Identification of coexistent load and damage based on virtual distortion method

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ABSTRACT

This paper presents a novel method to identify coexistent load and damage based on the idea of Virtual Distortion Method (VDM), which is significant for structural healthy monitoring. This method models a system with unknown damage and load by an equivalent undamaged system with the same load and certain virtual distortions, which are estimated stepwise via measured response. Then damage size can be computed by the estimated virtual distortions. It could be used for both off-line and online identification. A numerical experiment validates that two kinds of damage sizes can be identified as well as coexistent continuous and triangular loads. Moreover two methods (load shape function and initial system iterates) are proposed and incorporated to improve the computational accuracy and to reduce the numerical effort.

INTRODUCTION

Identification of coexistent load and damage refers to the reconstruction of unknown external loads and the damage size of a damaged structure. In recent years, either load or damage identification is an interesting topic. There are quite a lot of methods focusing them respectively. However in real applications, unknown damage and loads usually coexist and together influence system responses. Moreover, the damage can progress with external load. Therefore, coexistent load and damage identification is certain to be complex, but necessary.

For load reconstruction, frequency domain methods [1,2] and time domain methods [3,4] are popular and often used. Both basically reduce the identification to the deconvolution problem of the measured structural response and the impulse response or frequency response functions, which are estimated in advance. Therefore, they require a definite structure. Additionally, other interesting methods are unknown input estimation based on observer [5], and Inverse Structural Filter (ISF) method [6]; both also require determined structural parameters like mass, damping and stiffness. Thus, these methods seem infeasible for load identification in system with an unknown damage.

Popular methods used for damage identification are dynamic fingerprint method (belongs to the frequency-domain methods), model updating method, time-domain method as well as artificial intelligence-based methods [7]. Although some dynamic fingerprint and time series methods can identify the damage without known input information, the exact damage size is hard to tell via them.

Using the idea of Virtual Distortion Method (VDM) [8, 9], this paper presents an effective method to identify coexistent load and damage. In a numerical example two damage types (constant change of Young's modulus and a simple crack model) have been tested, their sizes could be successfully estimated, as well as the satisfied reconstruction of continuous and triangular loads. This approach can be used for both off-line and on-line identification.

VIRTUAL DISTORTION METHOD (VDM)

Identification of coexistent load and damage is based on the idea of VDM [8, 9]. The damaged structure is modeled by an equivalent distorted structure (i.e. original structure with the same external load and virtual distortions introduced in the damaged elements). Both structures are equivalent in the terms of identical strains ε_i and internal forces.

Let us confine the consideration to elastic truss structures first. If the small deformation case is assumed, the strain in the externally loaded equivalent structure can be expressed as a sum of the linear and residual parts:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \sum_{\tau=0}^{t}\sum_{j=1}^{\alpha} D_{ij}(t-\tau)\varepsilon_j^0(\tau) \tag{1}$$

where α is the number of damaged elements and ε_i^L is the response of the original structure to the external load. The virtual distortion ε_j^0 of the jth damaged element models its internal defects, while $D_{ij}(t-\tau)$ denotes an element of the dynamic influence matrix (transfer matrix), that is the original structural response at the location of the ith sensor at time t to the unit impulse of virtual distortion applying to the jth damaged element at time τ. A virtual distortion is modeled by a pair of self-equilibrated forces applied axially at the nodes of the concerned element so that in the static case it would be respectively strained.

The main postulate of VDM requires that element forces and strains respectively in the damaged and distorted structure be equal: $\hat{E}_i\hat{A}_i\hat{\varepsilon}_i(t) = E_iA_i\left(\varepsilon_i(t) - \varepsilon_i^0(t)\right)$, $\hat{\varepsilon}_i(t) = \varepsilon_i(t)$. According to this, if the cross-section is invariant (otherwise the response is influenced by the respective change of mass, which has to be modeled by virtual forces, see [8], the stiffness modification coefficient for each truss element i is the ratio of the modified Young's modulus $\hat{E}_i$ to the original one E_i:

$$\mu_i(t) = \frac{\hat{E}_i(t)}{E_i} = \frac{\varepsilon_i(t) - \varepsilon_i^0(t)}{\varepsilon_i(t)} \tag{2}$$

BASIC CONCEPT

Based on the idea of VDM, the information of the unknown external load and damage is by Eq.(1) reflected in the measured response, which could be structural displacement, strain or acceleration. In the paper, we temporarily choose the element strains as the measured response and assume the initial state equal to zero. The known nonzero state case will be discussed later. Eq.(1) can be also written as:

$$\varepsilon_i(t)=\sum_{\tau=0}^{t}\sum_{l=1}^{\beta}D_{il}^{p}(t-\tau)p_l(\tau)+\sum_{\tau=0}^{t}\sum_{j=1}^{\alpha}D_{ij}^{\varepsilon}(t-\tau)\varepsilon_j^0(\tau),$$

where matrices D_{il}^{p} and D_{ij}^{ε} are the above-mentioned dynamic influences matrices, which relate the discrete dynamic response of the *i*th sensor and the *l*th external load or the *j*th damaged element respectively.

Therefore, the relation between the measured discrete response $\boldsymbol{\varepsilon}^{\mathbf{M}}$, corresponding discrete force history $\mathbf{p}$ and time-dependent virtual distortion $\boldsymbol{\varepsilon}^{\mathbf{0}}$ could be expressed in matrix form:

$$\boldsymbol{\varepsilon}^{\mathbf{M}}=\mathbf{D}^{\mathbf{p}}\mathbf{P}+\mathbf{D}^{\varepsilon}\boldsymbol{\varepsilon}^{\mathbf{0}}=\begin{bmatrix}\mathbf{D}^{\mathbf{p}} & \mathbf{D}^{\varepsilon}\end{bmatrix}\begin{bmatrix}\mathbf{P}\\ \boldsymbol{\varepsilon}^{\mathbf{0}}\end{bmatrix}=\mathbf{Dz} \tag{3}$$

Where $\mathbf{D}^{\mathbf{p}}$ and $\mathbf{D}^{\varepsilon}$ are composed of Toeplitz matrices $\mathbf{D}_{il}^{p}$ and $\mathbf{D}_{ij}^{\varepsilon}$ respectively. $\boldsymbol{\varepsilon}^{\mathbf{M}}$ denotes the vector of discrete dynamic responses measured by all sensors, $\mathbf{z}$ collects the unknowns $\mathbf{p}$ and $\boldsymbol{\varepsilon}^{\mathbf{0}}$, and $\mathbf{D}=[\mathbf{D}^{\mathbf{p}}\ \ \mathbf{D}^{\varepsilon}]$.

If the dynamic influences matrices have been obtained in advance and the measured response is given, the unknown load and the virtual distortions modeling the unknown damage can be identified by solving Eq.(3).

However, because coefficient matrix $\mathbf{D}$ usually is singular or close to be singular, errors are certain to exist during the estimates of $\boldsymbol{\varepsilon}^0$ and $\boldsymbol{\varepsilon}$. So even actual damage factor μ is a constant, the estimated values at different time step are scatters around some constant. Therefore, μ is estimated by the least square method (LSM), which is expressed in Eq (4), (5).

In this paper, two kinds of damage types are considered.

I: The modified Young's modulus $\hat{E}_i$ of the damaged element is constant.

II: A stimulant crack model. When damaged element is under tension, $\hat{E}_i$ is reduced and equals to $\mu_i^a E_i$, while when it is in compression $\hat{E}_i$ equals to E_i. $\mu_i^a<1$ is a constant value. The identified $\mu_i(t)$ should have two values: $\mu_i=\begin{cases}\mu_i^a & \varepsilon_i(t)>0\\ 1 & \varepsilon_i(t)\le 0\end{cases}$.

In the constant damage case, μ_i is computed by LSM□

$$\mu_i=\frac{\left(\boldsymbol{\varepsilon}_i-\boldsymbol{\varepsilon}_i^0\right)^{\mathbf{T}}\boldsymbol{\varepsilon}_i}{\boldsymbol{\varepsilon}_i^T\boldsymbol{\varepsilon}_i} \tag{4}$$

For damage type II, the estimated strain $\boldsymbol{\varepsilon}_i(t)$ is classified by its sign into tensile strain $\boldsymbol{\varepsilon}_i^t(t)$ and compression strain $\boldsymbol{\varepsilon}_i^c(t)$. The homologous estimates of the virtual distortions are labeled $\boldsymbol{\varepsilon}_i^{0t}$ and $\boldsymbol{\varepsilon}_i^{0c}$ respectively.

$$\mu_i^t=\frac{\left(\boldsymbol{\varepsilon}_i^t-\boldsymbol{\varepsilon}_i^{0t}\right)^T\boldsymbol{\varepsilon}_i^t}{\boldsymbol{\varepsilon}_i^{tT}\boldsymbol{\varepsilon}_i^t}\qquad\qquad \mu_i^c=\frac{\left(\boldsymbol{\varepsilon}_i^c-\boldsymbol{\varepsilon}_i^{0c}\right)^T\boldsymbol{\varepsilon}_i^c}{\boldsymbol{\varepsilon}_i^{cT}\boldsymbol{\varepsilon}_i^c} \tag{5}$$

where μ_i^t is tensile damage factor, μ_i^c is press damage factor.

STEPWISE IDENTIFICATION

The main task of identifying the coexistent external load and damage is to solve Eq.(3), i.e. a deconvolution problem. The quality of the solution and computational effort depends mainly on the coefficient matrix $\mathbf{D}$. With the sampling time increasing,

$\mathbf{D}$ will have a big dimension. If the number of unknowns is n_z, number of sensors n_s and n_t time steps are considered, then the dimension of $\mathbf{D}$ is $n_s n_t \times n_z n_t$. With large n_t, n_z or n_s, each direct solution of Eq.(3) is time-consuming and prone to numerical errors . Moreover, it can be used only for off-line identification. Herein, a stepwise identification method utilizing superposition theorem of linear elastic structure is proposed to eliminate the drawbacks and to enable online identification.

Sampling time is divided into several parts with overlapping adjacent sections.

Sampling time is divided into several overlapping adjacent sections.The discrete measured response of distorted structure $\boldsymbol{\varepsilon}^{\mathbf{M}(i)}$ in the ith section can be considered as two parts: one $\bar{\boldsymbol{\varepsilon}}^{(i)}$ is free vibration caused by initial state of the ith section. The other part $\hat{\boldsymbol{\varepsilon}}^{(i)}=\boldsymbol{\varepsilon}^{\mathbf{M}(i)}-\bar{\boldsymbol{\varepsilon}}^{(i)}$ is caused only by external load and virtual distortions which are the input vectors of distorted structure. Given that the initial state of the ith section is known, then $\bar{\boldsymbol{\varepsilon}}^{(i)}$ can be obtained easily. In linear system, $\hat{\varepsilon}^{(i)}$ is proportional to input vectors, so the unknown parameters (the load and the virtual distortions) can be obtained by solving a reduced version of Eq. (3):

$$\tilde{\boldsymbol{\varepsilon}}^{(i)}=\mathbf{D}_s^{(i)}\mathbf{z}^{(i)} \tag{6}$$

where the superscript (i) denotes the number of the section. $\mathbf{D}_s^{(i)}$ is the coefficient matrix $\mathbf{D}$ reduced according to the length of the ith section.

The load and the virtual distortions estimated in the first section can be applied to the numerical model of the equivalent (distorted) structure, in order to obtain the strains $\boldsymbol{\varepsilon}$ of the distorted elements. Then the damage factor can be obtained by Eq.(4) or Eq.(5). Similarly, the states in each time step can be obtained to provide the initial state for the next section. In the like manner, the unknown parameters and the damage factor $\mu_i^{(j)}$ can be identified section by section.

ESTIMATION IN EACH SECTION

Solving Eq.(6) is an inverse and a well-known ill-posed problem, because the coefficient $\mathbf{D}_s^{(i)}$ is often ill-conditioned [10], hence a small disturbance of $\tilde{\varepsilon}^{(i)}$, like noise in the measured response, may cause a large change in identified parameters. Especially in the stepwise identify method, previous errors are accumulated and influence the identified results in the next sections, so it is more sensitive to noise. Thus concepts of load shape function and initial system iterates are proposed here to improve that, which are based on three vital points: i) improving the ill-conditioning of the coefficient matrix; ii) increasing the resistance against noise; iii) reducing errors in each section.

For notational simplicity, in this section of the paper the superscript (i) denoting the time section is neglected. The proposed methods can be applied in each time section independently.

Load Shape Function (LSF)

The idea of the LSF is analogous to that of a shape function in the Finite Element Analysis. The discrete time history of each unknown parameter $\mathbf{z}_j$ in the current section can be approximated based on 'vertical displacements' and 'rotations' of a limited set of evenly distributed 'time history nodes'. If the kth discrete unit load shape function is denoted by $\mathbf{n}_k$, then

$$\mathbf{z}_j = \mathbf{n}\boldsymbol{\alpha}_j \tag{7}$$

where $\boldsymbol{\alpha}_j$ is a vector of the coefficients α_{kj} corresponding to the kth load shape function, and the matrix $\mathbf{n}$ collects all load shape functions as column vectors. For the vector $\mathbf{z}$ of all unknowns, it can be also written as:

$$\mathbf{z} = \mathbf{N}\boldsymbol{\alpha} \tag{8}$$

where $\mathbf{N}$ is a diagonal block matrix consisting of n_x matrices $\mathbf{n}$, while $\boldsymbol{\alpha}$ is a column vector collecting all the individual coefficients $\boldsymbol{\alpha}_j$, j=1,2,…,n_x. Thus we have

$$\hat{\boldsymbol{\varepsilon}}^{(i)}=\mathbf{D}_s\mathbf{N}\boldsymbol{\alpha}=\mathbf{A}\boldsymbol{\alpha} \tag{9}$$

By comparing Eq. (6) and (9), it can be noticed that the coefficient matrix has been changed from $\mathbf{D}_s$ into $\mathbf{A}$, which has different dimension.

From the above considerations, it can be seen that the estimated load can be smoothed via load shape functions to some degree, so that the influence of the noise is weakened. In addition, the dimension of $\mathbf{A}$ is obviously less than that of $\mathbf{D}_s$, which reduces the computational effort.

Initial System Iterating

In each time section, the virtual distortion can be obtained by solving Eq.(6) or Eq.(9). However, the estimation accuracy is not sure. It was empirically observed that the accuracy can be improved if another equivalent (distorted) system S^2 is constructed, whose elements Young's modulus equal to the values $\mu_j^1 E_j$ estimated with the first initial system. Note that this system can be expected to model the investigated damaged system better. By constructing the dynamic influence matrices for this new system and by solving Eq.(6)or Eq.(9), the virtual distortions ε^{0_1} of the system S^2 can be estimated, which let's call temporarily 'comparative virtual distortions'. The 'comparative damage factor' $d\mu_j^1$ can be then computed by the Eq.(4) or Eq .(5) via its final strain and 'comparative virtual distortion'.

Based on the idea of VDM, If μ_j^1 is very close to the actual value, $\varepsilon_j^{0_1}$ should be close to zero, i.e. $d\mu_j^l$ should be very close to 1, which means $\delta^1 = \left\| d\mu^1 - 1 \right\|_2$ is confined in predefined accuracy range (1e-5 in the paper). Otherwise, we will let $\mu_j^2 = \mu_j^l d\mu_j^l$, and repeat the above options until δ^m satisfies the accuracy.

Iterating the initial system, when δ^{k-1} arrive at a certain accuracy (1e-3 in this paper), in the next iterative the comparative virtual distortion $\varepsilon_j^{0_k}$ can be considered as to be very close to zero. This can be enforced by estimating the unknown parameters by an augmented system.

$$\begin{bmatrix} \hat{\boldsymbol{\varepsilon}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_s^{p_k} & \mathbf{D}_s^{\varepsilon_k} \\ \mathbf{0} & \beta\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{P}^k \\ \boldsymbol{\varepsilon}^{0_k} \end{bmatrix} \tag{10}$$

where β is a regularization parameter. Eq.(10) is somehow similar to the Tikhonov regularization term.

Finally, the damaged factor can be estimated as:

$$\mu_j = \mu_j^m d\mu^m \tag{11}$$

where the superscript m denotes the values obtained in the last iteration.

The described iteration of the initial system gives the identified damage factor and the external load more correcting possibilities, which can play an important role in stepwise identification method based on VDM, especially in the case of a considerable noise. However the iterations are at the cost of the compute time. Sometimes the convergence of δ^m is very slow, mainly because of the influence of the numerical error or noise, so a predefined accuracy level and a limited iteration number are required as additional termination criteria.

NUMERICAL EXAMPLE

A 40-element cantilever truss model (Figure 1) hinged on one end is used to illustrate the effectiveness of methods proposed in this paper. The height of the model is 5 m, width is 5 m, Yong's modulus is 201 GPa, cross sections is 210 cm^2, ρ=6800 kg/m^3. Assume the elements 19 and 30 are damaged with damage factors 0.3 and 0.7 respectively for type I, while for type II they are 0.3(1.0) and 0.7(1.0) respectively. Two unknown external forces are applied in the 29th and 32th DOFs. Strain sensors are located on elements 20, 24, 30 and 34. A total of 202 measurement steps have been numerically simulated with the sampling frequency of 1024Hz. The exact simulated sensor responses have been contaminated with a Gaussian noise at the 10 % level:

$$\boldsymbol{\varepsilon}^{\mathbf{M}} \leftarrow \boldsymbol{\varepsilon}^{\mathbf{M}} + 0.1\eta\sqrt{\left\|\boldsymbol{\varepsilon}^{\mathbf{M}}\right\|_2^2 / n} \tag{12}$$

where η is a vector of n random values from the N(0,1) distribution and n is the number of all measurements $n = n_s n_t$.

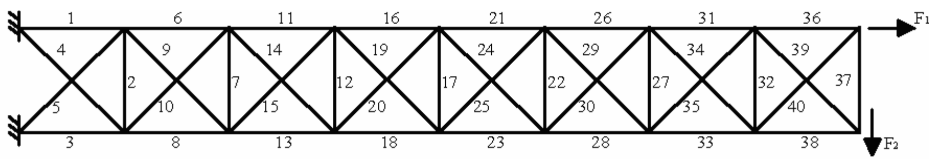

Fig.1 Truss model

Six identification cases are considered. If necessary, the solutions have been regularized by truncation of singular values. In the first five cases, the damage type I is considered. In the last case, both damage types are considered.

1□ Load and damage size were reconstructed by Eq.(3) using all the measured responses together, which simulates the off-line case or single time section. Neither load shape functions nor initial system iterates were used.

2□ Load and damage size were reconstructed by Eq.(9) using all the measured response and the LSF method (22 load shape functions).

3□ Load was reconstructed stepwise by solving Eq.(6).

4□ Load was reconstructed stepwise with LSF (10 load shape functions)

5□ Load was reconstructed stepwise using the initial system iterates.

6□ Load was reconstructed stepwise using both LSF (10 load shape functions) and the initial system iterates (Damage type I: Case 6a; Damage type II: Case 6b).

The number of time steps in each time section from case 3 to case 6 is 101. Overlapping time steps of adjacent sections is 50. The levels of singular value truncation (relative to the maximum singular value or the number of truncated values) are 0.5%, 1, 0.5%, 3, 3, 3 corresponding to case 1 ~ case 6. It is confirmed that the LSF method is robust to noise: truncating only the last few singular values was enough to relax the ill-conditioned problem.

For description concisely, only characteristic figures are give. In case 1 (figure 2a), the identified loads have obvious oscillation because of the noise. In case 2 (figure 2b), the loads are smoother and more accurate. Moreover, in case 2 the computational effort is much less than in case 1, because the dimension of the coefficient matrix is reduced from 808×808 to 808×88. In case 3 (figure 2c), the unknowns are estimated stepwise. Good results could be obtained by proper singular value truncation. Case 6 (figure 2d,e) demonstrates that by using the LSF method and initial system iterates, the identified loads are smoothed and the accuracy of both the estimated loads and the damage factors is further increased.

It can be noticed that at the end of the time interval, some identified loads have obvious deviation from the actual values, which are due to the accumulation of errors and lack of time to influence the response. However, this behavior exists at the end of every time section and is corrected by the overlapping part of the next time section.

Table I lists the damage factors estimated in cases 1 and 2. Table II lists the damage factors estimated stepwise, both the estimates in each of the time sections and the mean value. From the tables it can be seen that: 1) the results in the off-line cases have higher accuracy than the results of the stepwise identification, especially when the LSF method is adopted (case 2). because of the longer response time. Moreover, there is no error accumulation from the previous time sections; 2) For stepwise method, although the LSF method makes the identified loads look smoother with better accuracy, it does not improve significantly the accuracy of the estimated damage factor; 3) Initial system iterates method improves the damage factor estimation accuracy; 4) Different time sections can have different identification accuracy, which can be caused by random Gaussian noise.

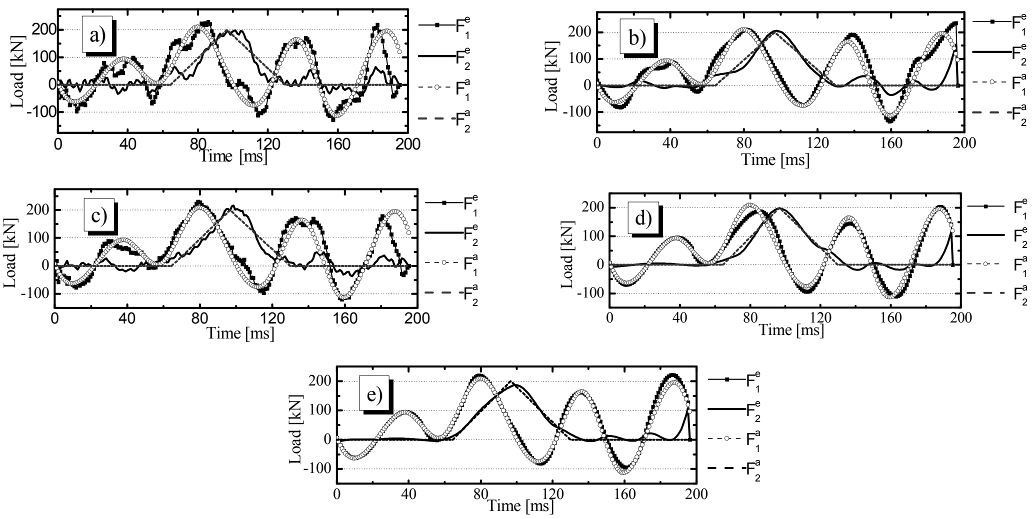

Fig.2 Comparison of the identified loads and actual loads. $F_i^e\left(i=1,2\right)$ denotes the *i*th estimated load, $F_i^a\left(i=1,2\right)$ denotes the *i*th actual load.

TABLE I ESTIMATED DAMAGE FACTOR IN OFF-LINE IDENTIFICATION

Case	element No.	estimated value
1	19/30	0.304/0.634
2	19/30	0.324/0.671

TABLE II. ESTIMATED DAMAGE FACTOR BY STEPWISE METHOD

Case	element No.	section 1	section 2	section 3	mean
3	19/30	0.192/0.629	0.297/0.682	0.395/0.653	0.295/0.655
4	19/30	0.126/0.877	0.288/0.715	0.330/0.654	0.248/0.749
5	19/30	0.227/0.720	0.288/0.637	0.307/0.692	0.274/0.683
6a	19/30	0.195/0.855	0.294/0.687	0.327/0.680	0.272/0.741
6b	19	0.206 (0.981)	0.252 (1.000)	0.184 (1.000)	0.214 (0.994)
	30	0.743 (1.000)	0.742 (1.000)	0.758 (0.7467)	0.748 (0.916)

CONCLUSION AND FURTHER STEPS

A novel methodology for identification of coexistent load and damage size was proposed in this paper. The method is based on the VDM and can be used both for off-line and on line problems. In a numerical experiment, the tested external continuous and triangular loads have been identified with a satisfying accuracy. Moreover, the sizes of damages of two types have been simultaneously estimated. The LSF method and the initial system iterate ideas have been proposed and adopted to improve the stepwise identification method. The LSF method makes it robust to noise and is helpful to identify smooth loads with higher accuracy. In addition, it reduces considerably the compute work, which is especially necessary in off-line case. The idea of initial system iterates also plays an important role in the stepwise identification method.

In this paper, the number of sensors equals to the number of unknowns. The underdetermined case (i.e. the number of sensors is less than the number of unknowns) is a subject of a further research. The idea of stepwise identification is flexible and can be used in both off-line and online identification in elastoplastic systems or in the presence of more complex damage types (like growing cracks).

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