Homework 4 (10 points)

Basics of numerics

Let

$$S(n) = \sum_{i=1}^{n} \frac{1}{i}.$$

In accurate arithmetic, the following three formulas are equivalent[1]:

$$s_1 = \frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{n}$$

$$s_2 = \frac{1}{n} + \frac{1}{n-1} + \ldots + \frac{1}{1}$$

$$s_3 = \frac{1}{n} + \frac{1}{n}\frac{n}{n-1} + \frac{1}{n}\frac{n}{n-1}\frac{n-1}{n-2} + \ldots + \frac{1}{n}\frac{n}{n-1}\cdots\frac{3}{2}\frac{2}{1}$$

[1] Assume that "+" is left-associative:

$$a + b + c \qquad \text{is interpreted as} \qquad (a + b) + c.$$

Homework 4 (10 points)

Basics of numerics

Consider the following implementations in single (float) arithmetic:

```
float s1 = 0;
for(int i=1; i<=n; ++i)
    s1 += 1/i;
```

```
float s2 = 0;
for(int i=n; i>=1; --i)
    s2 += 1/i;
```

```
float d = 1/n, s3 = d;
for(int i=n-1; i>=1; --i) {
    d *= (i+1)/i;
    s3 += d;
}
```

Homework 4 (10 points)

Basics of numerics

Compute $S(10^8)$ using all three algorithms.

1. The results are quite unexpected. Find and explain the reason.
2. Correct the error and repeat the computations. Explain the differences between the results.
3. How can you compute the exact value (up to five or six significant decimal digits)? What is it?

E-mail the answers to `ljank@ippt.pan.pl`.