Experimental verification of a methodology for simultaneous identification of coexistent loads and damages[1]

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Abstract

This paper develops and experimentally verifies a practical methodology for identification of coexistent loads and damages. Although both factors usually coexist in practice, there is not much investigation on the simultaneous identification. The main difficulty seems to lie in a very different type of the involved unknowns: excitations *vs.* structural parameters. Previously, the authors have proposed an approach based on the Virtual Distortion Method (VDM) that allows the unknowns to be treated in a unified way and which uses a single virtual distortion to model structural damage. This article extends the methodology into real structures with multiple element distortions and deduces the corresponding physical relation among damage extent, virtual and total distortions. Loads and virtual distortions are reconstructed simultaneously based on the measurements; the damage extent and type is then recovered by a comparison of the virtual and actual distortions (which essentially yields the stress-strain curve). The ill-conditioning, common in inverse problems, is effectively avoided by approximating the loads using load shape functions. A damaged cantilever aluminum beam is used in the experimental verification. Both load and damage (extent and type) are successfully identified. The identification is performed off-line as well as online by a repetitive application in a moving time window.

1. Introduction

External load and structural damages are the two crucial factors in Structural Health Monitoring (SHM), which provide indispensable guidance on maintaining structural integrity, as well as the evidence for forensic engineering. In recent years, many investigations have been performed on either load identification or damage identification. However, it seems that there is yet not much investigation on their simultaneous identification, although they usually coexist together in practice.

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Load identification is most often performed off-line in time domain[1,2] or in frequency domain[3]. Online load identification is usually carried out using observer techniques[4], the Inverse Structural Filter (ISF)[5] or Kalman filer[6]. All these methods are model-based and their identification accuracy relies on the model of the monitored structure.

Response of a damaged structure is influenced also by the damage, besides the excitation. As opposed to local, high-frequency ultrasonic scanning, this paper considers only global or low-frequency damage identification methods. A part of such vibration-based methods[7] requires a pre-defined, known excitation. Other methods, like some of the modal or time series methods, do not need the exact time history of the excitation, but are applicable in special conditions only (e.g. ambient excitation). In case of unknown coexistent load and damage, it is generally difficult to decouple the related identification problems and solve either one of them independently. Load and damage have essentially different nature, and thus a two-step iteration procedure is often adopted [8,9,10]: the excitations and structural parameters are updated separately in each iteration, so that the optimization process proceeds in an alternate manner. Zhang et al.[11] presents a method of simultaneous load and damage identification, which uses Chebyshev polynomials to parametrize the unknown force such that all the parameters related to the damage and excitation can be updated simultaneously in each iteration. Based on the Virtual Distortion Method (VDM)[12,13], Zhang et al.[14] proposed a method for simultaneous identification of load and multiple moving masses. A different approach for identification of coexistent load and damage is used by Zhang et al. in [15]. The damages are modeled by virtual distortions, which are identified along with the unknown excitation. Then both the type and extent of the damage are recovered by identified relation of element strains and stresses. In [15], a simple truss element, which has a single distortion state, is considered. This article extends the methodology into real structures with multiple element distortions and deduces the corresponding physical relation among damage extent, virtual and total distortions. The time history of the excitation force and the damage are simultaneously identified off-line and, if necessary, online by using a moving time window. An experiment with a cantilever aluminum beam is performed to verify the method.

2. Virtual Distortion Method (VDM)

The Virtual Distortion Method is a quick reanalysis method, applicable in both statics and dynamics[12,13]. The method introduces virtual distortion to model structural modifications, including damages, and physical nonlinearities like material yielding. The virtual distortions are additionally imposed on the involved elements of the original linear structure. As a result, the response of the damaged structure to an external load is represented by a combination of the linear responses of the intact structure to the same load and to certain response-coupled virtual distortions. For the sake of notational simplicity, this paper only considers stiffness-related damages. However, the methodology can be straightforwardly extended to other damage patterns like mass-related modifications or plastic yielding of the material[2,12,13]. The assumption of small deformations (geometric linearity) is used.

2.1 Virtual distortion and damage

Let μ_i denote the stiffness reduction of the *i*th element, that is the proportion ratio of the modified local stiffness matrix $\tilde{\mathbf{K}}_i$ to the original local matrix $\mathbf{K}_i$,

$$\tilde{\mathbf{K}}_i = \mu_i \mathbf{K}_i \tag{1}$$

Let $f(t)$ be an external excitation of the damaged structure. Using (1), the equation of motion of the damaged structure can be expressed as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \sum_i \mathbf{L}_i^{\mathrm{T}} \left(\mathbf{K}_i - \left(1-\mu_i\right)\mathbf{K}_i \right) \mathbf{u}_i\left(t\right) = \mathbf{f}\left(t\right) \tag{2}$$

where $\mathbf{L}_i$ is the transformation matrix from the global co-ordinate system to the local co-ordinate system of the *i*th element, $\mathbf{u}$ is the nodal displacement vector and $\mathbf{u}_i$ is the local nodal displacements vector of the *i*th element. By moving the modification terms to the right hand side, the equation can be equivalently stated as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \sum_i \mathbf{L}_i^{\mathrm{T}} \mathbf{K}_i \mathbf{u}_i\left(t\right) = \mathbf{f}\left(t\right) + \sum_i \left(1-\mu_i\right) \mathbf{L}_i^{\mathrm{T}} \mathbf{K}_i \mathbf{u}_i\left(t\right) \tag{3}$$

which is the equation of motion of the intact structure. It can been seen that the response of the damaged structure equals to the response of the intact structure to the same external load and to certain pseudo-load, which acts in the degrees of freedom (DOFs) of the damaged elements and which is implicitly related to the damage and response.

In the VDM, the pseudo-load turns out to be equivalent to certain virtual distortions imposed on the involved element of the intact structure. For a finite element, the number and form of its distortions can be analyzed via the eigenvalue problem of its local stiffness matrix $\mathbf{K}_i$. The matrix $\mathbf{K}_i$ is positive semidefinite, and hence it has two kinds of eigenvectors: unit distortion vectors that correspond to positive eigenvalues and unit rigid motion vector that correspond to zero eigenvalues. For example, the local stiffness matrix of a 2D beam element has three positive eigenvalues and thus such an element has three distortions: axial distortion, bending distortion and shear/bending distortion. The matrix $\mathbf{K}_i$ can be expressed by its positive eigenvalues λ_{ij} and the corresponding eigenvectors φ_{ij} :

$$\mathbf{K}_i = \sum_j \lambda_{ij} \varphi_{ij} \varphi_{ij}^{\mathrm{T}} \tag{4}$$

where φ_{ij} is the *j*th unit distortion of the *i*th element and is equivalent to the following local nodal unit pseudo-load:

$$\mathbf{n}_{ij} = \mathbf{K}_i \varphi_{ij} = \lambda_{ij} \varphi_{ij} \tag{5}$$

Due to (4) and (5), the total local nodal load of the ith element can be expressed as a linear combination of the local unit pseudo-loads $\mathbf{n}_{ij}$,

$$\mathbf{K}_i \mathbf{u}_i(t) = \sum_j \kappa_{ij}(t) \mathbf{n}_{ij} \tag{6}$$

where the combination coefficient

$$\kappa_{ij}(t) = \varphi_{ij}^{\mathrm{T}} \mathbf{u}_i(t) \tag{7}$$

denotes the total jth distortion of the ith element. Equations (3) and (6) yield

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \sum_{i,j} \kappa_{ij}(t) \mathbf{L}_i^{\mathrm{T}} \mathbf{n}_{ij} = \mathbf{f}(t) + \sum_{i,j} \kappa_{ij}^0(t) \mathbf{L}_i^{\mathrm{T}} \mathbf{n}_{ij} \tag{8}$$

where $\kappa_{ij}^0(t)$ is the jth *virtual distortion* of the ith element applied on the involved element of the intact structure,

$$\kappa_{ij}^0(t) = (1 - \mu_i) \kappa_{ij}(t) \tag{9}$$

Note that virtual distortions depend on the damage and are coupled with the total distortion (the response). If the virtual distortion $\kappa_{ij}^0(t)$ and total distortion $\kappa_{ij}(t)$ are identified, the damage extent μ_i can be recovered via (10).

2.2 Response of the damaged structure

Equations (3) or (8) confirm that, using the VDM, the response $y_\alpha(t)$ of a damaged structure at the αth sensor (linear sensor of any type, e.g. strain sensor, accelerometer, etc.) to external loads can be modeled by two parts: linear response of the intact structure to the same excitation $y_\alpha^{\mathrm{L}}(t)$ and the residual response to the virtual distortions imposed on the intact structure,

$$y_\alpha(t) = y_\alpha^{\mathrm{L}}(t) + \sum_{i,j} \int_0^t D_{\alpha ij}^{\kappa}(t-\tau) \kappa_{ij}^0(\tau) d\tau \tag{10}$$

where $D_{\alpha ij}^{\kappa}(t)$ is the corresponding impulse response of the undamaged structure, that is the response of the αth sensor to the unit impulse distortion φ_{ij} of the ith element. In practice, it often turns out that certain distortions types are practically not excited and can be neglected in (8) and (10), which decreases the numerical costs of the analysis.

3. Load and damage identification

3.1 Response of the damaged structure to unknown excitation

If the excitation *f*(*t*) is unknown, with the assumption of zero initial states (the nonzero initial state is discussed in Section 3.3), the response of the damaged structure (10) expands to

$$y_\alpha(t) = \sum_i \int_0^t D^{\mathrm{f}}_{\alpha i}(t-\tau) f_i(\tau) d\tau + \sum_{i,j} \int_0^t D^{\kappa}_{\alpha ij}(t-\tau) \kappa^0_{ij}(\tau) d\tau \tag{11}$$

where $D^{\mathrm{f}}_{\alpha i}(t)$ is the impulse response of the undamaged structure at the α th sensor to an impulsive load in the *i*th DOF.

In practice, the data are usually obtained from measurements or simulated using a finite element model, and hence are discrete. Equation (11) should be thus discretized for practical analysis,

$$\mathbf{y} = \mathbf{D}^{\mathrm{f}}\mathbf{f} + \mathbf{D}^{\kappa}\boldsymbol{\kappa}^0 = \begin{bmatrix} \mathbf{D}^{\mathrm{f}} & \mathbf{D}^{\kappa} \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\kappa}^0 \end{bmatrix} = \mathbf{D}\mathbf{z} \tag{12}$$

where the vector $\mathbf{z}$ includes all the time histories of the unknown loads and unknown virtual distortions. With a certain arrangement of the vector $\mathbf{z}$, the coefficient matrix $\mathbf{D}$ is a structured block matrix, of which each block is a Toeplitz matrix that relates a sensor with a distortion or with a load-exposed DOF.

3.2 Off-line identification of loads and virtual distortions

The information about excitations and damages is reflected in the structural responses. Therefore, the identification can be performed minimizing the mean square distance between the estimated responses $\mathbf{y}$ and the measured responses $\mathbf{y}^{\mathrm{M}}$, which is equivalent to finding the least squares solution to the following equation:

$$\mathbf{y}^{\mathrm{M}} = \mathbf{D}\mathbf{z} \tag{13}$$

To guarantee the uniqueness of the solution, the number of independent sensors should not be smaller than the total number of the load-exposed DOFs and the distortions of damaged elements. In practice, to limit the number of necessary sensors, the potential locations of unknown loads and damages should be known.

The dimension of the coefficient matrix $\mathbf{D}$ is proportional to the number of the considered time steps, and so, if the sampling time is long and the discretization of time steps is dense, the matrix $\mathbf{D}$ is large and the solution of (13) is hardly possible. With a dense time discretization, the time histories of loads and virtual distortions can be approximated using a limited number of certain basis functions to reduce the numerical effort. Here, load shape functions[16] are used and

$$\mathbf{y}^{\mathrm{M}} \approx \mathbf{DN\alpha} \tag{14}$$

Where $\mathbf{z} \approx \mathbf{N\alpha}$, and the approximation coefficients $\boldsymbol{\alpha}$ are much fewer in number than the unknowns $\mathbf{z}$.

Moreover, due to the structure of the matrix **D**, (13) and, to some extent, (14) are well-known ill-conditioned problems and a proper numerical regularization technique is required. Equation (13) can be solved quickly by the conjugate gradient method (CGLS), which has excellent regularizing properties. Other possibilities include the truncated singular value decomposition (TSVD) or the Tikhonov method[17,18]. In this paper (14) is used, which is of much smaller dimensions than (13), which allows the TSVD to be straightforwardly applied.

3.3 Online identification of loads and virtual distortions

The main task of the identification is to solve (13) or (14), which is essentially a deconvolution problem. The identification accuracy and the numerical effort depends mainly on the coefficient matrix **D** or **DN**. When sampling time is long, both matrices can be huge, even if approximation is used. Moreover, both equations can be used only for off-line identification. Here, the technique of a moving time window is used to overcome these drawbacks and to enable online identification

Equation (11) expresses the response of the damaged structure under the assumption of zero initial state. The response $y_\alpha^{(n)}(t)$ in the *n*th time window can be expressed in as:

$$y_\alpha^{(n)}(t) = y_\alpha^{(n)}(t) + \sum_i \int_0^t D_{\alpha i}^{\mathrm{f}}(t-\tau)\, f_i(\tau) d\tau + \sum_{i,j} \int_0^t D_{\alpha ij}^{\kappa}(t-\tau) \kappa_{ij}^0(\tau) d\tau \tag{15}$$

where the term $\bar{y}_\alpha^{(n)}(t)$ denotes the free vibration of the intact structure caused by the initial state of *n*th time window. The unknowns in the current time window can be obtained by comparing the computed response and the measured response $\mathbf{y}^{\mathrm{M}(n)}$:

$$\mathbf{y}^{\mathrm{M}(n)} - \bar{\mathbf{y}}^{(n)} = \mathrm{B}^{(n)} \mathbf{z}^{(n)} \tag{16}$$

where matrix $\mathbf{B}^{(n)}$ is a reduced version of the matrix **D** or **DN** according to the length of the *n*th window. Equation (16) is thus much smaller and easier to solve than (13) or (14). The initial state and the free vibrations in each section can be obtained straightforwardly, given the loads and distortions in the previous section. Therefore, the online identification is performed by a repetitive solution of (16) in successive time windows.

The identification in each section utilizes the previously identified unknowns, so that it is prone to accumulation of error from the previous sections. In practice, the effect of measurement error might be large. Moreover, the signal in each section is short and so it is more sensitive to noise than a long signal. Therefore, the following procedures are suggested to increase the identification accuracy in practical applications:

- The identification via load shape functions in each section can reduce the numerical cost and also improve the ill-conditioning of the inverse problem. Usually, only a few singular values need to be truncated to guarantee the required accuracy.
- The consecutive time sections overlap, thus the identification performed in the prior time section yields non-vanishing time histories of the loads and distortions for the overlapping part of the next section. These time histories are used, along with the initial state, to generate the initial response $\bar{y}_{\alpha}^{(n)}(t)$ in the current time section. In this way, there is less high-frequency components in the initial response. As exactly these components are significantly ill-conditioned in any deconvolution problem[18], the numerical accuracy is considerably improved.

3.4 Damage identification

If the loads and virtual distortions are identified off-line using (13) or (14), or online using (16), the corresponding structural response can be computed along with the total distortions of the damaged elements (equation (7)). The damage extent can be then recovered via (9), that is by comparing the total and virtual distortions:

$$\mu_i(t) = \frac{\kappa_{ij}(t) - \kappa_{ij}^0(t)}{\kappa_{ij}(t)} \tag{17}$$

Notice that the numerator in (17) is proportional to the stress, while the denominator is the strain. Thus, the curve obtained by plotting $\kappa_{ij}(t) - \kappa_{ij}^0(t)$ *vs.* $\kappa_{ij}(t)$ corresponds to the stress-strain curve. An examination of the graph can often reveal the type of the damage. For instance, a linear relation suggests that the damage is constant a constant reduction of stiffness (e.g. related to corrosion); a bilinear function would suggest that the damage is variable with respect to the stress of the element and can be e.g. a breathing crack model[19]. Assumed the damage model, the damage parameters can be obtained by fitting the graph.

4. Experimental Verification

An experiment of an aluminum cantilever beam is performed to verify the proposed method of identification of coexistent load and damage.

4.1 The structure

The experimental setup is shown in Figure 1. An aluminum beam has the length of 136.15 cm, with the rectangular cross-section of 2.7 cm x 0.31 cm. The Young's modulus is 70 Gpa, and the density is 2700 kg/m^3. The fixed end is mounted to a stable frame. The beam is slender, and thus the influence of the beam gravity is considered in the finite element model, as well as the presence of the piezoelectric actuator and strain sensors.

The beam is damaged by cutting even notches near to the fixed end on the section of a length of 10.23 cm. The stiffness of the damaged section is reduced to 42% of the original value, while the mass remains almost unchanged.

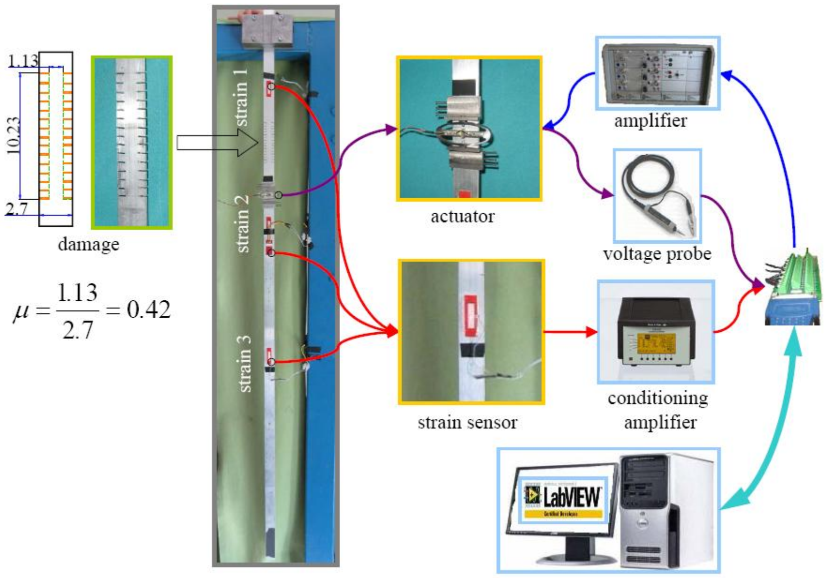

Figure 1. Experimental setup

4.2 Excitation and measurements

The excitation is applied using an Amplified Piezo Actuator (APA), which is fixed on the beam in such a way that is can be assumed to apply a pure moment load. The structural dynamic responses are measured by three piezoelectric patches (denoted as S1~S3), which are glued on the beam to measure the structural strain. The excitation and the responses are acquired by a LabVIEW system and stored in a PC. In addition, the strain and excitation signals are amplified respectively using Brüel&Kjaer charge amplifier and power amplifier.

A sample designed excitation is shown in the top plot of Figure 2. The sampling frequency is 2500 Hz to guarantee that no important information is lost. The measured responses are shown in the bottom plot of Figure 2.

4.2 Load and damage identification

The intact finite element model is assembled; the damaged section is assumed to be a single element. A generic 2D beam element has three virtual distortions. However, a pure moment load is applied here, which causes mainly bending distortions, so that the other two distortions (axial and shear/bending) are neglected in the analysis. Therefore, one external load and one damage-related virtual distortions are the unknowns to be identified, and at least two sensors are required to obtain the unique solution.

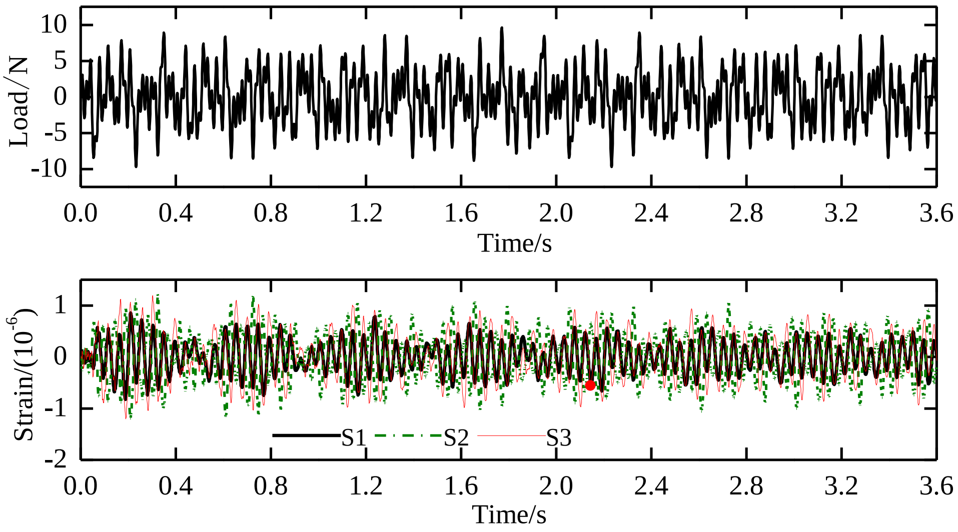

Figure 2. Measured excitation and the corresponding structural responses

The identification is performed using a moving time window. Each window has 400 time steps with 200 steps overlapping. A total of 8800 time steps is analyzed within the total time of 3.52 s, which is divided into 43 windows. In each window the unknown load and virtual distortion are identified using (16) and an approximation with a basis of forty two shape functions. Two or three sensors are used separately; the identified loads are shown in Figure 3.

Figure 4 shows the (scaled) strain-stress relationship of the damaged element, which is recovered using sensors S1 and S3. The relationship is close to a linear function, which suggests that the damage is a constant reduction of stiffness. The damage extent is estimated in each time window using (17), see Figure 5. In this way, the damage is monitored online. The average damage extent of all the windows $\bar{\mu}$ and the value μ identified off-line are listed in Table 1.

As expected, the identification accuracy is better when three sensors are used, although the results obtained with sensors S1 and S3 are also satisfactory. The results obtained with sensors S1 and S2 are not as accurate as with S1 and S3, which might be related to the fact that S2 is much closer to S1, the damaged element and the actuator than S3. The problem of optimum sensor placement for SHM, which is a challenging and widely-studied problem[20], is outside the scope of this paper.

5. Conclusions

An effective method for simultaneous identification of coexistent loads and damages is developed and experimentally verified. The methodology is based on the Virtual Distortion Method (VDM) and models stiffness-related damages with certain virtual distortions. Time histories of the virtual distortions and excitation loads are identified together via the measured response of the damaged structure in a deconvolution-type problem. The damage, including the type and extent, is then recovered by an analysis of the stress-strain relationship between virtual and total distortions of the damaged elements. Numerical efficiency of identification is increased by performing the identification online in a moving time window and by approximation of loads and distortions with load shape functions.

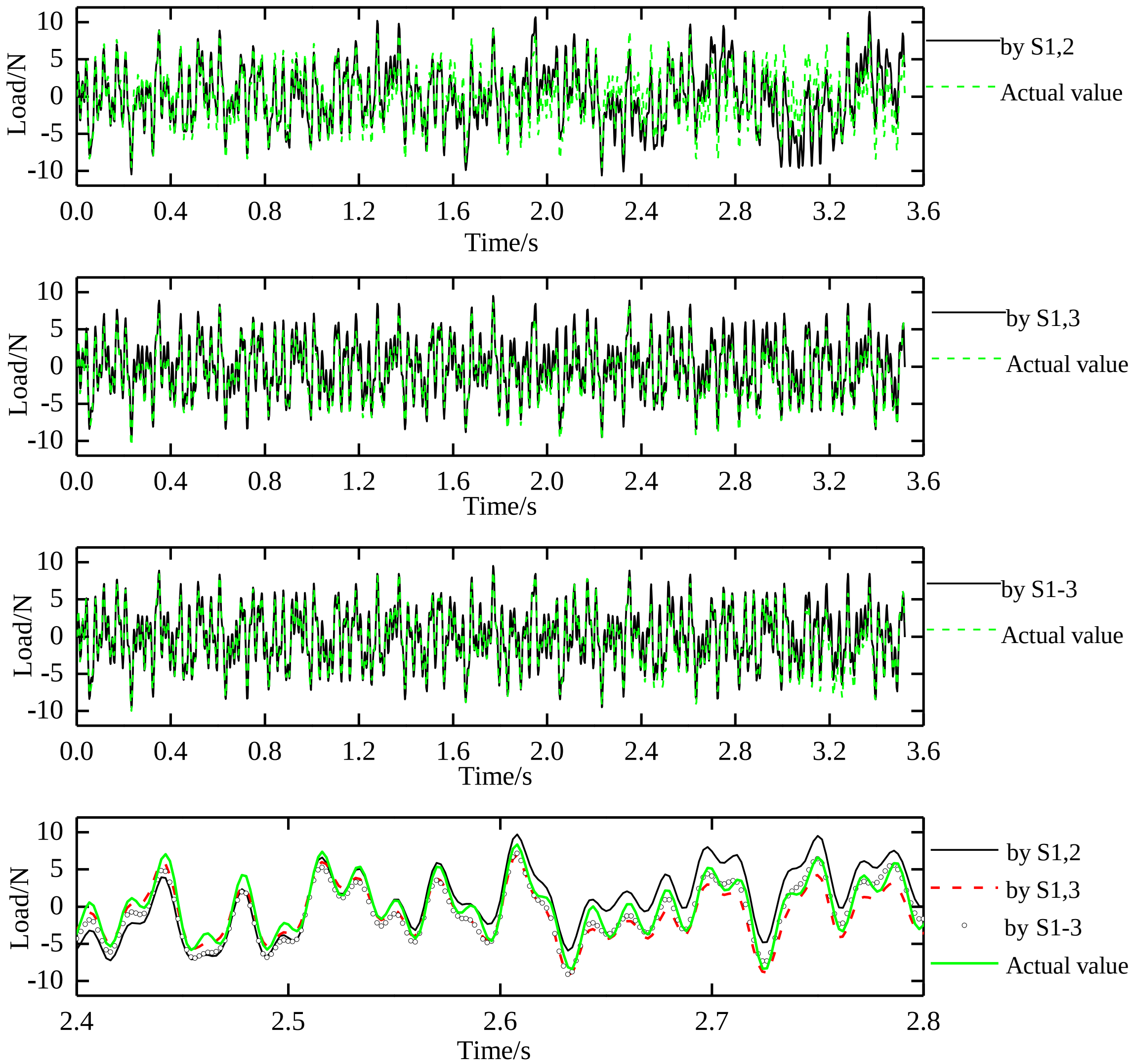

Figure 3. Identification of external excitation

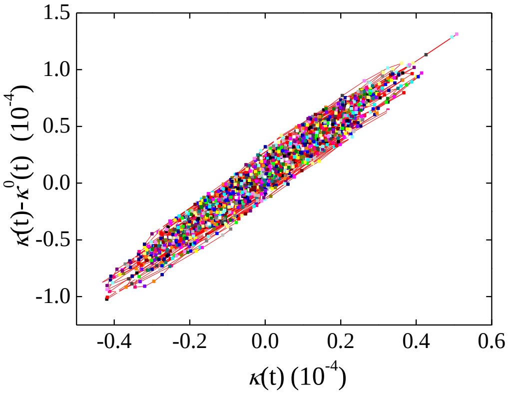

Figure 4. Identified (scaled) stress-strain relationship of the damaged element

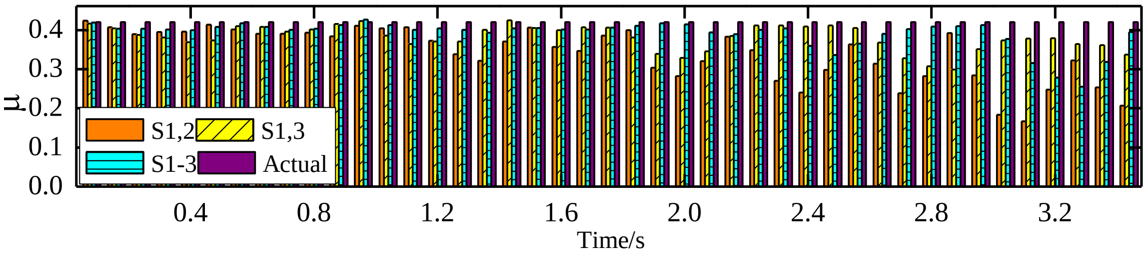

Figure 5. Identified damage extents for each time window

Table 1. Damage parameter: actual, identified off-line and online (the mean of the results in all time sections)

Actual value	$\overline{\mu}_{1,2}^{\text{online}}$	$\mu_{1,2}^{\text{offline}}$	$\overline{\mu}_{1,3}^{\text{online}}$	$\mu_{1,3}^{\text{offline}}$	$\overline{\mu}_{1-3}^{\text{online}}$	$\mu_{1-3}^{\text{offline}}$
0.420	0.340	0.327	0.388	0.381	0.391	0.388

Note: the subscripts denote the sensors used for identification

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