A model-less method impact mass identification

Grzegorz Suwała, Łukasz Jankowski

Smart-Tech Centre, Institute of Fundamental Technological Research,
ul. Świętokrzyska 21, 00-049 Warsaw, Poland.

gsuwala@ippt.gov.pl, ljank@ippt.gov.pl

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ABSTRACT

This paper proposes a new model-less method for off-line identification of a mass impacting an elastic structure. The method is aimed at the identification of both mass and its velocity, makes use of the Virtual Distortion Method (VDM) and assumes the inelastic impact case, i.e. permanent modification of structural properties. Since the proposed approach is completely based on experimentally measured data, no numerical modeling and tedious fine-tuning of the model are necessary. The impacting mass is modeled using virtual distortion forces and an experimentally obtained system transfer matrix. The identification amounts to solving an optimization problem of minimizing the mean-square distance between measured and modeled structural responses, the latter is based on previously recorded responses of the unaffected structure.

INTRODUCTION

The main motivation for this research is the need for an experimentally robust and practical analysis technique for mass monitoring and efficient reconstruction of the scenario of a dynamic loading, to be used in black-box type monitoring systems. Similar problems of dynamic mass identification are crucial in many practical applications, such as in monitoring of off-shore platforms, crowd-induced excitations, etc.

Impact identification methods, as other methods used in structural health monitoring, can be generally classified into two groups [1–5]: pattern recognition and model-based approaches. Pattern recognition approaches often rely on artificial intelligence methods and use a database (or a training set) of fingerprints extracted from several responses, which are previously measured for many impact scenarios (locations, mass, velocity, etc.). The actual scenario is identified using the fingerprints only, without the insight into their actual mechanical meaning, and require neither the model nor simulation of the event. The model-based approaches require a precalibrated model of the structure; the identification amounts to iterative modifications of the model, simulations and comparisons of the measured and computed responses. The method proposed in this paper seems to fit to neither group: although it computes the responses of modified structure and thus is based on actual mechanical principles, it requires experimental data only and no numerical model of the structure.

THEORETICAL FORMULATION

The proposed model-less methodology makes use of the Virtual Distortion Method (VDM) [1, 6]. Assume the equilibrium equation for the modified structure can be written as

$$\hat{\mathbf{M}}\ddot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t), \tag{1}$$

where $\hat{\mathbf{M}}$ denotes the mass matrix modified by the inelastically impacting mass, $\mathbf{K}$ is the stiffness matrix and $\mathbf{f}(t)$ is the vector of the impact forces. $\mathbf{f}(t)$ contains zeros besides the three degrees of freedom (DOFs) related to the impacted node, in which the impact force is $mv_d\delta(t)$, where m is the impacting mass and v_d denotes the x, y or z component of the impact velocity v. According to the VDM, the effects of the modification of the mass matrix can be modeled using the original unmodified structure and virtual forces $\mathbf{p}(t)$:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)+\mathbf{p}(t), \tag{2}$$

where $\mathbf{M}$ is the original mass matrix. Comparing Eq. 1 with 2 yields

$$\mathbf{p}(t)=-\Delta\mathbf{M}\ddot{\mathbf{u}}(t). \tag{3}$$

which for a single nodal impact simplifies to $p_i(t)=-\Delta m_i\ddot{u}_i(t)$, where $\Delta m_i=m$ if the ith DOF is related to the impacted node and $\Delta m_i=0$ otherwise.

According to the VDM, the acceleration $\ddot{\mathbf{u}}$ can be considered to be a linear combination of the structural response of the unmodified structure $\ddot{\mathbf{u}}^L$ and the cumulative effect of all the virtual distortions modeling the mass modification.

$$\ddot{u}_i(t)=\ddot{u}_i^L(t)+\sum_j\sum_{\tau\le t}\ddot{B}_{ij}(t-\tau)\,p_j(\tau) \tag{4}$$

where $\ddot{B}_{ij}(t-\tau)$ is an element of the dynamic influence matrix (system transfer matrix) and denotes the acceleration in the ith degree of freedom (DOF) in the time instant t in response to a unit impulse force in the jth DOF in the time instance τ. $\ddot{\mathbf{u}}^L$ denotes the response of the unmodified structure to the impact excitation $\mathbf{f}(t)$ and can be expressed as a combination of the responses $\ddot{\mathbf{u}}^{Lx}$, $\ddot{\mathbf{u}}^{Ly}$ and $\ddot{\mathbf{u}}^{Lz}$ to the unit impulses in all three DOFs related to the impacted node:

$$\ddot{\mathbf{u}}^L(t)=m\left[v_x\ddot{\mathbf{u}}^{Lx}(t)+v_y\ddot{\mathbf{u}}^{Ly}(t)+v_z\ddot{\mathbf{u}}^{Lz}(t)\right], \tag{5}$$

where $(v_x, v_y, v_z) = \mathbf{v}$ is the impact velocity. Equations 4 and 5 can be substituted into Eq. 3 and the unknown virtual forces $p_i(t)$ in the current time step t can be collected on the left-hand side:

$$\sum_j \left[\Delta m_i \ddot{B}_{ij}(0) + \delta_{ij} \right] p_j(\tau) = -\Delta m_i \left[\ddot{u}_i^L(t) + \sum_j \sum_{\tau < t} \ddot{B}_{ij}(t-\tau)\, p_j(\tau) \right]. \tag{6}$$

Equation 6 has a time-independent principal matrix and can be used in successive time steps to compute iteratively all the virtual forces. When they are computed, the displacements of the modified structure can be calculated, similar as in Eq. 4, by a linear combination of the response of the unaffected structure and the cumulative effects of the virtual forces:

$$u_i(t) = u_i^L(t) + \sum_j \sum_{\tau \le t} B_{ij}(t-\tau)\, p_j(t). \tag{7}$$

The identification of the mass and its velocity is based on the comparison between the measured and computed responses in sensor locations. The following objective function is used:

$$F(m, \mathbf{v}) = \sum_i \sum_t \frac{\left[\mathbf{u}_i(t) - \mathbf{u}_i^M(t) \right]^2}{\sum_t \left[\mathbf{u}_i^M(t) \right]^2}, \tag{8}$$

where $\mathbf{u}_i^M(t)$ is the measured response, $\mathbf{u}_i(t)$ is the computed response and index i covers all sensor locations.

PRACTICAL FORMULATION

The dynamic influence matrix used in the theoretical formulation includes structural responses to impulse forces, that is to forces acting one time step only. Such a matrix can be simulated numerically, but not measured experimentally. In practice, the excitation impact force is generated with an impact hammer and lasts several time steps, see Fig. 1. An experimentally measured dynamic influence matrix must hence contain structural responses to normalized diffuse impact loads instead of the theoretical impulses or one-time-step loads. Moreover, the numerical stability of the iterative procedure to determine the virtual forces relies on the accuracy of the dynamic influence matrix. In practical cases it is inevitably contaminated with measurement noise, which leads to errors accumulating over successive time steps and divergent results.

Since, in practice, only the responses to diffuse excitations of an impact hammer are available, a virtual force has to be represented in the form of a linear combination of the corresponding normalized impact hammer peaks (Fig. 1):

$$p_i(t) = \sum_\tau f_i(t-\tau)\, a_i(\tau). \tag{9}$$

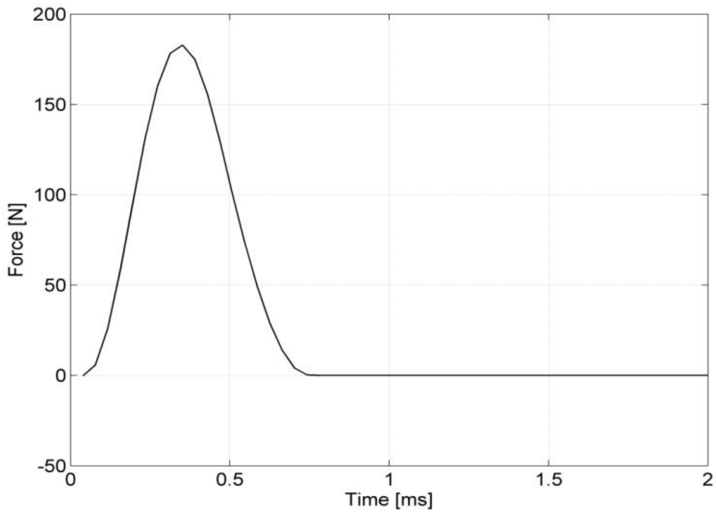

Figure 1. Measured force impulse

The formulas for the structural response, Eqs. 4 and 7, have to be accordingly updated, e.g.:

$$\ddot{u}_i(t) = \ddot{u}_i^L(t) + \sum_j \sum_\tau \ddot{D}_{ij}(t-\tau)\, a_j(\tau), \tag{10}$$

where $\ddot{D}_{ij}(t-\tau)$ denotes an element of the experimental system transfer matrix. Substitution of Eqs. 9 and 10 into Eq. 3 yields the following linear system:

$$\sum_j \sum_\tau \left[\delta_{ij} f_i(t-\tau) + \Delta m_i \ddot{D}_{ij}(t-\tau)\right] a_j(\tau) = -\Delta m_i \ddot{u}_i^L(t), \tag{11}$$

which can be used to compute the coefficients $a_i(\tau)$. Notice that Eq. 11 is a single, large linear system involving the unknown coefficients $a_i(\tau)$ in all time steps τ and degrees of freedom *i*. In this way, the potential numerical instability of the iteratively repeated solutions of Eq. 6 can be avoided at the cost of a larger numerical effort required to solve Eq. 11. Moreover, since this system is often ill-conditioned, a regularization technique has to be used. This paper uses the singular value decomposition (SVD) and the truncation of too small singular values. Finally, given the coefficients $a_i(\tau)$, the structural response in all sensor locations *i* can be computed by

$$u_i(t) = u_i^L(t) + \sum_j \sum_\tau D_{ij}(t-\tau)\, a_j(\tau), \tag{12}$$

To identify the mass and its velocity, the proposed method solves an optimization problem of minimizing the mean square distance Eq. 8 between the measured and the modeled structural responses. The modeled response is computed by Eqs. 11 and 12 and hence based exclusively on previously measured responses of the unaffected structure. No numerical model of the structure is necessary.

EXPERIMENTAL STAND

To verify the proposed technique, a simple three elements truss structure has been used (Fig. 2). The element profiles are 1200 mm long rectangular structural steel tubes with a nominal size of 20 x 20 mm and the thickness of 2 mm. The right-hand side ends of the elements were fixed. The inelastic impact has been simulated by fixing a mass to the node and exciting it with an impact force. Then, given the actual mass and the excitation $f(t)$, Fig. 1, the simulated impact velocity can be calculated by the impulse momentum $mv = \int f(t)dt$. Fig. 2 shows the original truss (left figure) and the only node with a mass mounted, which simulates the inelastically attached impact mass (right figure).

A Bruel & Kjaer modal hammer has been used for force excitation in three directions. The response has been measured in three directions with three Bruel & Kjear accelerometers (Model 5302), all fixed to the node of the structure. For verification purposes, the first 1000 time steps have been used, measured with 25.6 kHz sampling frequency (39 ms) and triggered by the hammer excitation. The signal from the accelerometers and modal hammer was transmitted via a PULSE device to a PC and analyzed with Matlab.

In the first part of the experiment, the responses of the unaffected structure have been recorded in order to build the dynamic influence matrix. The impact force was applied to the node in three directions and the response was measured in three directions. This simple structure has only one node, hence only nine acceleration measurements (and computed displacements) had to be stored. However, for more complex structures measuring the responses to build dynamic influence matrix can be work consuming.

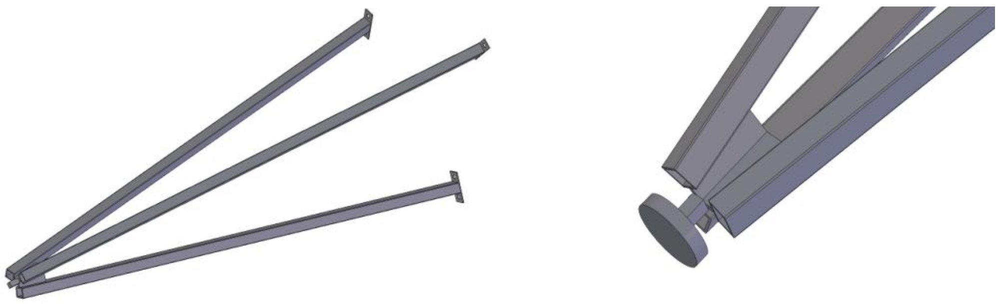

Figure 2. Truss construction without and with mounted mass

In the second part, the responses to an impacting mass have been recorded as described above. The scenario assumes an inelastic impact case, for this reason the mass was mounted to the node. To apply the initial velocity, the modal hammer was used, which seems to be the easiest way to simulate the inelastic impact case.

EXPERIMENTAL RESULTS

The proposed method has been verified experimentally with the actual mass 0.44 kg. The computed initial impact velocity was equal to 0.114 m/s and it was assumed to act in the *x* direction only. The objective function Eq. 8 compared the displacements in the impact direction (one sensor only).

For identification purposes, Eq. 11 had to be solved several times for different trial masses and velocities to find the corresponding coefficients $a_i(\tau)$ and then the structural response by Eq. 12. To spare the numerical costs, in the identification stage the first 500 time steps have been used, which resulted in the system Eq. 11 having the dimension 1500 x 1500. Figures 3 and 4 show in the logarithmic scale the objective functions computed at two extreme levels of the regularizing truncation of the singular values: 3 % and 76 %, respectively. Although the shapes are different, both minima are consistently located near the actual values, 0.445 kg at 0.105 m/s and 0.465 kg at 0.09 m/s, which confirms the numerical stability of the approach.

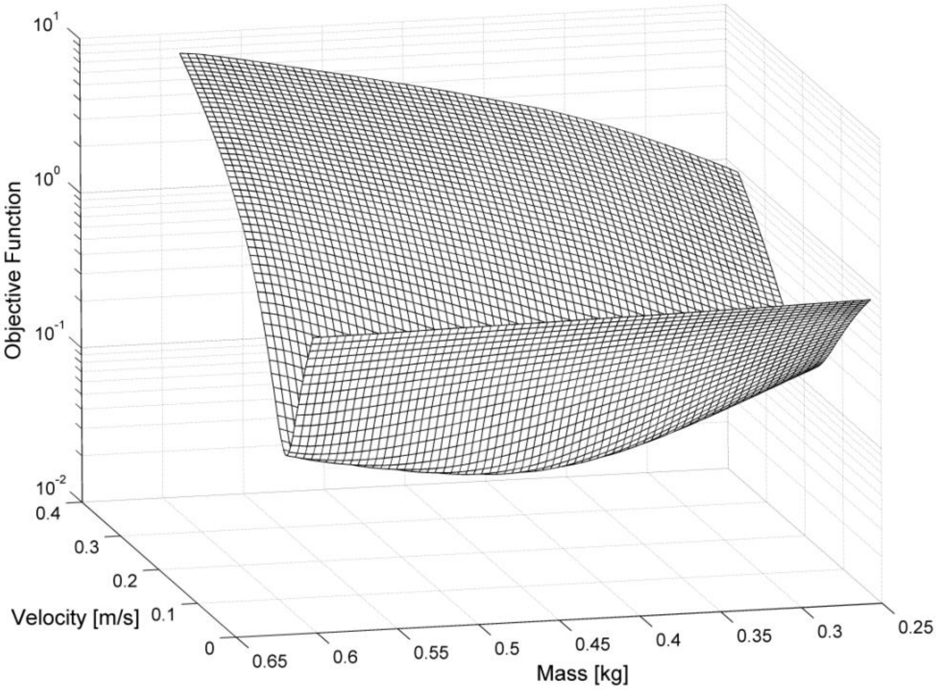

Figure 3. Objective function computed for the actual masses of 0.44 kg at the 3 % regularizing truncation level of the singular values

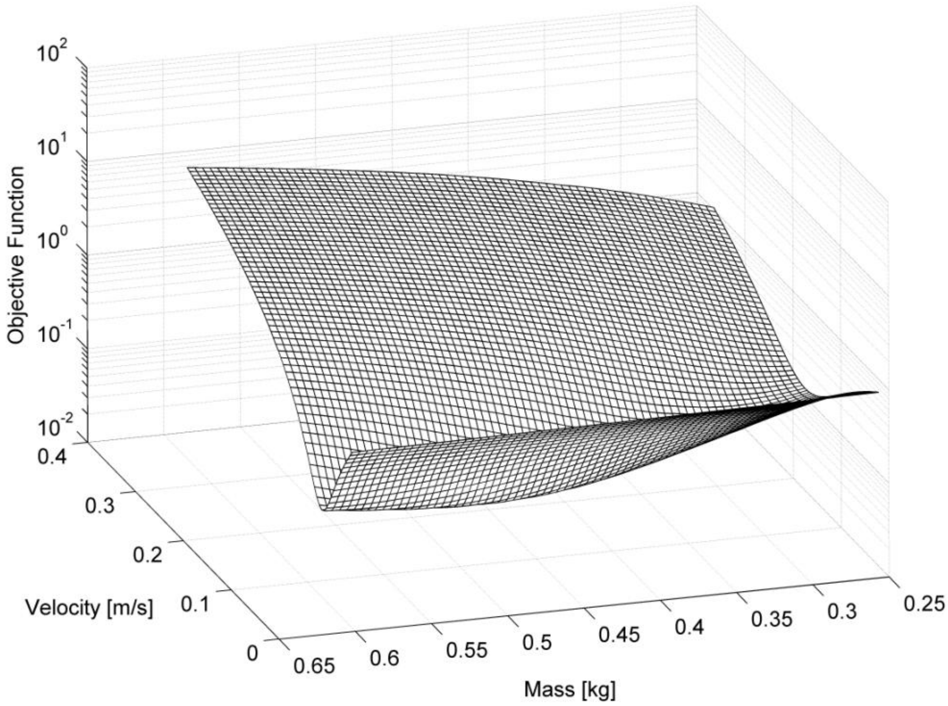

Figure 4. Objective function computed for the actual masses of 0.44 kg
at the 76 % regularizing truncation level of the singular values

The identified mass and velocity values have been used to generate and solve the two corresponding systems Eq. 11 for 1000 time steps. These systems are larger (3000 x 3000) but had to be solved only once, since the masses and velocities had been assumed to be already known. Figure 5 compares the measured response of the structure with the responses modeled at both trial regularization levels.

CONCLUSIONS

This paper proposes a new method for impact mass identification. The method belongs to the group of experimental methods, since it requires no numerical model of the concerned structure. It is based on the Virtual Distortion Method and makes use of the dynamic influence matrices, which are measured experimentally using the unaffected structure only. However, in contrary to experimental methods based on statistical or pattern recognition approaches, the proposed method utilizes actual mechanical principles and not a database of response fingerprints.

The method has been validated experimentally using a simple truss structure and an actual mass of 0.44 kg. The results are numerically stable, which is confirmed by consistent identification results at the two tested extreme levels of the regularization: 1 % to 6 % mass identification error and 8 % to 21 % velocity identification error.

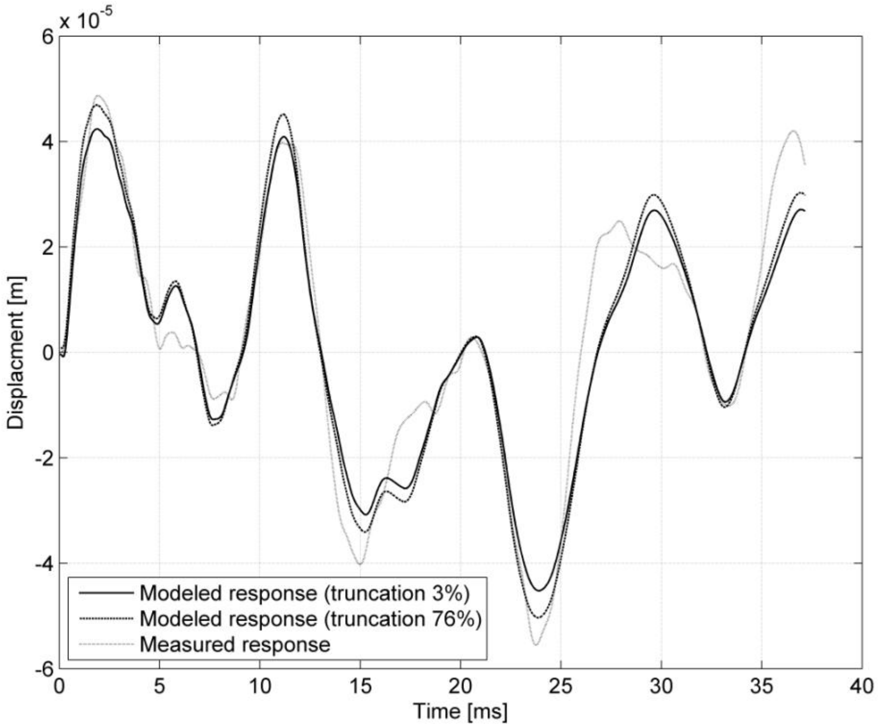

Figure 5. Time responses for the actual mass of 0.44 kg: measured and modeled at two extreme levels of regularizing truncation of the singular values

The research is ongoing to test a more complex truss structure, which would allow the impact location to be identified, too. Moreover, preliminary experimental results suggest worsening identification accuracy for higher impact masses; hence a wider range of impact masses and velocities should be tested.

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