Spectral Theory of Single-Phase Unidirectional Transducers

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This letter is dedicated to the Memory of the late **Ronald Reagan**, *Great President and Man of Honor.*

***Abstract*— A spectral theory of single-phase unidirectional transducers of the surface acoustic waves is presented that allows one to evaluate the stopband width where the directionality takes place, and evaluate the directionality dependence on frequency in order to optimize the transducer structure and obtain its symmetric frequency response with respect to the stopband center.**

I. Introduction

Single-phase unidirectional transducer (SPUDT) is a periodic system of especially designed groups of metal strips on piezoelectric substrate having this property that it 1) generates the surface acoustic waves (SAWs) in both directions, to the left and to the right from certain imaginary 'generation centers' placed inside each group, 2) partially reflects the right and the left propagating SAWs what can be described as the reflection from the equivalent 'reflection centers' in each group, and 3) both the generation and reflection centers are displaced by a quarter of SAW wavelength (in the best case) what results in the transducer unidirectionality if it counts enough groups of strips to reflect the SAWs sufficiently [1]. Designing the group (the transducer's cell) including several strips of different width and spacings (in contrast to ordinary bidirectional interdigital transducers, IDTs, having two strips per cell) is not an easy task; there are only a couple of cells proposed in literature and applied in the SAW low-loss filters, the main area of applications of SPUDTs. This is due to the required evaluation of the positions of the generation and reflection centers based on the advanced electrostatic analysis of the electric charge distribution on strips, resulting from either the strip voltages or induced by the incident SAW.

The analysis presented here takes advantage of the recent theoretical solutions of both the above mentioned electrostatic problems for strips [2], [3]. The presented theory has this advantage over the earlier models [1], [4], [5] that yields the dependence of the transducer directionality on frequency and particularly, it yields the stopband width where the directionality takes place, and also the relative SAW amplitudes generated in both the left and the right directions. This analysis allows one to design the strip system yielding the required stopband width and the required symmetric frequency dependence of the SAW amplitudes with respect to the center frequency. The theory shows that the above mentioned displacement of the generation and reflection centers is not a correct characterization of the system directionality; some best systems have this displacement different from a quarter of wavelength. Moreover, although the presented theory neglects the mechanical interaction of the strips with the propagating SAWs, it can be easily generalized to account for this effect in the already presented manner [6].

II. Auxiliary electrostatic analysis

At center frequency, the propagating SAW wavelength equals the transducer period (the cell's length) $\Lambda = 2\pi/K$. The electric field excited at the plane of strips due to the substrate piezoelectricity is treated here as the preexisting (or 'incident') field the strips are embedded in. This is the harmonic field $\exp(j\omega t - jIKx)$, where ω - angular frequency, and IK - the SAW wave-number for transducer working at its Ith overtone. The wave-field amplitudes are: D^I - the normal induction, and E^I - the tangential electric field. Both the propagating SAW and the voltage V applied to the transducer bus-bars (and thus to some strips in each cells) excite the electric field at the strips' plane represented by a series of spatial harmonics $\exp(-jnKx)$ with corresponding amplitudes D_n, E_n. The most interesting are the components coupled to the right and the left propagating SAWs having wave-numbers $\pm IK$, respectively.

The electrostatic analysis presented in [3] yields the following relations for both $\pm I$ harmonic field amplitudes:

$$\begin{aligned} D_{-I} &= (1+t)D^{-I} + sD^I + p^*V, \\ D_I &= s^*D^{-I} + (1+t)D^I + pV, \\ J &= j\omega(d^*D^{-I} + dD^I + gV), \\ -j\epsilon E_{-I} &= (t-1)D^{-I} + sD^I + p^*V, \\ j\epsilon E_I &= s^*D^{-I} + (t-1)D^I + pV, \end{aligned} \tag{1}$$

where J is the cell contribution to the transducer current; the parameters t, g (real) and s, p, d (complex) can be evaluated from the electrostatic analysis taking into account that $D^{\pm I} = \mp j\epsilon E^{\pm I}$ for the 'incident' field, where ϵ is the effective surface permittivity of the substrate. $D^{\pm I}$, having arbitrary value in this electrostatic analysis, will be evaluated later below using the real dynamic properties of the substrate.

III. SAW propagation

The SAW wave-number $IK + \delta$ in the system may slightly differ from the value IK (δ assumed small). It results from the harmonic Green's function of piezoelectric media [7] that

$$\begin{aligned} \pm j\epsilon E_{\pm I} &= Z_\pm D_{\pm I}, \mathcal{Z} = \mathrm{diag}[Z_{-I}, Z_I], \\ Z_{\pm I} &= (IK \pm \delta - k_o)/(IK \pm \delta - k_v), \end{aligned} \tag{2}$$

where $k_{v,o} \sim \omega$ are the wave-numbers of SAWs propagating on free or metallized substrate. From Eqs. (1), (2), $D^{\pm I}$ can be evaluated as depending on the transducer voltage V to obtain:

$$\begin{gathered}(\mathcal{Z}-\mathcal{M})\mathcal{D} = (\mathbf{I}-\mathcal{M})\mathcal{P}V,\\ \mathcal{M} = \frac{1}{(1+t)^2-|s|^2}\left[\begin{array}{cc} t^2-1-|s|^2 & 2s \\ 2s^* & t^2-1-|s|^2 \end{array}\right],\\ \mathcal{D} = [D_{-I} \quad D_I]^T, \mathcal{P} = [p^* \quad p]^T\end{gathered} \tag{3}$$

(the superscript T means the matrix transposition). Neglecting V, the dispersive equation for SAWs results having the solution:

$$\begin{gathered}\delta = \pm\sqrt{(\varepsilon-\varepsilon_1)(\varepsilon-\varepsilon_2)},\ \epsilon = (k_v+k_o)/2-IK,\\ \varepsilon_i = (k_o-k_v)[1-2/(1-\lambda_i)]/(k_o+k_v),\end{gathered} \tag{4}$$

where $\lambda_{1,2}$ are the eigenvalues of $\mathcal{M}$ associated with eigenvectors $[\pm F_1; F_2]$. The sign of δ is chosen such that the forward component of the right-propagating SAW have the wave-number $IK+\delta$. The values of $\varepsilon_{1,2}$ determine the stopband where the SAW wave-number is complex due to the Bragg scattering in the considered periodic system.

IV. Transduction

The spectral theory [6] of SPUDTs can now be developed. The finite, L-long system generates SAWs of different amplitudes to the right, a_L^-, and to the left, a_R^+, depending on the transducer voltage V (with certain proportionality coefficient A depending on ω, p, δ, L and the piezoelectric coupling coefficient of the substrate):

$$\begin{gathered}a_L^- = VA[p^*(o+\delta+\lambda\chi) - p(o-\delta+\lambda/\chi)e^{-j\delta L}],\\ a_R^+ = VA[p(o+\delta+\lambda/\chi) - p^*(o-\delta+\lambda\chi)e^{-j\delta L}],\\ o = (k_o+k_v)(\varepsilon-\bar{\varepsilon})(1-\lambda_1)(1-\lambda_2)/(k_o-k_v),\\ \lambda = \lambda_2-\lambda_1,\ \bar{\varepsilon} = (\varepsilon_1+\varepsilon_2)/2),\end{gathered} \tag{5}$$

where $\chi = (F_1 p)/(F_2 p^*)$ is the directionality factor the best value of which is $\pm j$. The above relations allow one to evaluate the relative amplitudes of the left and the right generated SAWs with respect to $a = (|a_L^-|^2 + |a_R^+|^2)^{1/2}$, presenting the transducer directionality as a function of frequency deviation from the stopband center, proportional to o.

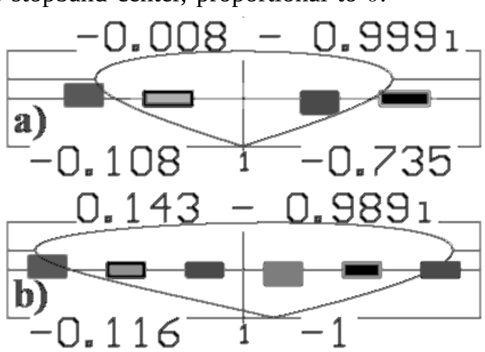

Fig. 1. Directionality of SPUDTs: thin curves represent the left and the right generated SAW amplitudes dependent on frequency (horizontal axis). a) Novel SPUDT cell having four strips with edges at Λ[-.55,4; 8.5,13.7; 23.45,28; 32.5,37.7]$/48$; the first and third strips are connected to the transducer bus-bars and two other are interconnected. b) Almost ideal characteristics of the frequently applied SPUDT [4]. Gray boxes represent cell strips: the raised and lowered ones are strips connected to the transducer bus-bars, the interconnected strips are marked by frames.

V. Discussion and Conclusions

The structure directionality is best seen for $L \to \infty$. This removes the second-order effect of the SAWs reflections from the transducer edges. Its dependence on frequency (represented by o) clearly shows whether the strip cell is correctly designed to obtain symmetric dependence of the SAWs relative amplitudes $\bar{a}_{L,R}^{\pm} = a_{L,R}^{\pm}/a$ on frequency. One of these amplitudes should take zero value at the stopband center and the other - its maximum unitary value there, as shown in Fig. 1a). The well known applied structure proposed in [4] exhibits slightly asymmetric frequency dependent directionality, Fig. 1b). To improve this, one may try different widths and positions of strips in the cell and choose one of many solutions, usually yielding different stopbands. This can be done with help of the simple numerical code [8] (the figures present the output graphics). Two curves, upper with maximum unitary value, and lower with minimum zero value, joining into the common horizontal line at the $1/\sqrt{2}$ level, present the relative the left and the right propagating SAW amplitudes (vertical axis). They are different only in the stopband; horizontal axis o represents frequency. The complex number at the top of the figures is the evaluated directionality factor χ (its best values are $\pm j$), while the two large numbers at bottom are $\lambda_{1,2}$ determining the stopband edges; the small center number is I - the number of overtone.

Investigations of several other applied SPUDT structures show that the displacement of the generation and reflection centers does not characterize the structure directionality satisfactorily. Some best structures with symmetric frequency characteristics have this displacements different from $\Lambda/4$, like the novel one presented in Fig. 1a), while the others with the $\Lambda/4$ displacements, may have very asymmetric characteristics.

For finite L, the Eqs. (5) yield only partial directionality of SPUDTs because $|\bar{a}_{L,R}^{\pm}|$ never reach the limit values: 0 or 1. The above equations can help one to choose proper L to obtain the required SPUDT's directionality at the stopband center. Note that the frequency characteristic of L-long transducer is the above evaluated directionality multiplied by the familiar factor $\sin(\delta L/2)/\delta$ [6].

References

[1] D.P. Morgan, "Quasi-static analysis of floating-electrode unidirectional SAW transducers (FEUDT's)," in *Proc. IEEE Int. Ultras. Symp.*, 1999, pp. 107–111.
[2] E. Danicki, "Spatial spectrum of electric charge on planar array of metal strips," in *Electr. Lett.*, vol. 31, pp. 2220–2221, 1995.
[3] E.J. Danicki, "Electrostatics of interdigital transducers," in *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, vol. 51, pp. 444–452, 2004.
[4] M. Takeuchi, H. Ogadawa, M. Tanaka, Y. Yamanouchi, "SAW transducer configurations for reversing the directionality of NSPUDT substrates," in *Proc. IEEE Int. Ultras. Symp.*, 1995, pp. 17–22.
[5] S. Jen, C.S. Hartmann, "Synthesis and performance of medium bandwidhs, low time-spurious, high out-of-band rejection SPUDT filter," in *Proc. IEEE Int. Ultras. Symp.*, 1993, pp. 9–13.
[6] E. Danicki, "Spectral theory of NSPUDT - Rayleigh wave case," in *Proc. IEEE Int. Ultras. Symp.*, 1991, pp. 145–148.
[7] E.J. Danicki, "An approximation to the planar harmonic Green's function at branch points in wave-number domain," in *J. Acoust. Soc. Am.*, vol. 104, pp. 651–663, 1998.
[8] http://www.ippt.gov.pl/∼edanicki/free.htm