Non-hertzian contact model in wheel/rail or vehicle/track system

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Abstract

In the paper the spring-mass system describing the moving load, determined with the Hertz theory, was replaced with the spring-mass system with an inertial part being in contact with the beam, rail, or a track. Computational problems can be reduced significantly. Results are qualitatively and quantitatively improved, especially at higher range of the speed, related to critical values.

Keywords: wheel/rail contact, train-track system, moving loads

1. Introduction

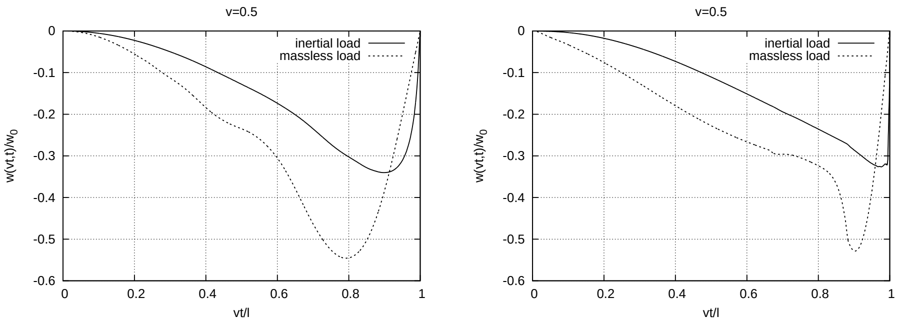
Numerical dynamic analysis of engineering problems nowadays successfully replace experimental or real scale investigations. Unfortunately there exist several problems that can not be easily treated by computer means. Such are problems with moving loads, especially with inertial moving loads. There are two main reasons of difficulties. The first one concerns problems with numerical description of inertial particles moving along finite elements. Despite the massless load can be applied to the system in an extremely simple way, the inertia attached to the moving force vector require modification of matrices in the system of differential equations describing the motion of the whole system. The second problem occurs when we compute frequencies of the response of the structure subjected to a moving load system with required accuracy. In engineering practice quantitative results start to be essential in numerical simulations. This is a fundamental reason why we push our research from cost experiments towards numerical analysis.

Preliminary calculations exhibit significant difference of results obtained with the use of a massless load acting to the track and with the use of the inertial moving load. The point mass increasing the inertia of the continuous track significantly changes the dynamic response. The fundamental question then is what part of a wheel or wheelset can be attached to the track to give more realistic response. More realistic means in our case more accurate amplitudes of displacements or accelerations, and more accurate frequencies of vibrations under the load moving at high speed, comparing with experimental measurements. We consider the speed in the near critical range, both under and over-critical. The contributing numerical observations must last a few seconds of the real time to enable the vehicle pass a few hundred meters. The dynamic analysis of a long-term response of the track to a moving vehicle or a train can be successfully carried on with the use of multi-scaled numerical computations. The analysis of the wheel-rail system allowed us to determine the partition of the wheel between the part moving along the rail, being in contact with the rail, and a part that subjects the rail through an elastic massless element, considered as a part of the spring-mass system that describes the vehicle.

In the paper we intend to demonstrate the dynamic analysis of vehicle–track system, with the influence of the inertia of the load. The monograph by Fryba [1] treats problems of structures only under moving massless loads. The theoretical analysis of the problem of an inertial load is presented in [2, 3, 4]. Numerical analysis could not be previously efficiently performed. Existing examples of a beam vibrations published in literature concern relatively low moving velocity and even in this case exhibit significant errors. At higher speed presented solutions differ significantly with accurate results. In the case of pure hiperbolic differential equations which describe a string or a bar vibrations, integrated numerically by the step-by-step schemes resulting solutions diverge. In a series of papers [5, 6, 7] we explain how to derive elemental matrices that carry a moving mass particle and apply them to the finite element method or space-time finite element method. In all cases displacements of the contact point in the static equilibrium state are equal, although dynamic responses differ.

The next important feature is related to the interesting property of the differential equation describing the Timoshenko beam or simpler case, the string. The detailed analysis of the solution exhibits discontinuity of the inertial particle trajectory in the neighbourhood of the rear support, in the case of simply supported span. The phenomenon was first analysed in the case of a string, mathematically proved and published in [2]. It is also observed in the case of of a Timoshenko beam or a thick plate. We can discuss whether this effect of the shock is noticably in reality. Practics gives the positive answer. Examples of the effect can be noticed in the case of electric cables of the train traction being in contact with a moving pantograph power receiver. As another example we can consider road plates. During the motion of vehicles significant force jumps are registred at final stage of the passage.

Let us compare trajectories of contact points under the load. Figure 1 depicts the comparison of mass and massless load moving on the Euler and Timoshenko beam. In this comparison dimensionless unitary data were assumed. Left-hand side plot exhibit the mass trajectory while the right-hand side plots depict deflection in the middle of the span. Results obtained for the case of both types of beams differ considerably. Other structures, i.e. strings, plates etc. exhibit similar differences in



a b

Figure 1: Comparison of the load trajectories on the (a) Euler and (b) Timoshenko beam in the case of inertial and massless load for the speed v=0.5.

the case on pure force and the force with a point mass as a load.

In the paper we will demonstrate, that the computational model should be assumed with attention. Detailed analysis of one phenomenon in micro-scale could be less valuable if the model of a whole structure is far from the real one. We will demonstrate that results obtained with different numerical tools differ significantly and, what is more, differ from the real registered signal. The classical track will be considered. However, the identical analysis can be performed for the ballast-less track and track with Y-type sleepers.

2. Analysis of the attached mass

Classical approach to the whel-rail contact analysis is based on the Hertz theory. The contact between the rail and the wheel is then nonlinear and is massless (Figure 2a). The contact stiffness between the wheel ring and the rail head depends on the type of the wheel and equals 500—580 MN/m. The wheel disc has the rigidity equal 500 MN/m in the case of tyred wheel and overpass 900 MN/m in the case of the monoblock wheel. The averaged stiffness of the entire wheel equals 250 MN/m for tyred wheel and 355 MN/m for monoblock wheel.

At the first stage we must establish the percentage of the wheel that influences the track motion. The aim is not easy. We should solve the inverse problem to determine parameters of the problem: attached mass, sprung mass, and spring stiffness. Moreover, the identification of parameters depends on the velocity of the motion and may be influenced by other vehicle and track parameters. We must emphasize here that the contact between the wheel and the rail is non-linear.

We solved two problems to determine unknown parameters m_L, m_U, and the stiffness k of the alternative simplified model of the wheel placed on the rail (Figure 2). In the first one we assume the velocity v=0. In this case displacements in time of the contact point A $u_A(t)$ and accelerations $\ddot{u}_A(t)$ are registered. In the simplified model excited with the same initial conditions appropriate quantities $\tilde{u}_A(t)$ and

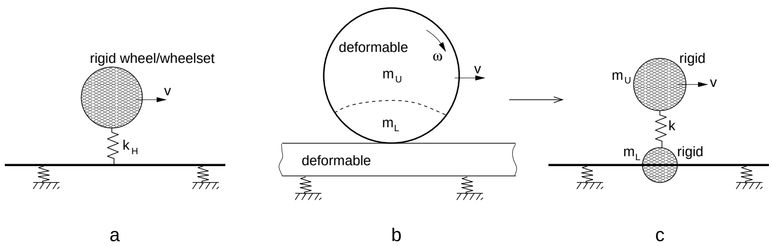


Figure 2: Replacement of a continuous system with a rigid-body system: a) classical Hertz contact model, b) and c) proposed approach.

$\tilde{\ddot{u}}_A(t)$ are measured in the same point, i.e. in the point of a beam at the center of the mass m_L. The objective function that estimates the quality of the selection of parameters is as follows:

$$I_1 = \int_0^{t_f} \alpha(t) \left[u_A(t) - \tilde{u}_A(t)\right]^2 \mathrm{d}t + \int_0^{t_f} \beta(t) \left[\ddot{u}_A(t) - \tilde{\ddot{u}}_A(t)\right]^2 \mathrm{d}t \tag{1}$$

$\alpha(t)$ and $\beta(t)$ are weight functions that determine validity of consideration of displacements and accelerations in time.

In the second problem we assume rolling of a wheel. Appropriate objective function is similar:

$$I_2 = \int_0^{t_f} \alpha(t) \left[u(vt,t) - \tilde{u}(vt,t)\right]^2 \mathrm{d}t + \int_0^{t_f} \beta(t) \left[\ddot{u}(vt,t) - \tilde{\ddot{u}}(vt,t)\right]^2 \mathrm{d}t \tag{2}$$

3. Numerical model

We assume the spring-beam system model of a vehicle. This is a simplified model, however it sufficiently represents the dynamic properties of the real vehicle. Proper stiffness, inertia and damping allows us to obtain dynamical response coinciding with a real response.

The track model can be composed of plates, beam, grid or frame elements and springs. Simple or complex track structure can be considered. Below we will consider the simplest classical track, built of grid elements placed on the elastic Winkler foundation, springs which model elastic pads and grid elements describing the geometry of rails, straight or curved.

4. Examples

Now let us have a look at a real example of vibrations of a carriage moving on a classical track. We use custom computer software implementing the numerical approach presented in this paper. We assume geometric and material data from [8]. In the Figure 4 in the case of a non-inertial load (lower diagrams) we can notice the

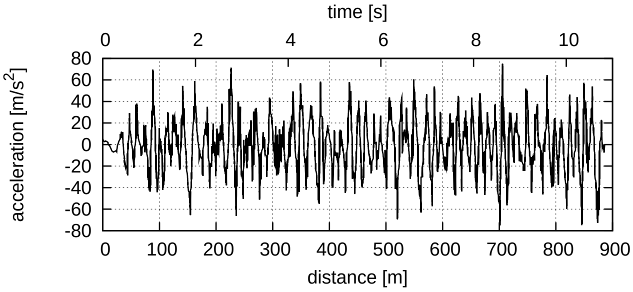


Figure 3: Acceleration of the axle box 290 km/h taken from [8].

strong influence of the sleepers. In the case of the inertial load (upper plots) this influence is moderate and the dynamic response is more realistic.

We can compare our results with the reference [8], Figure 3. Both the inertial load in Figure 4 and Figure 3 exhibit a similar range of accelerations of the axle box. The signal in Figure 3 shows a low frequency mode which is difficult to explain. The response of our simulation has the same level of accelerations and is characteristic of more realistic higher frequency oscillations.

5. Conclusions

In the paper we explained why the massless load should not be taken to computer simulations. Moreover, the rigid Hertz spring in computational practice is usually chosen arbitrary. Qualitative results can be obtained, but they differ quantitatively in all ranges of vehicle velocity. The inertial load in rigid body models is recommended.

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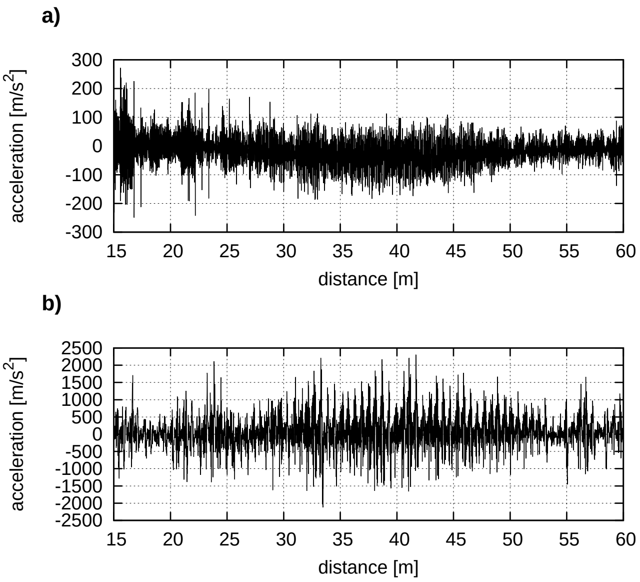

Figure 4: Vertical acceleration of the axle box at 290 km/h with inertial and non-inertial load assumed in the model with rigid ballast: a) inertial load, b) non-inertial load.

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Niehertzowski model kontaktu w układzie koło/szyna i pojazd/tor

W pracy zaproponowano zastąpienie układu sprężysto-masowego z parametrami teorii Hertza, realizującego ruchome obciążenie, układem sprężysto-masowym z częścią inercyjną będącą w kontakcie z belką, szyną lub torem. Zadania ulegają znacznemu uproszczeniu przy jednocześnie lepszych jakościowo i ilościowo wynikach. W szczególności dotyczy to przejazdu pojazdu szynowego z dużą prędkością.