) and (13), which yields the following formulas of the discrete adjoint variable method:

$$F_i(\Delta\mathbf{M}) = \boldsymbol{\lambda}^{\mathrm{T}}\Delta\mathbf{M}_i\ddot{\mathbf{u}},$$

where $\boldsymbol{\lambda}$ is a vector of the discrete adjoint variables that can be obtained at the cost of a single solution of the discrete adjoint equation

$$\mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{B}^{\mathrm{T}}\mathbf{d}, \tag{15}$$

which is the discrete counterpart of Eq.(13). By a similar discretization, the Hessian of the objective function can be computed as

$$F_{ij}(\Delta\mathbf{M}) = \mathbf{p}_i^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{B}\mathbf{p}_j + \boldsymbol{\lambda}^{\mathrm{T}}\left[\Delta\mathbf{M}_i\ddot{\mathbf{B}}\mathbf{p}_j + \Delta\mathbf{M}_j\ddot{\mathbf{B}}\mathbf{p}_i\right], \tag{16}$$

where $\boldsymbol{\lambda}$ denotes the discrete adjoint vector defined by Eq.(15), and $\mathbf{p}_i$ is the vector of the first-order derivatives of force distortions, which has to be obtained by solving the discrete counterpart of Eq.(11),

$$\mathbf{A}\mathbf{p}_i = -\Delta\mathbf{M}_i\ddot{\mathbf{u}}, \tag{17}$$

separately for each considered parameter of mass modification. The time complexity of Hessian computation is linear with respect to the number of unknowns, instead of the quadratic complexity of the direct method. As before, it is assumed in Eq.(16) that $\Delta\mathbf{M}_{ij}$ vanish, which happens for example when the modification parameters are the mass modifications, as it is usually the case in practice.

6.3. Numerical solution

Theoretically, given the discrete versions of the direct and the inverse problems, identification of mass modifications is straightforward and amounts to an iterative minimization of the objective function. In each iteration, two square linear systems have to be solved (Eq.(14) and the adjoint system Eq.(15)); if a second-order optimization method is used, then a separate solution of Eq.(17) for each optimization variable is also necessary.

However, all responses are stored and processed in time domain. This can result in large dimensions of the system matrix $\mathbf{A}$, which is a $3n \times 3n$ block matrix with $T \times T$ blocks, where T is the number of the time steps and n is the number of the nodes related to mass modifications; the total dimensions are thus $3nT \times 3nT$. In case of a longer time interval or a non-localized modification, the matrix can become huge and unmanageable by standard numerical techniques. Moreover, as can be expected from the Toeplitz structure of its blocks [11], the matrix is significantly ill-conditioned, and a regularization technique has to be used in order to obtain meaningful solutions. Therefore, it is usually advantageous to use the quick iterative algorithm of conjugate gradient least squares (CGLS, sometimes also called CGNE for conjugate gradient normal equations) [12] to solve all involved systems, since

- the CGLS method has good regularizing properties. The role of the regularization parameter is played by the number of iterations: the more iterations, the more exact but less regularized (that is more influenced by the measurement error) is the solution;
- the method uses the system matrix $\mathbf{A}$ only in the form of the matrix-vector products $\mathbf{Ax}$ and $\mathbf{A}^{\mathrm{T}}\mathbf{y}$, so that no costly matrix decomposition and even no direct access to its elements are necessary.

Moreover, the structure of the system matrix $\mathbf{A}$ can be also exploited:

- Each block of $\mathbf{A}$ is a $T \times T$ lower triangular Toeplitz block, and hence can be stored in computer memory in a reduced form using only T elements instead of T^2.
- Matrix-vector product for each $T \times T$ Toeplitz matrix can be computed in frequency domain using the FFT in time $O(T \log T)$ instead of $O(T^2)$, which makes the CGLS method especially appealing, as it makes use of $\mathbf{A}$ only in the form of matrix-vector products.

7. Experimental verification

7.1. Experimental test stand

To verify the proposed method, a 24-node and 70-element truss structure has been used, see Figure 1. The structure is 4 m long, the elements are 500 mm or 707 mm long circular steel tubes with a radius of 22 mm and a thickness of 1 mm; the total weight of the structure is approximately 32 kg. The two left-hand side nodes have been fixed, while the two opposite right-hand side nodes have been deprived of two degrees of freedom and are free to move only longitudinally.

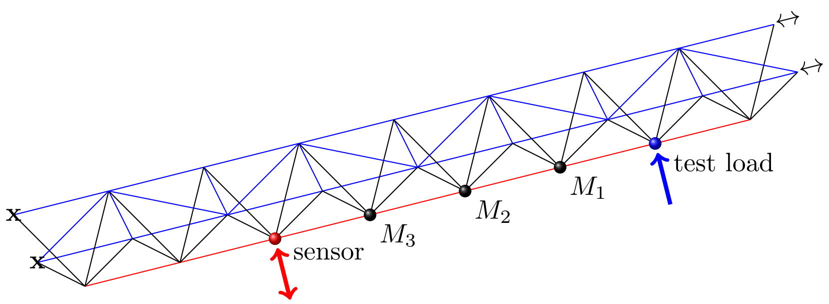

Figure 1: Schema of the truss structure used for experimental verification

Only nodal mass modifications have been considered, their locations are marked in Figure 1. Two modification scenarios have been investigated:

1. single nodal mass modification in node M_2 only and
2. modification of two nodal masses in nodes M_1 and M_3.

Additionally, Figure 1 shows the location of the test excitation and the sensor that have been used to identify the modifications.

An impact hammer has been used for the test excitation as well as for building the influence matrices in order to store the responses to impulse force excitations in all DOFs of nodes M_1, M_2 and M_3. The responses have been measured with accelerometers, the measurements were triggered by the hammer

excitation and sampled at 65.5 kHz. The signals from the sensors and the hammer were collected via a Brüel & Kjær PULSE system to a desktop PC for analysis. The acquisition system has internally double-integrated the acceleration responses to obtain the displacements.
Note that for the influence matrices $\mathbf{D}(t)$ and $\ddot{\mathbf{D}}(t)$ the impulse excitation is necessary, while a modal hammer can generate only excitations that last several time steps. Therefore, the recorded responses $\mathbf{D}^{\text{measured}}(t)$ and $\ddot{\mathbf{D}}^{\text{measured}}(t)$ had to be deconvoluted with the measured hammer excitation $\mathbf{f}(t-\tau)$ by solving several linear systems of the following form:

$$\sum_{\tau=0}^{t} f_j(t-\tau) D_{ij}(\tau) = D_{ij}^{\text{measured}}(t).$$

For the deconvolution, the CGLS method has been used.

7.2. Single nodal mass modification

In the first considered scenario, a single mass modification occurs in the node M_2. Four different masses have been attached to the node and tested: 1.36 kg, 2.86 kg, 3.86 kg and 5.36 kg. For the identification, the first 10 000 time steps have been used, which corresponds to the analysis time interval of approximately 150 ms and results in the matrix $\mathbf{A}$ in Eq.(14) having the dimensions $30\,000 \times 30\,000$. Note that these dimensions, if stored in computer memory in the full form using standard 8-byte double numbers, correspond to a hardly manageable 6.7 GB of data. On the other hand, in the reduced form that exploits the Toeplitz structure of the blocks, the matrix requires only 703 KB of storage.
Figure 2 (left) shows the results of the identifications. In computations, the Newton optimization algorithm with exact second derivatives has been used. In all four cases the relative identification errors are less than 5%. Note that the relative error increases with the increasing mass, which seems to be the result of the relative weighting of the two left-hand side terms in Eq.(8): if the modification $\Delta\mathbf{M}$ is small with respect to $\mathbf{M}$, the weighting coefficient $\mathbf{I} + \Delta\mathbf{M}\mathbf{M}^{-1}$ of the direct term $\mathbf{p}(t)$ is approximately constant, while the weighting coefficient of the convolution term is simply $\Delta\mathbf{M}$. The ill-conditioning is introduced by the latter term, hence the larger the modification, the more ill-conditioned is the identification task and the more information is lost below the measurement error level.

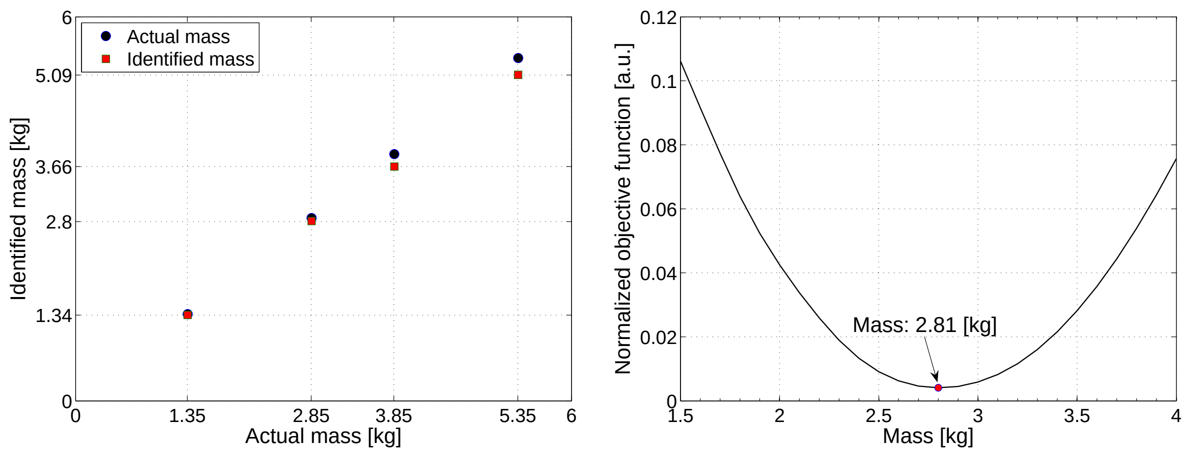

Figure 2: Single nodal mass modification: (left) identification results: identified mass vs. actual mass; (right) objective function computed for the actual modification mass of 2.86 kg

As an example, Figure 2 (right) shows the objective function computed for the actual mass of 2.86 kg; the minimum is found at 2.81 kg, which corresponds to the relative identification error of 1.7%. Figure 3 compares the three related responses of interest: the measured response of the original unmodified structure, the measured response of the modified structure (2.86 kg) and the response modeled for the identified mass of 2.81 kg. The computed response seems to model the measured response well, including its fine higher frequency characteristics.

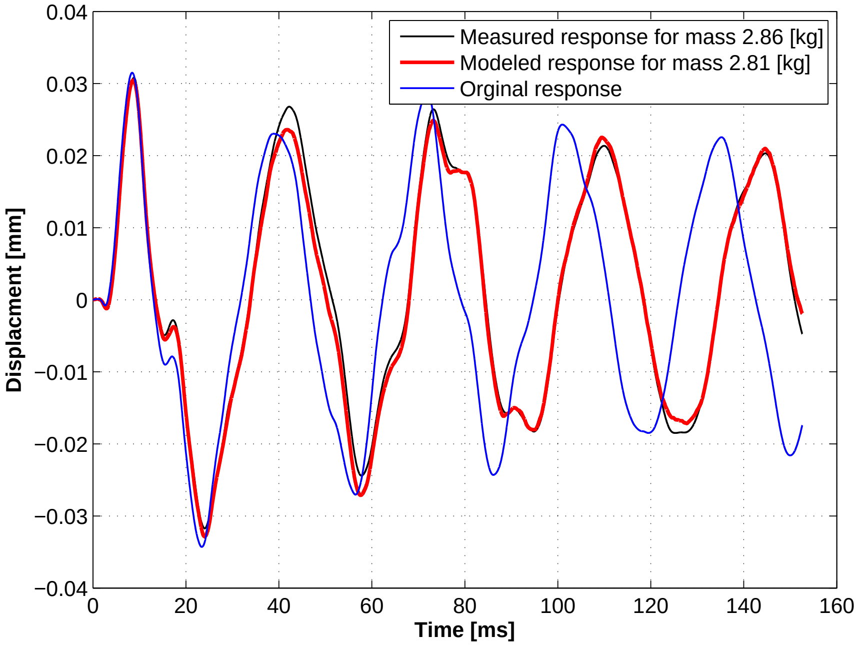

Figure 3: Single nodal mass modification: measured response at the actual mass 2.86 kg, modeled response for the identified mass 2.81 kg and the response of the original unmodified structure

7.3. Modification of two nodal masses

In the second scenario, mass modifications occur simultaneously in two nodes of the structure (M_1 and M_3). Again, four different cases have been tested, that is all four combinations of $M_1 \in \{1.36\,\text{kg}, 2.86\,\text{kg}\}$ and $M_3 \in \{1.39\,\text{kg}, 2.89\,\text{kg}\}$. The system matrix $\mathbf{A}$ in Eq.(14) is a 6×6 block matrix, where each block row and block column corresponds to one of the six DOFs of the two considered nodes. In order to retain the same dimensions of the matrix as in the previous case, the first 5 000 time steps are used in computations. The structure of the matrix $\mathbf{A}$ is illustrated in Figure 4.

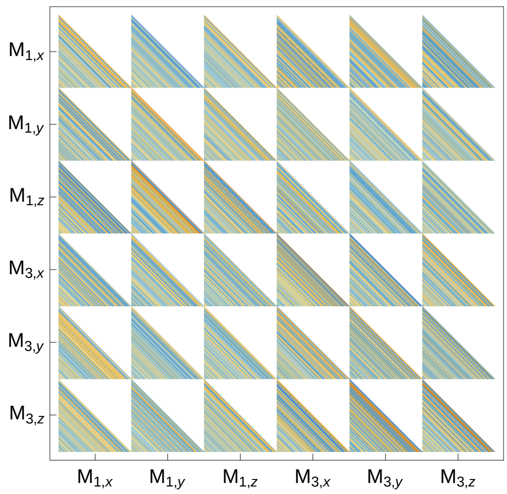

Figure 4: Modification of two nodal masses: structure of the system matrix $\mathbf{A}$ in Eq.(14)

Figure 5 (left) shows the results of the identifications, which have been obtained using the Newton optimization algorithm and exact Hessians. The relative identification errors vary between 1.8% and 13.7%. Again, the relative errors are generally larger for larger modifications.

As an example, Figure 5 (right) shows the objective function computed for the actual masses $M_1 = 1.36\,\mathrm{kg}$ and $M_3 = 2.89\,\mathrm{kg}$. The minimum is found at $1.54\,\mathrm{kg}$ and $2.79\,\mathrm{kg}$, respectively; the respective relative errors are 13.7% and 3.6%. Note that the objective function forms a long and rather narrow valley. It is relatively easy to find the bottom line of the valley (which corresponds approximately to the constant sum of the two masses), while the exact location of the minimum along the bottom line is sensitive to measurement and numerical errors. Figure 6 compares the three responses related to the considered case: the measured response of the original unmodified structure, the measured response of the modified structure and the response modeled for the identified masses.

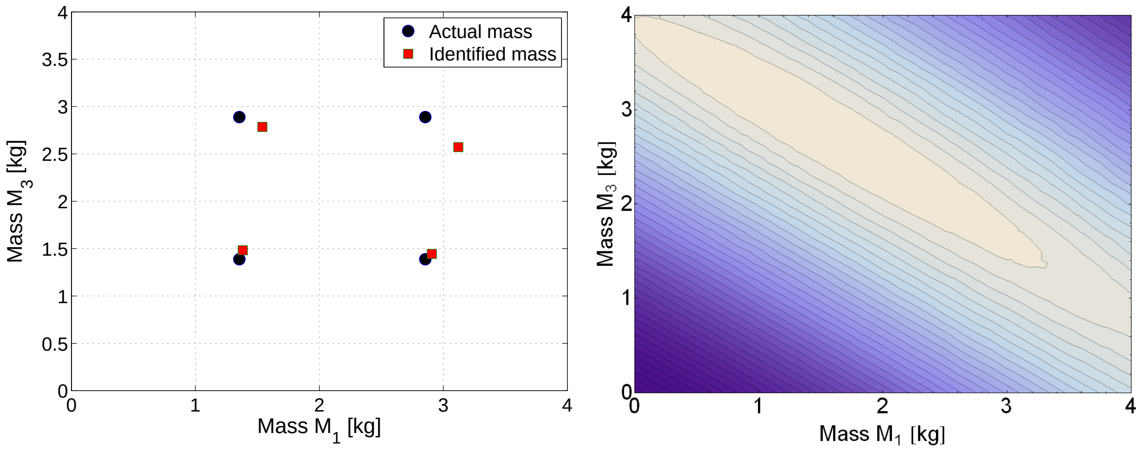

Figure 5: Modification of two nodal masses: (left) identification results; (right) objective function computed for the actual masses $M_1 = 1.36\,\mathrm{kg}$ and $M_3 = 2.89\,\mathrm{kg}$

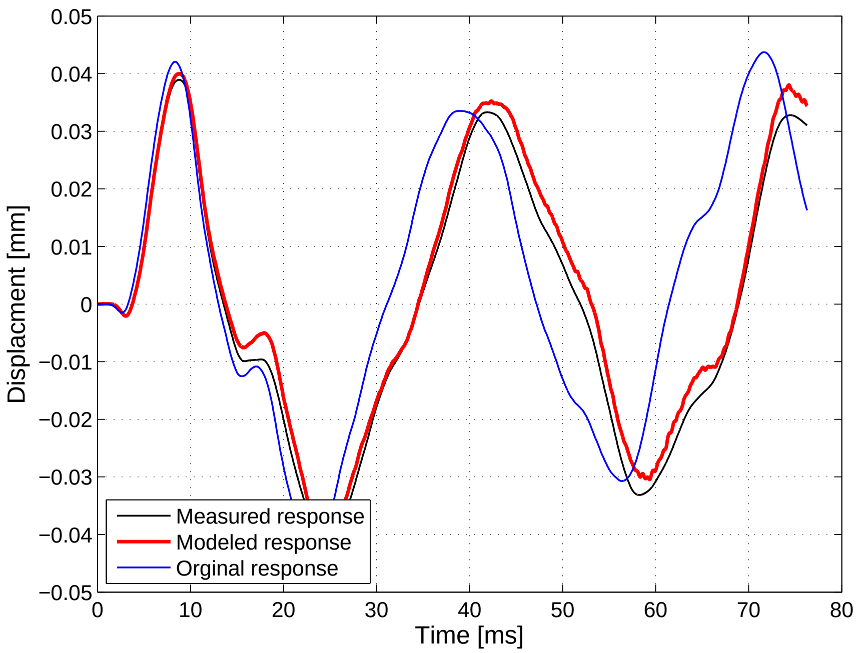

Figure 6: Modification of two nodal masses: measured response at the actual masses $(M_1, M_3) = (1.36\,\mathrm{kg}, 2.89\,\mathrm{kg})$, modeled response for the identified masses $(1.54\,\mathrm{kg}, 2.79\,\mathrm{kg})$ and the response of the original unmodified structure

8. Conclusions

In this paper, a new model-free method for added mass identification is proposed. The method requires no numerical model of the monitored structure, is based on the virtual distortion method (VDM) and makes use of experimental data, which are measured using the unmodified structure only. However, contrary to pattern-recognition experimental methods of identification, the proposed method utilizes directly actual mechanical principles and not a database of purely numerical response fingerprints.

The method has been validated experimentally using a 70-element 3D truss and nodal mass modifications of 4% – 18% of the total mass. Single and double modifications have been tested and both successfully identified using a single sensor with the relative errors less than 5% and 14%, respectively.

The long-term motivation behind this research is the need for an experimentally robust analysis technique for mass and damage identification that could be used in black-box type monitoring systems and for reconstruction of the scenario of a mass-induced loading. The research is ongoing to generalize the approach to identification of structural damages and dynamic excitations, and to study the problem of optimum test excitement and sensor placement.

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