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A VDM-based method for fast reanalysis and identification of structural damping

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1. Abstract
This paper proposes a new approach for modeling and identification of damping in linear structures. The approach is based on the Virtual Distortion Method (VDM), which is a reanalysis methodology for fast modeling and identification of structural parameters. The VDM is extended in the sense of considering structural damping in frequency domain. The assumed damping model is a modified version of the proportional damping, which allows to distinguish between and to modify independently the damping properties of each element and in each degree of freedom. In this way, a precise formulation of the task of remodeling of damping is possible. Moreover, the proposed approach is used to state and solve the (basically nonlinear) inverse problem of identification of material damping by decomposing it into two linear subproblems.
2. Keywords: damping identification, structural reanalysis, Virtual Distortion Method (VDM)

3. Introduction
The energy of vibrations in real structures is always dissipated and the nature of this process is complex. There are various mechanisms of the damping phenomena, discussed f.e. in [1,2,3]. One class is material damping in high damping alloys, composite materials and viscoelastic materials. Another class is interface friction also called boundary damping associated with junctions or interfaces between substructures. Further there are viscous effects due to the fluid which is in contact with a structure as well as non material damping like acoustic radiation damping or linear air pumping. The most commonly used model is the classical damping model. It is based on the assumption that linear system possesses classical normal modes and there are non-zero elements only on the diagonal of modal damping matrix. In non-classically damped systems the off-diagonal elements are ignored. This assumptions make the calculation easier because allows for decoupling of equations of motion. In some cases it is not possible to make such simplifications and off-diagonal elements which couple equations of motion have to be taken into account [2,4]. In this paper only classical, viscous damping model is considered. Methods of viscous damping identification in frequency domain can be divided into three groups [5,6]: matrix methods which use full frequency response functions to obtain mass, stiffness and damping matrices, modal methods based on complex mode shapes and natural frequencies received from modal testing and enhanced methods which are based on the two previous methods in order to improve accuracy or reduce the number of measurements. Adhikari presented a generalized proportional damping method for damping modeling in which damping coefficients vary with the frequency [7]. The method requires measurements of natural frequencies and modal damping factors.

The approach proposed in this paper is based on the Virtual Distortion Method (VDM), which is a fast reanalysis method for modeling and identification of structural parameters [8,9,10,11]. At its beginnings, the VDM was used in statics for remodeling and optimal design using only one stiffness-related field of distortions. It has been soon applied to dynamic reanalysis in time and in frequency domains, as well as extended to model both mass and stiffness modifications. The method has been widely applied for damage identification and optimal design of elastic structures, for dynamic load adaptation of elastoplastic structures, but also for identification of water leakages or damages in electric networks, etc [12,13]. Up to now, however, reanalysis of damping was not considered. As a frequency-dependent feature, it is pursued here in frequency domain, which leads to a quasi-static formulation.

The VDM allows a structure to be quickly reanalyzed based only on the Finite Element Method solution of the unmodified structure, without the need to recompute the mass, stiffness and damping matrices. It is possible, provided that the response of the original structure can be combined with its responses to certain local distortions that model the modifications. These responses, that is the influence of local distortions on the global solution, are stored in influence matrices which are the crucial tool of the VDM. Such an approach, anticipating that partial responses can be scaled and added together, is valid only for linear systems. As a result, the VDM allows the damping properties of a structure or its substructure to be quickly reanalysed and identified, without the need for a repeated analysis of the full structure.

In general, the relation between the damping parameters and the response is clearly nonlinear. However, the response depends linearly on the damping-modeling distortions. Moreover, in the task of damping identification, the relation between the distortions and the damping parameters turns out to be also linear, provided the response is measured in suitable points. Hence, a temporary change of variables from the damping parameters to the distortions significantly simplifies the (basically nonlinear) identification problem by decomposing it into two simple linear subproblems. The proposed process of damping identification proceeds in two stages. First, the space of all complex distortions corresponding to the measured response is found. This solution is often nonunique, as the governing matrix is usually singular. Hence, second, the solution is limited to the distortions corresponding to physically meaningful real damping parameters, which in practical cases can be in this way determined uniquely.

The paper is structured as follows: the next (fourth) section specifies the generalized damping model, the fifth section states the general VDM-based formulation of the reanalysis problem, while the inverse problem of damping identification is considered in the sixth section. The seventh section illustrates the proposed approach in a numerical example.

4. Damping model

A generalized Rayleigh damping model is used. The standard model decomposes the structural damping into two components that are related to environmental and material factors [15] and weighted by the coefficients α and β, respectively. In the standard model, the damping matrix $\mathbf{C}$ is a linear combination of the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$, which are assumed in this paper to be known (or identified beforehand),

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}. \tag{1}$$

The standard Rayleigh model is generalized here to allow independent modeling of material damping in the structural elements and environmental damping in the degrees of freedom (DOFs). For notational simplicity it is assumed here that the considered structure is a truss; however, the approach can be straightforwardly applied for other structures (see e.g. [14,11] for VDM-based reanalysis of frames and plates). In case of a truss, the stiffness matrix can be directly related to the diagonal matrix $\mathbf{S}$ of element stiffnesses $E_i A_i$ by

$$\mathbf{K} = \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{G}, \tag{2}$$

where $\mathbf{L}$ is the diagonal matrix of element lengths l_i and $\mathbf{G}$ is the geometric (or displacement-strain) matrix, which transforms the global displacements $\mathbf{u}$ to local element strains, that is $\boldsymbol{\varepsilon} = \mathbf{G}\mathbf{u}$. Independent modeling of damping in all elements and in all DOFs is possible by assuming that

$$\mathbf{C} = \mathbf{C}_\alpha\mathbf{M} + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{C}_\beta\mathbf{G}, \tag{3}$$

where $\mathbf{C}_\alpha$ and $\mathbf{C}_\beta$ are diagonal matrices of the environmental and material damping factors related respectively to the DOFs and to the elements of the structure,

$$\begin{aligned} \mathbf{C}_\alpha &= \operatorname{diag}\{\alpha_1, \ldots, \alpha_{N_{\mathrm{DOF}}}\}, \\ \mathbf{C}_\beta &= \operatorname{diag}\{\beta_1, \ldots, \beta_{N_{\mathrm{el}}}\}, \end{aligned}$$

where N_{DOF} and N_{el} denote the numbers of the DOFs and of the elements.

5. Reanalysis of damping properties

The analysis is performed in the frequency domain. A harmonic excitation $\mathbf{f}e^{\mathrm{i}\omega t}$ is considered, where $\mathbf{f}$ is a vector of the complex excitation amplitudes in all DOFs.

In agreement with the general idea of the VDM [9], modifications of environmental and material damping are modeled by two respective fields of virtual distortions imposed on the original unmodified structure. The response of the modified structure is thus modeled as the response of the unmodified structure distorted by the influences of the virtual distortions.

5.1. Original unmodified structure

The response of the structure to the harmonic excitation $\mathbf{f}$ is also harmonic; it will be denoted by $\mathbf{u}^{\mathrm{L}}$

(displacements) or $\boldsymbol{\varepsilon}^{\mathrm{L}}$ (strains). It is assumed that the response is known, that is obtained by solving the linear equation of motion of the original unmodified structure, which yields the following quasi-static formulation:

$$\left[-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K}\right]\mathbf{u}^{\mathrm{L}} = \mathbf{f} \tag{4}$$

or, by Eq.(2) and Eq.(3),

$$-\omega^2\mathbf{M}\mathbf{u}^{\mathrm{L}} + \mathrm{i}\omega\mathbf{C}_\alpha\mathbf{M}\mathbf{u}^{\mathrm{L}} + \mathrm{i}\omega\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{C}_\beta\boldsymbol{\varepsilon}^{\mathrm{L}} + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\boldsymbol{\varepsilon}^{\mathrm{L}} = \mathbf{f}. \tag{5}$$

5.2. Modified structure

Consider a modification of the damping properties that is characterized by certain (diagonal) modifications to the original matrices of damping parameters,

$$\Delta\mathbf{C}_\alpha := \hat{\mathbf{C}}_\alpha - \mathbf{C}_\alpha,$$
$$\Delta\mathbf{C}_\beta := \hat{\mathbf{C}}_\beta - \mathbf{C}_\beta,$$

where $\hat{\mathbf{C}}_\alpha$ and $\hat{\mathbf{C}}_\beta$ denote the modified matrices. The response $\mathbf{u}$ of the modified structure can be computed directly by solving the quasi-static form of the equation of motion,

$$-\omega^2\mathbf{M}\mathbf{u} + \mathrm{i}\omega\hat{\mathbf{C}}_\alpha\mathbf{M}\mathbf{u} + \mathrm{i}\omega\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\hat{\mathbf{C}}_\beta\boldsymbol{\varepsilon} + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\boldsymbol{\varepsilon} = \mathbf{f}. \tag{6}$$

However, Eq.(6) relates the response $\mathbf{f}$ to the damping modifications $\Delta\mathbf{C}_\alpha$ and $\Delta\mathbf{C}_\beta$ in a nonlinear way, and hence any direct solution of the inverse problem corresponding to Eq.(6) can be numerically costly. Moreover, the formulation of Eq.(6) involves all DOFs and thus requires a solution of the full system, which is impractical in case of a large structure and localized modifications.

5.3. Modeled structure

The modifications $\Delta\mathbf{C}_\alpha$ and $\Delta\mathbf{C}_\beta$ of the environmental and material damping are modeled by two respective fields of harmonic virtual distortions: force distortions (or pseudo forces) acting in the affected DOFs and strain distortions imposed on the affected elements. The complex amplitudes of these distortions are denoted by $\mathbf{d}^0$ and $\boldsymbol{\phi}^0$, respectively. Both fields distort the original unmodified structure,

$$-\omega^2\mathbf{M}\mathbf{u} + \left[\mathrm{i}\omega\mathbf{C}_\alpha\mathbf{M}\mathbf{u} - \mathbf{d}^0\right] + \left[\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}(\mathrm{i}\omega\mathbf{C}_\beta\boldsymbol{\varepsilon} - \boldsymbol{\phi}^0)\right] + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\boldsymbol{\varepsilon} = \mathbf{f}, \tag{7}$$

in such a way that the responses and the member forces in the modified and the modeled structures are equal. Therefore, Eq.(6) and Eq.(7) yield together the following system of two linear equations:

$$-\mathrm{i}\omega\Delta\mathbf{C}_\alpha\,\mathbf{M}\mathbf{u} = \mathbf{d}^0, \tag{8a}$$
$$-\mathrm{i}\omega\Delta\mathbf{C}_\beta\,\boldsymbol{\varepsilon} = \boldsymbol{\phi}^0. \tag{8b}$$

Note that Eqs.(8) express the distortions $\mathbf{d}^0$ and $\boldsymbol{\phi}^0$ in an implicit way, since the responses $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ on the left-hand side depend on the distortions.

The formula Eq.(7), rewritten in the form

$$-\omega^2\mathbf{M}\mathbf{u} + \mathrm{i}\omega\mathbf{C}_\alpha\mathbf{M}\mathbf{u} + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{C}_\beta\boldsymbol{\varepsilon} + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\boldsymbol{\varepsilon} = \mathbf{f} + \mathbf{d}^0 + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\boldsymbol{\phi}^0$$

and compared to Eq.(5), proves that the response $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ of the modeled structure depends linearly on the virtual distortions and hence can be stated in the following form:

$$\mathbf{u} = \mathbf{u}^{\mathrm{L}} + \mathbf{B}^{\mathrm{f}}\mathbf{d}^0 + \mathbf{B}^{\varepsilon}\boldsymbol{\phi}^0, \tag{9a}$$
$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{L}} + \mathbf{D}^{\mathrm{f}}\mathbf{d}^0 + \mathbf{D}^{\varepsilon}\boldsymbol{\phi}^0, \tag{9b}$$

where $\mathbf{B}^{\mathrm{f}}$, $\mathbf{B}^{\varepsilon}$, $\mathbf{D}^{\mathrm{f}}$ and $\mathbf{D}^{\varepsilon}$ are so-called quasi-static influence matrices, which describe in frequency domain the response of the structure to the unit harmonic force in each DOFs ($\mathbf{B}^{\mathrm{f}}$ and $\mathbf{D}^{\mathrm{f}}$) and to the unit harmonic distortion imposed on each element ($\mathbf{B}^{\varepsilon}$ and $\mathbf{D}^{\varepsilon}$). In other words, $\mathbf{B}^{\mathrm{f}}$ is the $N_{\mathrm{DOF}} \times N_{\mathrm{DOF}}$ FRF matrix computed at the considered ω and

$$\mathbf{D}^{\mathrm{f}} = \mathbf{G}\mathbf{B}^{\mathrm{f}},$$
$$\mathbf{B}^{\varepsilon} = \mathbf{B}^{\mathrm{f}}\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S},$$
$$\mathbf{D}^{\varepsilon} = \mathbf{G}\mathbf{B}^{\mathrm{f}}\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}.$$

The formulas Eq.(9) can be substituted into Eq.(8) in order to yield the following single square linear system:

$$\mathbf{F}\mathbf{x}^0 = \mathbf{b}, \tag{10}$$

where the principal matrix $\mathbf{F}$ is given by

$$\mathbf{F} = \begin{bmatrix} \mathbf{I} + \mathrm{i}\omega\Delta\mathbf{C}_\alpha\mathbf{M}\mathbf{B}^\mathrm{f} & \mathrm{i}\omega\Delta\mathbf{C}_\alpha\mathbf{M}\mathbf{B}^\varepsilon \\ \mathrm{i}\omega\Delta\mathbf{C}_\beta\mathbf{D}^\mathrm{f} & \mathbf{I} + \mathrm{i}\omega\Delta\mathbf{C}_\beta\mathbf{D}^\varepsilon \end{bmatrix},$$

the vector $\mathbf{x}^0$ collects the virtual distortions,

$$\mathbf{x}^0 = \begin{bmatrix} \mathbf{d}^0 & \boldsymbol{\phi}^0 \end{bmatrix}^\mathrm{T},$$

and the right-hand side vector is given by

$$\mathbf{b} = \begin{bmatrix} -\mathrm{i}\omega\Delta\mathbf{C}_\alpha\mathbf{M}\mathbf{u}^\mathrm{L} \\ -\mathrm{i}\omega\Delta\mathbf{C}_\beta\boldsymbol{\varepsilon}^\mathrm{L} \end{bmatrix}.$$

Given the response of the unmodified structure ($\mathbf{u}^\mathrm{L}$ and $\boldsymbol{\varepsilon}^\mathrm{L}$) and the assumed damping modifications ($\Delta\mathbf{C}_\alpha$ and $\Delta\mathbf{C}_\beta$), the corresponding response of the modified structure ($\mathbf{u}$ and $\boldsymbol{\varepsilon}$) can be computed by first solving Eq.(10) for the distortions, and then by the direct substitution of the computed distortions into Eq.(9).

5.4. Remarks

Theoretically, the square system Eq.(10) in the full form is large: the number of unknowns equals the total number of the distortions, that is the total number of both DOFs and elements of the considered structure. Indeed, if all structural matrices $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}_\alpha$ and $\mathbf{C}_\beta$ were available and if damping was globally modified, the direct solution of Eq.(6) would be numerically less costly. However, as in all other VDM-based approaches, in most of the applications the proposed here distortion-based approach can be preferable because of two common reasons:

1. *Reduced-size local analysis:* If modifications or identification of damping in only a limited number of DOFs or elements is considered, the virtual distortions in all other localizations vanish, and the dimensions of the VDM-based formulation become respectively smaller. Moreover, in such a case only the respective parts of the full response vectors $\boldsymbol{\varepsilon}$ and $\mathbf{u}$ are necessary, which can simplify a practical implementation.
2. *Linearity of the inverse problem:* The formula Eq.(6), as well as Eq.(10), relates the response to the damping modifications in a nonlinear way, and hence any direct solution of the corresponding inverse problem can be numerically costly. However, the proposed formulation of Eq.(8) and Eq.(9) allows the identification problem to be decomposed into two simple linear subproblems, provided the response is measured in suitable points. This is described in the next section.

6. Identification of material damping

In this section, the problem of identification of element-specific coefficients of the generalized material damping model is considered. It is assumed that the mass and stiffness matrices are known or identified beforehand. The problem can be thus stated as follows: *given the mass and stiffness matrices* $\mathbf{M}$ *and* $\mathbf{K}$*, a harmonic excitation* $\mathbf{f}$ *and the strain response* $\boldsymbol{\varepsilon}$ *in all considered elements, find the corresponding element-specific coefficients of material damping* β_i. A straightforward solution would require a minimization of the residual of Eq.(6) with respect to all coefficients β_i, which is clearly a nonlinear problem that can be numerically costly, especially in case of a large structure. In this section, an alternative and significantly simpler approach is proposed.

Let $\boldsymbol{\varepsilon}^\mathrm{L}$ denote the theoretical reference response corresponding to vanishing material damping $\beta_i = 0$; the response $\boldsymbol{\varepsilon}^\mathrm{L}$ can be directly computed by Eq.(4) with the damping matrix $\mathbf{C}$ equal to zero (or to an initial approximation). The original nonlinear inverse problem is decomposed into two simple linear steps:

1. Identification of the virtual distortions $\boldsymbol{\phi}^0$ that correspond to the response $\boldsymbol{\varepsilon}$. This amounts to the solution of the following version of the linear equation Eq.(9b):

$$\mathbf{D}^\varepsilon\boldsymbol{\phi}^0 = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^\mathrm{L}, \tag{11}$$

where, since identification of only material damping is considered, it has been assumed that the force distortions $\mathbf{d}^0$ vanish.

2. Computation by Eq.(8b) of the element-specific coefficients β_i of material damping. Since the response ε_i in each element is known, Eq.(8b) is decoupled into the following set of simple linear equations in one unknown:
$$-\mathrm{i}\omega\beta_i\varepsilon_i = \phi_i^0, \qquad i = 1, 2, \ldots, N_{\mathrm{el}}. \tag{12}$$

6.1. Step 1: Identification of equivalent virtual distortions

The aim of the first step is to solve the square linear system Eq.(11) in order to find the virtual distortions $\boldsymbol{\phi}^0$ that result in the response $\boldsymbol{\varepsilon}$. Since in statically indeterminate structures the influence matrix $\mathbf{D}^\varepsilon$ is singular, there is an infinite number of distortions that solve Eq.(11) in the least-squares sense; they will be collectively called the equivalent distortions. These distortions form a linear subspace that can be computed using the (unique) singular value decomposition (SVD) [16] of the influence matrix, defined as

$$\mathbf{D}^\varepsilon = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}, \tag{13}$$

where $\mathbf{U}$ and $\mathbf{V}$ are certain complex unitary matrices, $\mathbf{V}^{\mathrm{H}}$ denotes the conjugate transpose of $\mathbf{V}$, and $\boldsymbol{\Sigma}$ is a diagonal matrix with real positive elements on the diagonal, which are ordered non-increasing and are called the singular values of $\mathbf{D}^\varepsilon$. The equivalent virtual distortions can be represented in the following form:
$$\boldsymbol{\phi}^0 = \boldsymbol{\phi}^{\mathrm{SVD}} + \mathbf{W}\boldsymbol{\alpha}, \tag{14}$$
where the columns of the matrix $\mathbf{W}$ generate the null space of $\mathbf{D}^\varepsilon$, the vector $\boldsymbol{\alpha}$ contains arbitrary complex numbers and $\boldsymbol{\phi}^{\mathrm{SVD}}$ is the least-squares solution of Eq.(11). Therefore, the matrix $\mathbf{W}$ is the submatrix of $\mathbf{V}$ that is formed by its last columns that correspond to all vanishing singular values, while the vector $\boldsymbol{\alpha}$ linearly combines the columns of $\mathbf{W}$. The least-squares solution $\boldsymbol{\phi}^{\mathrm{SVD}}$ can be computed using the Moore-Penrose pseudoinverse of $\mathbf{D}^\varepsilon$ as

$$\boldsymbol{\phi}^{\mathrm{SVD}} = \mathbf{V}\boldsymbol{\Sigma}^+\mathbf{U}^{\mathrm{H}}\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{L}}\right),$$

where the diagonal matrix $\boldsymbol{\Sigma}^+$ is obtained from $\boldsymbol{\Sigma}$ by replacements of its all nonvanishing elements with their reciprocals.

6.2. Step 2: Computation of the coefficients of material damping

Given the equivalent distortions by Eq.(14), the element-specific damping parameters β_i can be computed by Eq.(12). Since all β_i are required to be real numbers, thus the set of complex equations Eq.(12) yields a twofold larger set of real equations,

$$\begin{aligned} \beta_i &= -\mathrm{Re}\,\frac{\phi_i^{\mathrm{SVD}} + \sum_j W_{ij}\alpha_j}{\mathrm{i}\omega\varepsilon_i}, \\ 0 &= -\mathrm{Im}\,\frac{\phi_i^{\mathrm{SVD}} + \sum_j W_{ij}\alpha_j}{\mathrm{i}\omega\varepsilon_i}, \end{aligned}$$

where the complex combination coefficients α_j cease to be arbitrary and instead have to satisfy together with the damping coefficients β_i a real linear system, which in the block matrix form can be stated as follows:
$$\begin{bmatrix} \mathrm{Re}\,\mathbf{W} & -\mathrm{Im}\,\mathbf{W} & -\omega\,\mathrm{diag}\,\mathrm{Im}\,\boldsymbol{\varepsilon} \\ \mathrm{Im}\,\mathbf{W} & \mathrm{Re}\,\mathbf{W} & \omega\,\mathrm{diag}\,\mathrm{Re}\,\boldsymbol{\varepsilon} \end{bmatrix} \begin{bmatrix} \mathrm{Re}\,\boldsymbol{\alpha} \\ \mathrm{Im}\,\boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} = \begin{bmatrix} -\mathrm{Re}\,\boldsymbol{\phi}^{\mathrm{SVD}} \\ -\mathrm{Im}\,\boldsymbol{\phi}^{\mathrm{SVD}} \end{bmatrix} \tag{15}$$
and in the case of overdeterminacy should be solved in the least-square sense. The system Eq.(15) has $2N_{\mathrm{el}}$ equations and $N_{\mathrm{el}} + 2N_{\mathrm{s}}$ real unknows, where N_{el} denotes the number of the considered i-indexed damping coefficients and N_{s} denotes the number of the j-indexed vanishing singular values in Eq.(13). An obvious lower bound constraint has usually to be imposed on all coefficients β_i in order to guarantee that the system is dissipative.

6.3. Remarks and possible generalizations

The identification procedure outlined in this section can be straightforwardly generalized in order to use the formulas Eqs.(8) and (9) in their full form. Hence, the DOF-specific environmental damping coefficients α_i can be identified in the same way as the material damping coefficients β_i. Moreover, the response measured in DOFs u_i can be also taken into account besides the strain response of elements ε_i. Note however that the response has to be measured in all DOFs and all elements considered for damping identification, so that the second step of the identification subproblem, which is related to Eq.(8), remains linear.

In order to guarantee the uniqueness of the identification, or as a least-square safeguard against an inevitable measurement noise, it is also possible to use

- surplus measurements in DOFs or elements which are not considered for damping identification. It would increase the number of equations in the first identification step of Eq.(9), reduce the dimensionality of the space of equivalent distortions, and thus increase the overdeterminacy of Eq.(15);
- more excitations than a single harmonic excitation $\mathbf{f}$ described above. In this case, the first identification step of Eq.(9) will have to be repeated for each measurement separately, since the equivalent distortions are excitation-specific. However, the damping coefficients remain the same, hence in the second identification step the overdeterminacy of Eq.(15) will be increased.

Although the identification methodology proposed here stems from the general approach of the VDM and hence uses a very different nomenclature, in the final form it turns out to be relatively similar to the method of estimation of damping matrix proposed by Chen et al. [17]. However, the method proposed here seems to be more general, as it allows for a reduced-size local identification and a more flexible treatment of damping parameters of various origin.

7. Numerical example

7.1. The structure

The effectiveness of the identification procedure was demonstrated with a fifteen-element truss structure shown in Figure 1. Truss elements of equal cross-sectional area $A = 0.56\,\mathrm{cm}^2$ were made of steel with Young's modulus 205 GPa and the material density of $7850\,\mathrm{kg/m}^3$. Additionally, there were concentrated masses in each node, $m = 0.8\,\mathrm{kg}$. The length of the horizontal elements was 0.75 m and the length of the vertical elements was 0.4 m. The structure was excited with a harmonic force with the amplitude of 100 N and the frequency of 100 Hz. The stiffness $\mathbf{K}$ and the mass $\mathbf{M}$ matrices have been computed based on the assumed structural parameters. The damping matrix has been initially approximated using the original Rayleigh model Eq.(1) with coefficients which corresponds to structural damping of the lowest modes at 5% of the critical damping. Based on the proposed model of the material damping, the actual damping (which is to be identified in the following) has been assumed to be increased twofold in elements no. 5 and 12 as threefold in element no. 10, see the slanted numbering of the elements in Figure 1, which marks also the various considered scenarios for the harmonic force application.

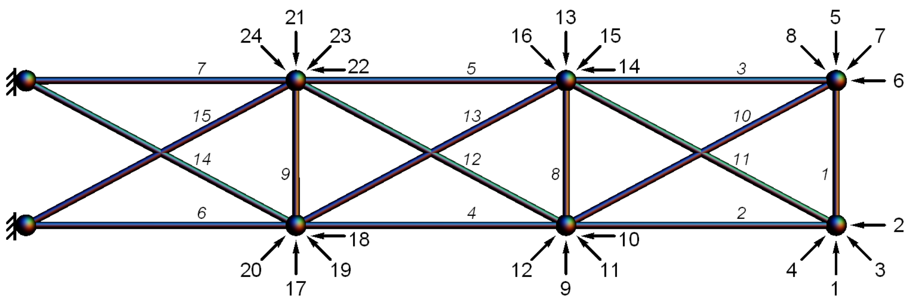

Figure 1: Truss geometry and the considered load cases

The task of identification of the actual damping amounts to

1. Computing the influence matrix $\mathbf{D}^{\varepsilon}$ of the *original structure*, that is using the known $\mathbf{M}$, $\mathbf{K}$ and the assumed initial approximation of $\mathbf{C}$.
2. Choosing the number and the form of the harmonic test excitations $\mathbf{f}_k$.
3. Computing the corresponding theoretical response $\boldsymbol{\varepsilon}_k^{\mathrm{L}}$ of the original structure using the known and assumed data.
4. Making the corresponding measurements $\boldsymbol{\varepsilon}_k$ of the actual structure, which in the numerical example can be computed using the assumed actual damping (which is to be identified later).

5. Identification of the modifications of the material damping coefficients in all truss elements by means of the Eq.(11) and Eq.(12) or Eq.(15).

It has been assumed that the (simulated) measurement setup is accurate enough to trace the shift between the responses $\Delta\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{L}}$ that is used to identify the damping. In order to make the results more reliable, the simulated measured response $\boldsymbol{\varepsilon}$ has been contaminated with uncorrelated Gausian noise at 10% rms level of $\Delta\boldsymbol{\varepsilon}$. The noise has been numerically generated as a vector of pseudorandom complex numbers with normally distributed amplitudes and the phases distributed uniformly between 0 and 2π.

7.2. Optimum choice of test excitations

Figure 1 depicts the cases of single test excitations $\mathbf{f}_k$, which have been assumed to be possible. Let $\mathcal{K}$ denote a subset of all these excitations, which is to be used for the identification. Due to practical reasons, the set $\mathcal{K}$ should be as small as possible and contain preferably only few excitations. In order to choose it, an optimality criterion is necessary. Since the identification accuracy depends mainly on the accuracy of the solution of Eq.(15), hence its condition number κ has been used as a measure of optimality. The exact condition number depends on the matrix $\mathbf{W}$, as well as on the measurements $\boldsymbol{\varepsilon}_k$ and hence on the unknown damping. However, the $\mathbf{W}$-related part of the system matrix in Eq.(12) is composed of unit vectors and thus should not introduce much ill-conditioning. Therefore, the condition number κ is estimated here by neglecting the $\mathbf{W}$-related part of the matrix and assuming $\boldsymbol{\varepsilon}_k \approx \boldsymbol{\varepsilon}_k^{\mathrm{L}}$, so that

$$\kappa(\mathcal{K}) \approx \left[\frac{\max_i \sum_{k\in\mathcal{K}} |\varepsilon_{k,i}^{\mathrm{L}}|^2}{\min_i \sum_{k\in\mathcal{K}} |\varepsilon_{k,i}^{\mathrm{L}}|^2} \right]^{\frac{1}{2}}, \tag{16}$$

where $\varepsilon_{k,i}^{\mathrm{L}}$ is the response of the ith element to the excitation $\mathbf{f}_k$. Note that the proposed approximation of κ is intuitive, as its minimization ensures that no element is too weakly excited with respect to other elements.

All 1-, 2- and 3- element subsets of the 24 possible single force excitations have been considered. By minimization of Eq.(16), the corresponding optimum excitation sets $\mathcal{K}$ have been found to be up to symmetry $\mathcal{K}_1 = \{\mathbf{f}_2\}$, $\mathcal{K}_2 = \{\mathbf{f}_2, \mathbf{f}_6\}$ and $\mathcal{K}_3 = \{\mathbf{f}_2, \mathbf{f}_6, \mathbf{f}_{18}\}$, respectively. Figure 2 plots in the logarithmic scale the value of the proposed optimality criterion for all 1- and 2-element excitation sets.

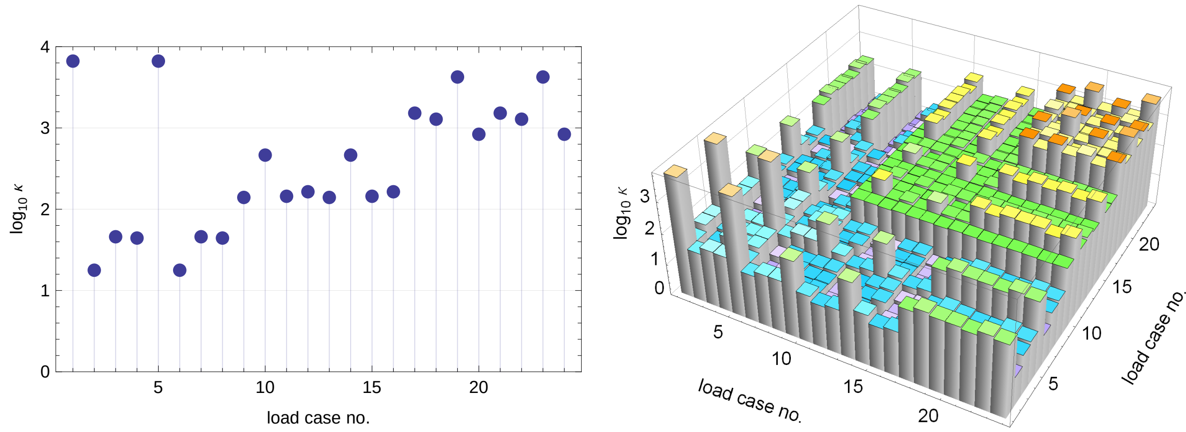

Figure 2: Optimality criterion for test excitations: (left) one excitation; (right) two excitations

7.3. Identification

The identification has been performed three times for the determined three optimum excitations sets $\mathcal{K}_1$, $\mathcal{K}_2$ and $\mathcal{K}_3$. In the first step, the equivalent distortions have been obtained by Eq.(11). The base of the null-space of the influence matrix consists of three vectors, which build the matrix $\mathbf{W}$. In the second step, the three system matrices of Eq.(15) have been constructed; for the two considered multi-excitation sets of $\mathcal{K}_2$ and $\mathcal{K}_3$, its structure had to be expanded, as described in Section 6.3. The structure of the matrix in the case of $\mathcal{K}_3$ is illustrated in Figure 3. Both the four diagonal parts related to $\operatorname{Im}\boldsymbol{\varepsilon}_1$, $\operatorname{Re}\boldsymbol{\varepsilon}_1$, $\operatorname{Im}\boldsymbol{\varepsilon}_2$ and $\operatorname{Re}\boldsymbol{\varepsilon}_1$ and the block parts composed of $\operatorname{Re}\mathbf{W}$ and $\operatorname{Im}\mathbf{W}$ are clearly recognizable.

The final identification results are shown in Figure 4. A single harmonic excitation turned out to be insufficient for an accurate identification, results obtained for two and three excitations seem to be comparable.

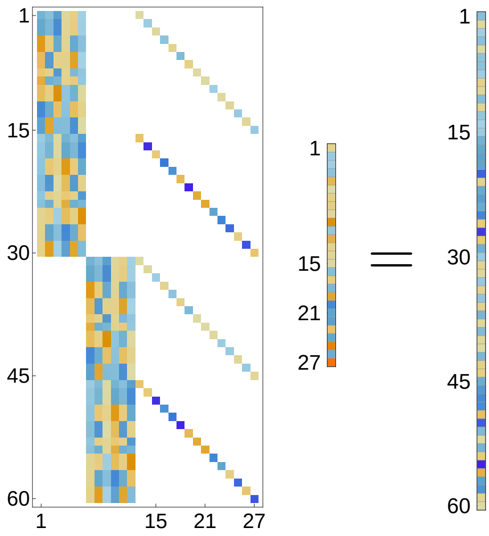

Figure 3: Visual representation of Eq.(15) expanded for the case of two excitation vectors

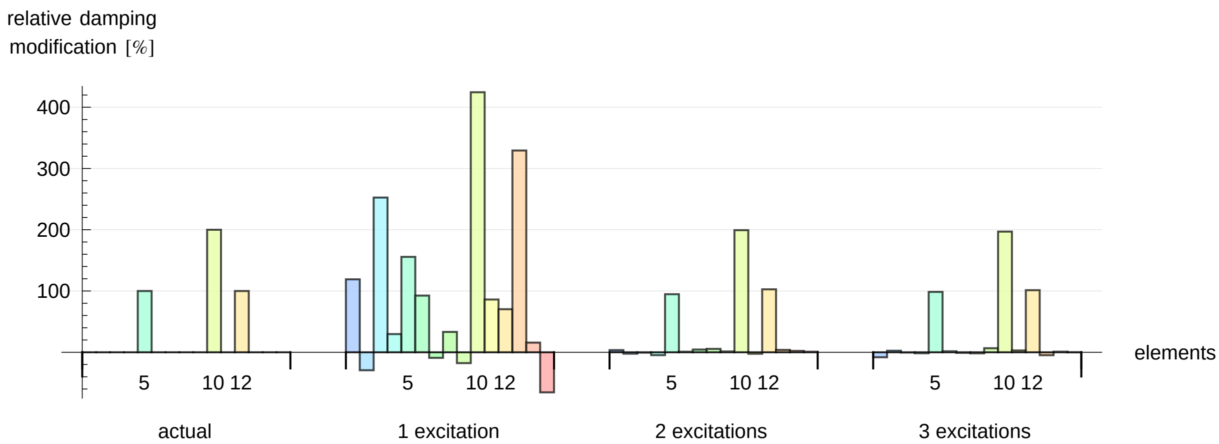

Figure 4: Actual damping modifications and values identified using one, two and three excitations

8. Summary and conclusions

The presented method of reanalysis and identification of material damping is based on the VDM method. The classical, viscous damping model is considered. The standard Rayleigh model is generalized here to allow independent modeling of the material damping in structural elements and the environmental damping in degrees of freedom (DOFs). The method requires that the mass **M** and stiffness **K** matrices are known, as well as the structural response of the modified system. The proposed approach is used to state and solve the (basically nonlinear) inverse problem of identification of material damping by decomposing it into two linear subproblems. The effectiveness of the identification procedure was demonstrated with a numerical example. The identification has been performed three times for the determined three optimal excitation sets. In order to make the results more reliable, the simulated measured response $\boldsymbol{\varepsilon}$ has been contaminated with the uncorrelated Gausian noise. Basing on the generated results it can be concluded that a single harmonic excitation is not sufficient in order to obtain a satisfactory solution. If two harmonic excitations are used then the obtained solution complies well with the actual damping modifications. Further increase in number of applied excitations does not, however, enhance the solution considerably.

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