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Dynamics of Railway Track

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Abstract

The classic and reinforced railway track is composed of two infinite rails separated from sleepers by visco-elastic pads. There are numerous assumptions leading to different simplifications in railway track modelling. The rails are modelled as infinite Timoshenko beams, sleepers by lumped masses or elastic bodies and ballast as a visco-elastic foundation.

Nowadays the interest of engineers is focused on the Y-shaped sleepers. The fundamental qualitative difference between the track with classic or Y sleepers is related to local longitudinal symmetric or antymetric features of railway track. The sleeper spacing influences the periodicity of elastic foundation coefficient, mass density (rotational inertia) and shear effective rigidity. The track with classical concrete sleepers is influenced much more by rotational inertia and shear deflections than the track with Y sleepers. The increase of elastic wave velocity in track with Y sleepers and more uniform load distribution will be proved by the analysis and simulations.

The analytical and numerical analysis allows us to evaluate the track properties in a range of moderate and high speed train. However, the correct approach is not simple, since the structure of the track interacts with wheels, wheelsets, boogies and vehicles, depending on the complexity of the analysis.

1. Introduction

Nowadays high speed trains and increasing load carrying capacity are the main reasons of damages of track and noise emission. The noise reduction must be done on both stages: elimination of sources and protection of humans. Each source of vibrations generates noise of different frequency spectrum. Each frequency has different intensity, penetrates the environment with various decay and finally effects human body with various power. Main sources of the noise are: periodicity of track (sleepers), dynamic coupling and interaction between wheelsets and vehicles moving along the track, friction (stick and sleep zones) between the rail and the wheel, breaks, creepage of wheels on curves.

Dynamic phenomena in railway transportation can be divided into two groups: (1) vibrations of the track and vehicle systems, (2) wave phenomena (wave

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propagation in the track, oscillation in the rail/wheel contact with friction, wave coupling of travelling multi point load). In the first group we consider dynamic properties of the track, i.e. rails, sleepers, elastic pads and the ballast, rigidity of wheelsets, vehicles and the train. The phenomena of the second group occur intensively in the case of high speed trains. The research in several centres is carried out intensively. However, in engineering practice the attention and understanding of wave phenomena is poor. Wave phenomena can not be neglected since they result in increased wear of wheels and rails, noise and even accidents. We must recall a group of scientific publications which deal with this problem (Knothe, 1983; Bogacz and Ryczek, 2003; Bajer, 1998; Bogacz and Kowalska, 2001).

The noise frequency strongly depends of the track type: classical sleepers, Y-type sleepers, systems with continuous or periodic supports of rails.

Periodicity of the track

Measurements and numerical simulations proved lower noise emission of the track with modified sleepers. The Y-type track with steel sleepers is especially efficient and allows reducing considerably the noise emission. Up to now there are several hundred kilometres of experimental tracks in the world (majority in Germany and Poland). However, the proper selection of parameters for simulation strongly depends on the local track properties (sort of ballast, foundation etc.).

Dynamic coupling

The set of inertial loads moving along the track also generates vibration of acoustic level. In a given case there exist ranges of parameters (for example speed or track rigidity) for which waves are transmitted with increasing intensity, aside of bands which exhibit decay properties.

2. Influence of sleeper features on dynamics of railway track

The conventional and reinforced railway track is composed of two infinite rails mounted to sleepers by means of elastic pads. There are various assumptions leading to the different simplification in railway track modelling. The two-dimensional periodic model of the track consists of two parallel infinite Timoshenko beams (rails) coupled by means of visco-elastic foundation (or equally spaced sleepers). The qualitative difference in track modelling with classic or *Y sleepers* concerns the local longitudinal symmetric or antymetric features of railway track. The dynamical analysis of both tracks models as periodic structures can be based on Floquet's theorem. The Timoshenko beam model placed on an elastic or visco-elastic foundation can also be used to describe the vertical or lateral track motion. In such a case sleeper spacing influences the periodicity of elastic foundation coefficient, mass density (rotational inertia) and shear effective rigidity. The track with classic concrete sleepers is influenced stronger by rotational inertia and shear deflections than the track with *Y sleepers*. The increase of elastic wave velocity in track with *Y sleepers* and more uniform load distribution will be proved in analysis and simulations.

Let us consider the Timoshenko beam motion described by the following set of partial differential equations:

$$\begin{aligned} &\frac{\partial}{\partial x}\left(K\left(\frac{\partial w}{\partial x} - f\right)\right) - mA\frac{\partial^2 w}{\partial x^2} - cw = p(x,t) \\ &EI\frac{\partial^2 f}{\partial x^2} + K\left(\frac{\partial w}{\partial x} - f\right) - mI\frac{\partial^2 f}{\partial t^2} = 0 \end{aligned} \tag{1}$$

where*:* EI – flexural rigidity, K – shear coefficient, G – shear modulus of elasticity, A – cross-sectional area with moment of inertia I, w – displacement, f – angle of the beam rotation, c – coefficient of elastic foundation and m – mass density.

The set of equations (1) is equivalent to the one of the following dimensionalless (fourth order) equation:

$$\begin{aligned} &\frac{\partial^4 w}{\partial x^4} - 4(a+b)\frac{\partial^4 w}{\partial x^2 \partial t^2} + 16ab\frac{\partial^4 w}{\partial t^4} - 8da\frac{\partial^3 w}{\partial x^2 \partial t} + 32abd\frac{\partial^3 w}{\partial t^3} - 4a\frac{\partial^2 w}{\partial x^2} + \\ &+ (16ab+4)\frac{\partial^2 w}{\partial t^2} + 8d\frac{\partial w}{\partial t} + 4w = 0 \end{aligned} \tag{2}$$

In the simple case of the technical equation of beam motion (Bernoulli-Euler beam), we have:

$$EI\frac{\partial^4 w}{\partial x^4} - mA\frac{\partial^2 w}{\partial t^2} + cw = cq \tag{3}$$

The classic track shown in Fig. 1 (left) is defined by following parameters: $E = 2.1 \cdot 10^{11}$ N/m^2, $I = 3.052 \cdot 10^{-5}$m^4, $m = 60.31$ kg/m, $c = 2.6 \cdot 10^8$ N/m, $l = 0.6$ m, $b = 6.3 \cdot 10^4$ N·s/m, sleeper mass $M = 145$ kg, visco-elastic foundation $C = 1.8 \cdot 10^8$ N/m and $B = 8.2 \cdot 10^4$ N·s/m.

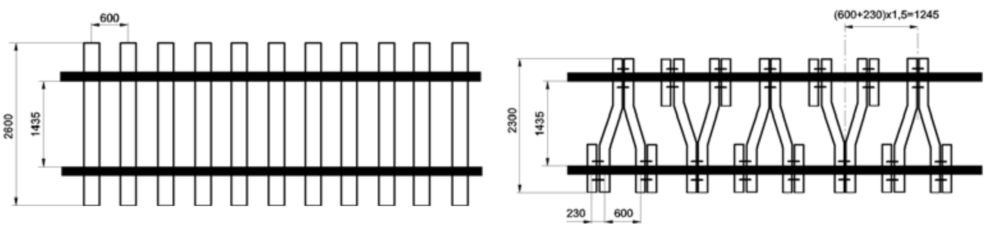

Fig. 1. Classic track (left) and reinforced track with Y sleepers (right)

The equation of sleeper motion for the case of symmetric (in phase) rails vibration is as follows:

$$M\ddot{q} + B\dot{q} + Cq = 2b(\dot{w} - \dot{q}) + 2c(w - q) \tag{4}$$

In the case of antimetric rails vibration we have the following equation of sleepers motion

$$J\ddot{p} + B_0 \dot{p} + C_0 p = bl(\dot{w} - \dot{p}) + cl(w - p) \tag{5}$$

Looking for the solution in the following form of travelling waves

$$w = W_0 \exp ik(x - vt), \qquad q = Q_0 \exp ik(x - vt) \tag{6}$$

the elastic wave speed dependent on the wave number *k* is given by the formula:

$$v = f(m, M, EI, c, C, k) \tag{7}$$

In the case of rail motion described by the Timoshenko beam parameters fulfilling inequality (8) the minimum of the elastic wave speed in rails is smaller then shear wave speed ($V_{cr}<V_G$). The dependence of the elastic wave velocities *v(k)* in such a case is shown in Fig. 2 (curve V_f''). This dependence in an alterative case when $F(V_G, V_E)>0$, is shown in Fig. 2 (curve V_f').

$$F(V_G, V_E) = E - KG(KGA^2 + Ic) < 0 \tag{8}$$

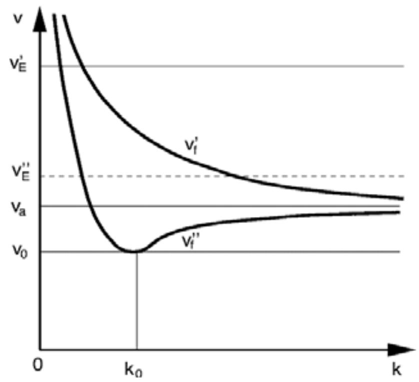

Fig. 2. Elastic waves velocities *v* versus wave number *k* for two different values of the Young modulus (E_1 and E_2)

The critical speed and elastic wave velocities in the track for two cases of sleepers modelling can be obtained by using of graphical or numerical methods. In the case of *in phase* vibrating rails, described by the equations (2) and (4) and in the case of *out of phase* motion (eqs. (2) and (5)) the displacement-pressure ratio f_1 and f_2 (pressure between sleepers and rails) versus the wave velocities for an elastic case is shown in Fig. 3. It is visible that the point of resonance in the case of *in phase* motion is obtained at grater velocity than in the case *out of phase* motion.

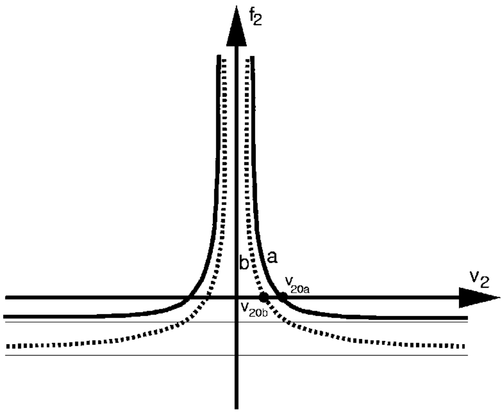

Fig. 3. Wave velocities in conventional track for the *case a* – when both rails are vibrating *in phase* and *case b* – when the rails are vibrating *out of phase*

The crossing points P_1, P_2, P_3 determine the wave velocities in the track.

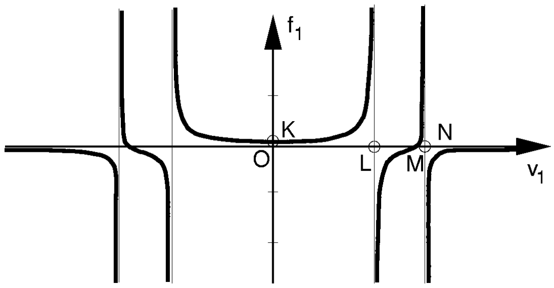

Fig. 4. Pressure – displacement ratio for the case of sleepers' motion *in phase*

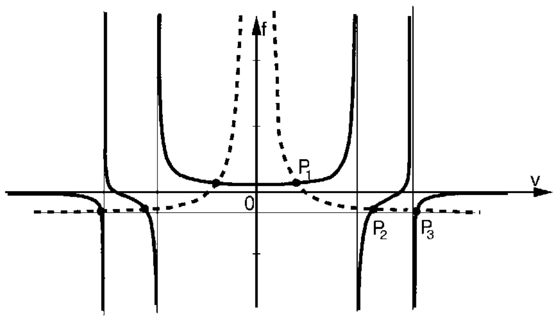

Fig. 5. Wave velocities in conventional track for the case when both rails vibrate *out of phase* and rails – sleepers vibrate *in phase*

In the case shown in Fig. 5 the minimum speed of elastic waves in the track v is determined by point P_1. The value of the speed is smaller as half of share wave velocity V_G. The determination of the velocities of elastic waves (P_1, P_2, P_3) in the track make it possible to estimate maximal speed of the train. The response of the track subjected to moving and oscillating wheelset motion is possible in analytical way using Floquet's technique similar as in ref.: Bogacz et al. (1995) or Bogacz (1995). The numerical analysis of the track response will be presented in the next part of the study by using space-time element method.

3. Numerical modelling

Numerical track model was composed of grid and bar finite elements (Fig. 6). Both rails and sleepers were modelled as a grid separated by visco-elastic pads assumed as bar elements. The Winkler type foundation was modelled by visco-elastic springs. The total length of the track was 20 m. Both ends were fixed. Significant damping allowed us to reduce the influence of boundary conditions. The vehicle was built as a mass and spring 3-dimensional system combined with frame elements. The distance between wheelsets was equal to 250 cm. We must emphasize that the coupling of displacements in right and left contact points was performed both by wheelsets and the vehicle frame and was stronger than in the case of the coupling between leading and hind wheelsets.

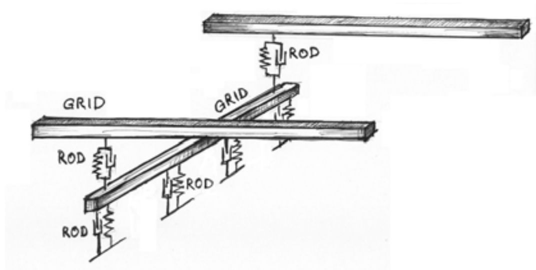

Fig. 6. Model of the track used in computational analysis

The complete system motion can be described by in a matrix form

$$\begin{bmatrix} \mathbf{A}_t & 0 & \mathbf{A}_{tc} \\ & \mathbf{A}_v & \mathbf{A}_{vc} \\ \text{(sym)} & & \mathbf{A}_c \end{bmatrix} \begin{bmatrix} \mathbf{v}_{ti} \\ \mathbf{v}_{vi} \\ \mathbf{v}_{ci} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_t & 0 & \mathbf{B}_{tc} \\ & \mathbf{B}_v & \mathbf{B}_{vc} \\ \text{(sym)} & & \mathbf{B}_c \end{bmatrix} \begin{bmatrix} \mathbf{v}_{t,i+1} \\ \mathbf{v}_{v,i+1} \\ \mathbf{v}_{c,i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{ti} \\ \mathbf{F}_{vi} \\ \mathbf{F}_{ci} \end{bmatrix} + \begin{bmatrix} \mathbf{s}_{ti} \\ \mathbf{s}_{vi} \\ \mathbf{s}_{ci} \end{bmatrix} \tag{9}$$

$\mathbf{A}_t$, $\mathbf{B}_t$ are related to the track system and $\mathbf{A}_v$, $\mathbf{B}_v$ to the vehicle. $\mathbf{A}_c$ corresponds to degrees of freedom common for track and vehicle. $\mathbf{A}_{tc}$, $\mathbf{B}_{tc}$, $\mathbf{A}_{vc}$, $\mathbf{B}_{vc}$ are matrices composed of coefficients topologically connected to common track-vehicle degrees of freedom. In our case the coupling of subsystems v and t is solved iteratively while each subsystem is solved directly. The step-by-step scheme is the following

$$\mathbf{A}\mathbf{v}_n+\mathbf{B}\mathbf{v}_{n+1}+\mathbf{s}_n=\mathbf{F}_n \tag{10}$$

A, **B** are the square matrices, which in a particular case can be obtained by multiplying the classic finite element rigidity matrix by coefficients proportional to the procedure parameter α, **v** is the velocity vector, **F** is the vector of external forces and **s** – the vector of nodal potential forces, computed at the end of the preceding time step.

The numerical dissipation is performed by modifying the formula for displacements

$$\mathbf{x}_{n+1}=\mathbf{x}_n+h[(1-\beta)\mathbf{v}_n+\beta\mathbf{v}_{n+1}], \quad \beta = 1-\alpha/(1+\gamma) \tag{11}$$

The important advantage of the method (10) is that it can be directly employed both to dynamic and quasistatic analysis. For $\alpha = 1$ the same procedure can be used even if the mass density is equal to zero. In such a case the kinematic boundary conditions should force the motion. In the case of positive mass density the unconditional stability is obtained for $\alpha \geq \sqrt{2}/2$.

The vehicle and the track represented by systems of algebraic equations were solved independently by a direct method. The coupling was ensured iteratively. In practice 3–6 iterations per time step provided sufficient precision. We can say that in spite of simplicity of the approach the results obtained were highly satisfactory.

Two examples demonstrate the difference between both tracks. In the first case the perfect wheel is rolling along the track. The vehicle was subjected to gravity forces. The initial stage of rolling into the rails was a sufficient excitation of the system. The response of the wheelset/track system depends on the velocity. In higher range we can notice significant influence of sleepers spacing (Fig. 7).

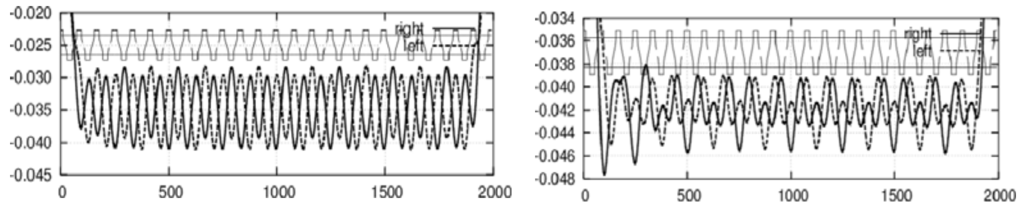

Fig. 7. Vertical displacements of contact points in the case of the speed 30 m/s (left) and 50 m/s (right)

In the second test the contact point was additionally subjected to eccentric wheels load. Such a case usually occurs in practice. It can be considered as a periodic load which acts to a wheelset together with the periodicity of the track structure at the speed 40 m/s. The simulation proved significantly lower vibration level of the track with Y-type sleepers than in the classic case of the track (Fig. 8).

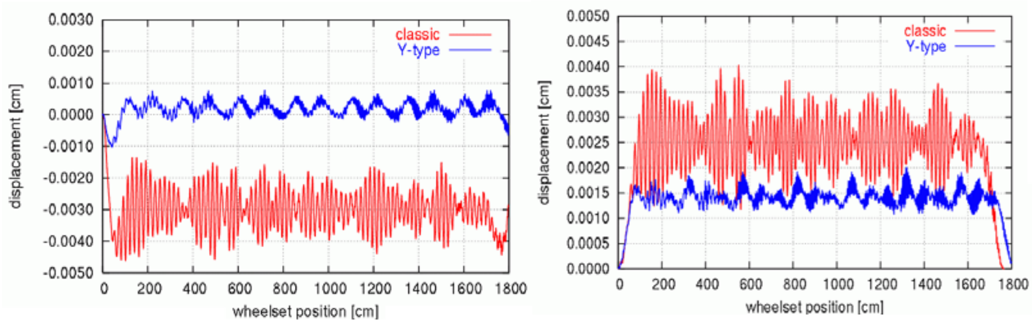

Fig. 8. Vertical displacements registered 120 cm (left) and 180 cm (right) in front of the contact points of the buggy for classic and *Y-type* track at the speed 40 m/s

Vibration measured in a specified distance in front of the wheelset is important in the case of coupling of interactions of successive vehicles travelling over straight or waved rail. The comparison of both tracks in the whole period of simulation is depicted in Fig. 9. We can see the waved time-space surface in the case of the classic track. The interesting phenomenon of waves travelling towards the source (Bogacz et al., 2002) can be observed. The *Y*-type track exhibits considerably lower level of amplitudes.

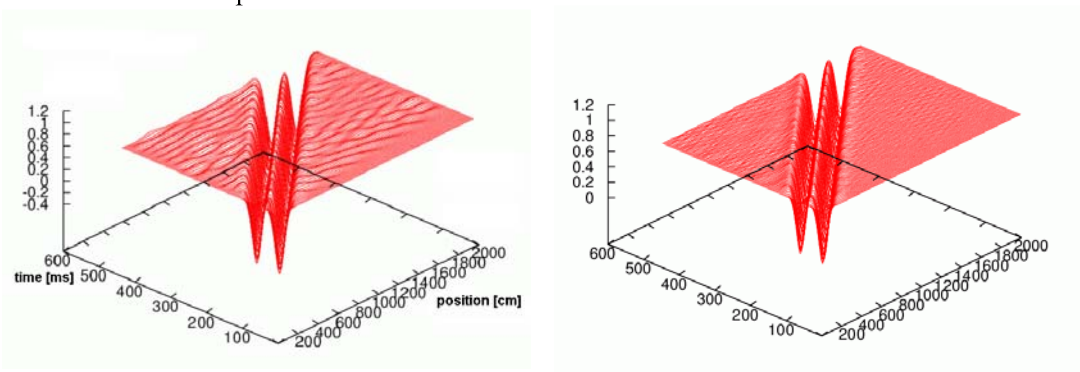

Fig. 9. Vertical displacements of a classic track (left) and *Y type* track (right) in time-space domain subjected by buggy moving with the speed 40 m/s

4. Conclusions

Analytical investigations give the qualitative relations between the track vibrations, especially bending and shear waves, and the speed of the travelling inertial coupled load. The existence of resonance regions was described in several papers devoted to the dynamics of the beam under moving load (eg. Bogacz et al., 1995). Quantitative analysis in practice can be performed numerically. However, the coincidence of obtained results with real measurements requires precise values of the track parameters. Especially, the system is sensible to the elasticity of the elastic/visco-elastic pad. The rigidity of springs in the vehicle has minor importance. The important question is the modelling of the wheel/rail contact. The analysis with the wheel assumed as a continuous 2 or 3-dimensional disk discretized by finite elements is relatively simple. In the case of vehicle made as a spring-mass system we can not determine exactly what amount of the wheel mass should attache the rail in vertical motion. However, numerical analysis proved analytical calculations.

Advantages of Y-type sleepers are significant for practical use. They are characteristic of lower amplitude level and lower acoustic emission. The wear (for example corrugations) should decrease since the contact force does not oscillate as strongly as in the case of the classic track.

References

Bajer, C., 1998, The space-time approach to rail/wheel contact and corrugations problem. *Comp. Ass. Mech. Eng. Sci.*, **5**, 267–283.

Bogacz, R., 1995, Residual stresses in high-speed wheel/rail system; Shakedown and corrugations, In A.N. Kounadis, Ed. *Proc. of 1st European Conference on Steel Structures EUROSTEEL '95*, Athens, A.A. Balkema, 331–343.

Bogacz, R., Bajer, C., 2002, *On modelling of contact problems in railway engineering. In: Recent Advances in Applied Mechanics*, J.T. Katsikadelis, D.E. Beskos and E.E. Gdoutos, Eds., Nat. Techn. Univ. of Athens, Greece, 77–86.

Bogacz, R., Dżuła, S., 1998, Dynamics and stability of a Wheelset/Track Interaction Modelled as Nonlinear Continuous System, *Machine Dynamics Problems*, **20**, 23–24.

Bogacz, R., Kocjan, M., Kurnik, W., 2002, Dynamics of wheel-tyre subjected to moving oscillating force, *Machine Dynamics Problems*.

Bogacz, R., Kowalska, Z., 2001, Computer simulation of the interaction between a wheel and a corrugated rail, *Eur. J. Mech. A/Solids*, **20**, 673–684.

Bogacz, R., Krzyżyński, T., Popp, K., 1995, Application of Floquet's theorem to high-speed train/track dynamics, *Advance automotive technologies, ASME Congress*, 55–61.

Bogacz, R., Ryczek, B., 2003, On active damping of stick-slip self-excited vibration for history dependent dry friction model, *Proc., Int. Conf. on Modelling and Sim. of the Fric. Phenom. in the Phys. and Techn. Syst.– 'Friction 2002'*, Warsaw, 54–61.

Knothe, K., 1983, *Rail Corrugations*, ILR Bericht 56, Berlin.

Kowalska, Z., Bogacz, R., 2001, *Computer simulation of the dynamic wheel/rail system, in Simulation in Research and Development*, R. Bogacz, Z. Kołodziński and Z. Strzyżakowski, Eds., Warsaw.