A model-less method for added mass identification

Grzegorz Suwała[1,a], Łukasz Jankowski[1,b]

[1]Smart-Tech Centre, Institute of Fundamental Technological Research
ul. Świętokrzyska 21, 00-049 Warsaw, Poland

[a]gsuwala@ippt.gov.pl, [b]ljank@ippt.gov.pl

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Abstract. This paper present and validates experimentally a model-less methodology for off-line identification of modifications of nodal masses. The proposed approach is entirely based on experimentally measured data; hence no numerical modeling and tedious fine-tuning of the model are necessary. The influence of the added mass is modeled using virtual distortion forces and experimentally obtained system transfer matrices. The identification amounts to solving an optimization problem of minimizing the mean square distance between measured and modeled structural responses, the latter is based on previously recorded responses of the unaffected structure.

Introduction

The main motivation for this research is the need for an experimentally robust and practical analysis technique for mass monitoring and reconstruction of the scenario of a mass-induced loading, to be used in black-box type monitoring systems. Moreover, related problems of dynamic mass identification are crucial in many practical applications, such as in monitoring of crowd-induced excitations, off-shore platforms, etc.

The methods used in structural health monitoring, including mass identification methods, can be generally classified into two main groups [1-4]: pattern recognition and model-based approaches. Pattern recognition approaches often rely on statistical or artificial intelligence methods and use a database (or a training set) of fingerprints extracted from several responses. The database has to be previously constructed by experimental measurements of several scenarios, which are to be identified later (impact locations, mass modifications, damages, etc.). Given the database, the actual scenario is identified using the fingerprints only, without the insight into their actual mechanical meaning, which requires neither the model nor simulation of the event. On the contrary, the model-based approaches require usually a pre-calibrated numerical model of the structure; the identification amounts to iterative modifications of the model, simulations of the corresponding response and its comparison with the experimentally measured response of the modified/damaged structure or the response gathered during the even being identified. The method proposed in this paper seems to be located in between both groups: although the response of the modified structure is computed numerically based on actual mechanical principles, it uses experimentally measured data only and no numerical model of the structure is necessary.

Theoretical formulation

The proposed model-less methodology makes use of the Virtual Distortion Method (VDM) [1, 5]. Assume the equilibrium equation for the modified structure can be written as

$$\hat{\mathbf{M}}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{1}$$

where $\hat{\mathbf{M}}$ denotes the modified mass matrix, $\mathbf{K}$ the stiffness matrix and $\mathbf{f}(t)$ are the external test forces. According to the VDM, the effects of the modification of the mass matrix can be modeled using the original unmodified structure and the so-called virtual forces $\mathbf{p}(t)$:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)+\mathbf{p}(t), \tag{2}$$

where is $\mathbf{M}$ is the original mass matrix. Comparison of Eqs. 1 and 2 yields the formula

$$\mathbf{p}(t)=-\Delta\mathbf{M}\ddot{\mathbf{u}}(t), \tag{3}$$

which in the case of nodal mass modifications simplifies to

$$p_i(t)=-\Delta m_i\ddot{u}_i(t), \tag{4}$$

where i denotes the degree of freedom and Δm_i the corresponding mass modification. According to the VDM, the acceleration $\ddot{u}_i(t)$ is a linear combination of the structural response of the unmodified structure $\ddot{u}_i^L(t)$ and the cumulative effect of all the virtual distortions $p_j(\tau)$, $\tau \le t$, modeling the mass modification, which can be stated explicitly as

$$\ddot{u}_i(t)=\ddot{u}_i^L(t)+\sum_j\sum_{\tau\le t}\ddot{B}_{ij}(t-\tau)\,p_j(\tau), \tag{5}$$

where $\ddot{B}_{ij}(t-\tau)$ is an element of the dynamic influence matrix (system transfer matrix) and denotes the acceleration in the ith degree of freedom (DOF) in the time instant t in response to a unit impact force in the jth DOF in the time instance τ. Eq. 5 can be substituted into Eq. 4 and the unknown virtual forces $p_i(t)$ in the current time step t can be collected on the left-hand side:

$$\sum_j\left[\Delta m_i\ddot{B}_{ij}(0)+\delta_{ij}\right]p_j(\tau)=-\Delta m_i\left[\ddot{u}_i^L(t)+\sum_j\sum_{\tau<t}\ddot{B}_{ij}(t-\tau)\,p_j(\tau)\right]. \tag{6}$$

Equation 6 has a time-independent principal matrix and can be used in successive time steps to compute iteratively all the virtual forces. When they are computed, the displacements of the modified structure can be calculated, as in Eq. 5, by a linear combination of the response of the unaffected structure and the cumulative effects of the virtual forces:

$$u_i(t)=u_i^L(t)+\sum_j\sum_{\tau\le t}B_{ij}(t-\tau)\,p_j(t), \tag{7}$$

where $\ddot{B}_{ij}(t-\tau)$ is the corresponding dynamic influence matrix.

The identification of the mass is based on the comparison between the measured and computed responses in sensor locations. The following objective function is used:

$$F(\Delta m)=\sum_i\frac{\sum_t\left[u_i(t)-u_i^M(t)\right]^2}{\sum_t\left[u_i^M(t)\right]^2}, \tag{8}$$

where $u_i^M(t)$ is the measured response, $u_i(t)$ is the computed response and index i covers all sensor locations.

Practical formulation

The dynamic influence matrices used in the theoretical formulation are composed of discretized structural responses to unit impact forces, that is to forces acting in one time step only. Such an excitation can be easily simulated numerically, but not implemented experimentally. In practice, the excitation impulse force is usually generated with an impact hammer and lasts several time steps, see Fig. 1.

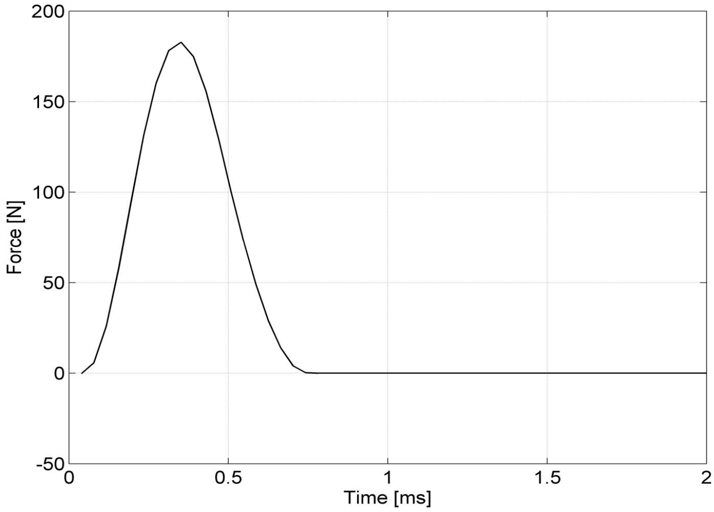

Figure 1. Experimentally measured force impulse

An experimentally measured dynamic influence matrix must hence contain structural responses to normalized diffuse impact loads instead of the theoretical one time step loads. Moreover, the numerical stability of the iterative procedure to determine the virtual forces relies on the accuracy of the dynamic influence matrix. In practical cases it is inevitably contaminated with measurement noise, which leads to errors accumulating over successive time steps and divergent results.

The differences between the theoretical formulation and the practical limitations result in different modeling of the virtual distortion forces. Since, in practical cases, only the responses to diffuse excitations of an impact hammer are available, a virtual force has to be represented in the form of a linear combination of the corresponding normalized impact hammer peaks (Fig. 1):

$$p_i(t) = \sum_{\tau} f_i(t-\tau)\, a_i(\tau). \tag{9}$$

The formulas for the structural response, Eqs. 5 and 7, have to be accordingly updated, e.g.:

$$\ddot{u}_i(t) = \ddot{u}_i^L(t) + \sum_j \sum_\tau \ddot{D}_{ij}(t-\tau)\, a_j(\tau), \tag{10}$$

where $\ddot{D}_{ij}(t-\tau)$ denotes an element of the experimental system transfer matrix. Substitution of Eqs. 9 and 10 into Eq. 4 yields

$$\sum_\tau f_i(t-\tau)\, a_i(\tau) = -\Delta m_i \left[\ddot{u}_i^L(t) + \sum_j \sum_\tau \ddot{D}_{ij}(t-\tau)\, a_i(\tau) \right], \tag{11}$$

which leads to the following linear system

$$\sum_{j}\sum_{\tau}\left[\delta_{ij}f_i(t-\tau)+\Delta m_i\ddot{D}_{ij}(t-\tau)\right]a_j(\tau)=-\Delta m_i\ddot{u}_i^L(t), \tag{12}$$

which can be used to compute the coefficients $a_i(\tau)$. Notice that Eq.12 is a single, large linear system involving the unknown coefficients $a_i(\tau)$ in all time steps τ and degrees of freedom *i*. In this way, the potential numerical instability of the iteratively repeated solutions of Eq. 6 can be avoided at the cost of a larger numerical effort required to solve Eq. 12. Moreover, since this system is often ill-conditioned, a regularization technique has to be used. This paper uses the singular value decomposition (SVD) and the truncation of too small singular values. Finally, given the coefficients $a_i(\tau)$, the structural response in all sensor locations *i* can be computed by

$$u_i(t)=u_i^L(t)+\sum_{j}\sum_{\tau}D_{ij}(t-\tau)a_j(\tau). \tag{13}$$

To identify the mass, the proposed method solves an optimization problem of minimizing the mean square distance Eq. 8 between the measured and the modeled structural responses. The modeled response is computed by Eqs. 12 and 13 and hence based exclusively on previously measured responses of the unaffected structure. No numerical model of the structure is necessary.

Experimental test stand

To verify the proposed method, a simple three elements truss structure has been used (Fig. 2). The elements are 1200 mm long rectangular (20 x 20 mm) steel tubes with the thickness of 2 mm. The right-hand side ends of the elements were fixed. Fig. 2 shows the original unmodified truss (left figure) and the only node with an additional mass mounted (right figure).

Figure 2. Truss construction without and with the additional mass

A Brüel & Kjær impact hammer has been used for force excitation in three directions. The response has been measured in three directions with three accelerometers (Brüel & Kjær, Model 5302), all fixed to the node of the structure. For verification purposes, the first 1000 time steps have been used, measured with 25.6 kHz sampling frequency (39 ms) and triggered by the hammer excitation. The signals from the sensors and the hammer were collected via a PULSE system to a desktop PC for analysis.

In order to build the experimental dynamic influence matrices, the responses of the unaffected structure had to be recorded. The impact force was applied to the node in three directions and the responses were measured in three directions. This simple structure has only one node, hence only nine acceleration measurements (and the corresponding computed displacements) had to be stored.

However, for more complex structures measuring the responses necessary to build the dynamic influence matrices can be work consuming.

Experimental verification

The proposed method has been verified experimentally with two masses (0.44 kg and 1.14 kg), which have been mounted to the node. As a test excitation source an impact hammer has been used. The objective function Eq. 8 compared the displacements in the impact direction (one sensor only).

For identification purposes, Eq. 12 had to be solved several times for different trial masses to find the corresponding coefficients $a_i(\tau)$ and then the structural response by Eq. 13. To spare the numerical costs, in the identification stage the first 300 time steps have been used, which resulted in the system Eq. 12 having the dimension 900 x 900. Fig. 3 shows the objective functions computed for the masses 0.44 kg (left figure) and 1.14 kg (right figure). The minima are found respectively at 0.47 kg and 1.29 kg, which correspond to the identification errors of 7 % and 13 %.

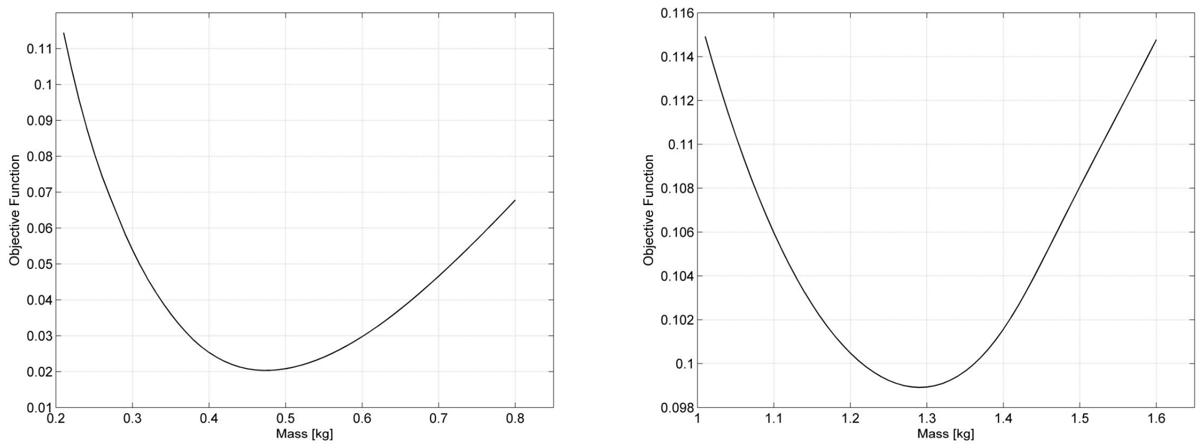

Figure 3. Objective functions computed for the actual masses of 0.44 kg (left) and 1.14 kg (right)

The two identified mass values have been used to generate and solve the two corresponding systems Eq. 12 for 1000 time steps. These systems are larger (3000 x 3000) but had to be solved only once, since the masses had been assumed to be already known. Figures 4 and 5 compare the measured response of the unaffected structure, the measured response of the modified structure and the response modeled for the identified mass values.

Conclusions

This paper proposes a new method for added mass identification. The method belongs to the group of experimental methods, since it requires no numerical model of the concerned structure. It is based on dynamic influence matrices, which are measured experimentally using the unaffected structure only. However, in contrary to experimental methods based on statistical or artificial intelligence approaches, it utilizes actual mechanical principles and not a database of response fingerprints.

The method has been validated experimentally using a simple truss structure and two masses of 0.44 kg and 1.14 kg. The mass identification errors were 7 % and 13 %, respectively.

The research is ongoing to generalize the approach to identification of inelastic impacts, including identification of the impacting mass, velocity and impact location.

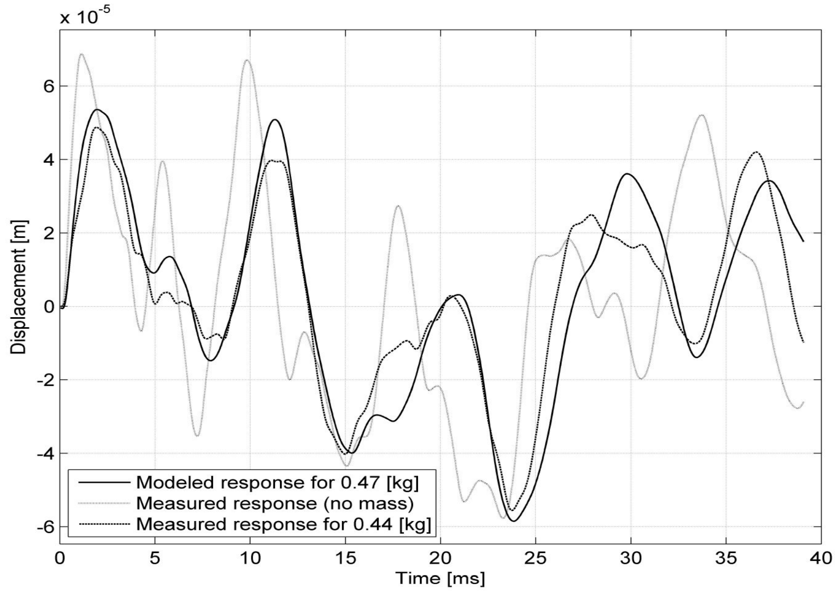

Figure 4. Time responses for the actual mass of 0.44 kg

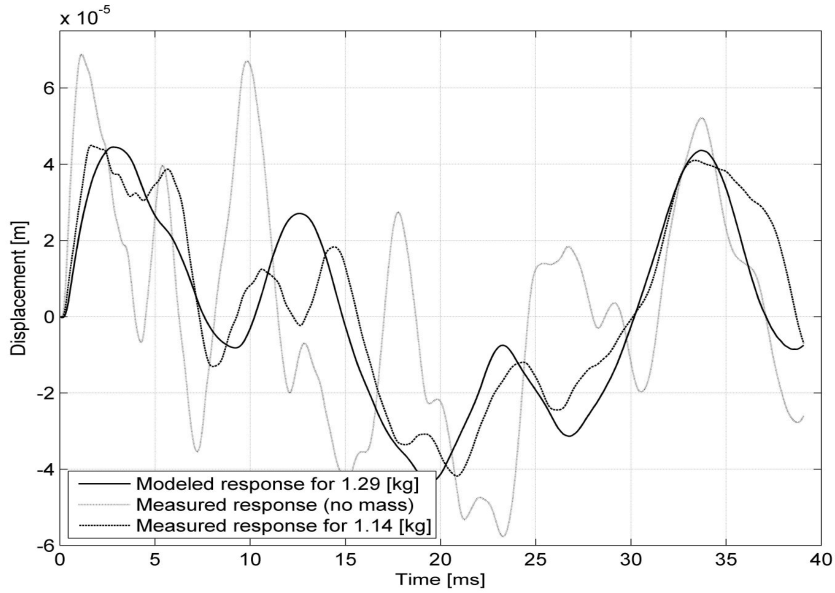

Figure 5. Time response for the actual mass of 1.14 kg

References

[1] *Smart Technologies for Safety Engineering*, edited by J. Holnicki-Szulc, John Wiley & Sons (2008).

[2] P. Kolakowski, L. E. Mujica Delgado and J. Vehi: J. Intell. Mater. Syst. Struct. Vol. 17(1) (2006), pp. 63-79.

[3] L. E. Mujica Delgado, *A hybrid approach of knowledge-based reasoning for structural assessment*, Ph.D. thesis, University of Girona (2006).

[4] R. Le Riche, D. Gualandris, J. J. Thomas and F. Hemez: J. Sound Vibrat. Vol. 248(2) (2001), pp. 247-265.

[5] J. Holnicki-Szulc and J. T. Gierliński, *Structural Analysis, Design and Control by the Virtual Distortion Method*, John Wiley & Sons, Chichester (1995).