Semi-active stabilization of smart structures subjected to impact excitation

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Abstract

In the work, a novel control method to stabilize vibrations of high structures is presented. The control is realized by changes of the stiffness parameters of the structural couplers. A seismic pulse excitation applied to the structure is submitted as a kinematic excitation. For such a representation the designed control law provides the best rate of the energy dissipation. Performance in different structural settings is studied by means of the stability analysis. Then, the efficiency of the proposed strategy is examined via numerical simulations. In terms of the assumed energy metric, the controlled structure outperforms its passively damped equivalent by over 50 percent.

Keywords: structural control, semi-active control, smart materials, smart buildings, stabilization

1. Introduction

Elimination of vibration is essential in the period of rapid technological development. Although theoretical basis is extensively elaborated, final solutions are not sufficiently implemented. Structures and technical devices are more complex nowadays and are subjected to extreme loads. It occurs in crash engineering, structures exposed to explosions, damages from seismic or paraseismic vibrations, etc. Classical methods that enhance the load carrying capacity do not increase sufficiently the resistance to incidental extreme loads. Active or semi-active techniques exhibit further possibilities in development. In the work, we design and examine a semi-active control method to reduce the vibration of high structures, i.e., buildings, chimneys, masts etc., subjected to short term excitations or abrupt load. Seismic pulse excitation applied to a bending structure will be submitted as a kinematic excitation. The proposed approach can also be addressed to structures subjected to the danger of damage of vital elements, for example, a guy rope in a mast.

Reduction of amplitudes of vibration of slender structures were considered in numerous publications. Examples of applications in automotive and aerospace industry, civil engineering structures, etc. are countless. Modern buildings are getting higher, and structures become more slender. Increasing height of skyscrapers is accompanied by increased flexibility and a lack of sufficient internal damping of vibrations. A new system can provide the most effective way of controlling structural response to environmental loads such as earthquake excitations [1, 2]. When the structural damping is insufficient, supplementary damping devices can be introduced. The auxiliary systems accomplish different strategies of energy dissipation. Indirect energy dissipation is the most popular reduction method of wind-induced vibrations but it can also be successfully applied to seismic technology. These methods utilize inertial effects and generally are called Tuned Mass Dampers (TMDs) [3, 4]. In many cases, insufficient space in buildings precludes the use of traditional TMD. Therefore, alternatives devices such as pendulums, inverted pendulums, slide-platform and rubber or hydrostatic bearings can be used. Review of the various auxiliary damping devices can be found in [5, 6].

In the work, we propose an efficient method for semi-active stabilization of transverse motion of beams induced by a kinematic impulse motion of the support. The stabilization is done by changes of the stiffness parameters of the structure's couplers. We derive the strategy of the control of switched elastic connectors that mistune the motion of the structure and remove the peak energy in time intervals. The efficiency is examined by numerical experiments.

2. Mathematical model

Our goal is to reduce the physical model of high buildings to a relatively simple structure. Two coupled cantilever beams that describe high structures, i.e., masts, towers or high buildings were assumed. The transverse motion, perpendicular to the length direction, is composed of bending and shearing state. Especially skyscrapers constructed as cores surrounded by the walls of the enclosure exhibit significant shear. We opted for the Timoshenko model of beams since it allows to contribute high shear stiffness.

For the sake of the stability analysis and control law design we will introduce a simplified equivalent of the considered system. Each of the beams will be represented as a simple oscillator (see Fig. 1). The parameters of the oscillators are computed such to mimic the dynamics of the first natural modes of the beams.

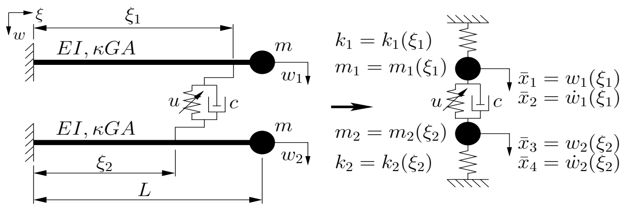

Figure 1: Cantilever beams system reduced into two coupled simple oscillators

3. Control law design

Introducing the state vector $\bar{\mathbf{x}} = [\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4]^T$ the dynamics of a simplified system is governed by

$$\dot{\bar{\mathbf{x}}} = \bar{\mathbf{A}}\,\bar{\mathbf{x}} + u\,\bar{\mathbf{B}}\,\bar{\mathbf{x}}\,. \tag{1}$$

For such a representation we consider the energy function given by $V = \frac{1}{2}\bar{\mathbf{x}}^T\,\mathbf{Q}\,\bar{\mathbf{x}}$, where $\mathbf{Q} = \mathrm{diag}(k_1, m_1, k_2, m_2)$. Its time derivative is of the form $\dot{V} = \bar{\mathbf{x}}^T\,\mathbf{Q}\,\bar{\mathbf{A}}\,\bar{\mathbf{x}} + u\,\bar{\mathbf{x}}^T\,\mathbf{Q}\,\bar{\mathbf{B}}\,\bar{\mathbf{x}}$. Substitution of the system matrices yields

$$\dot{V} = -c\,(\bar{x}_2 - \bar{x}_4)^2 - u\,(\bar{x}_1 - \bar{x}_3)\,(\bar{x}_2 - \bar{x}_4)\,. \tag{2}$$

We use the control u^* providing the best instantaneous energy dissipation, namely

$$\forall t\ :\quad u^*(t) = \mathrm{argmin}_{u\in[u_{min},u_{max}]}\,\dot{V}(\bar{\mathbf{x}}(t))\,. \tag{3}$$

From (2) we conclude that

$$u^*(t) = \begin{cases} u_{max} & \text{if } (\bar{x}_1(t) - \bar{x}_3(t))\,(\bar{x}_2(t) - \bar{x}_4(t)) > 0\,,\\ u_{min} & \text{otherwise}\,. \end{cases} \tag{4}$$

Analysing the structure of $\dot{V}$, we observe that the control (4) guarantees a permanent decrease of the energy, except for the time instants when $\bar{x}_2 = \bar{x}_4$. The asymptotic stability is assured only if the states remains desynchronized until the origin is reached. We can also observe that larger desynchronization provides better rate of decrease of the energy. All these facts motivate the analysis on the dynamics of the synchronization. Our goal is to answer two key questions: How the control given by (4) does impact on the system synchronization? How can we protect against the synchronization to keep a desired performance? The synchronization is studied by means of the analysis on the dynamics of the relative state

$$\dot{\epsilon} = \mathbf{A}_\epsilon\,\epsilon + u\,\mathbf{B}_\epsilon\,\epsilon + F(\bar{\mathbf{x}})\,. \tag{5}$$

Here $\epsilon_1 = \bar{x}_1 - \bar{x}_3$ and $\epsilon_2 = \bar{x}_2 - \bar{x}_4$. We will examine the energy function corresponding to the relative dynamics $V_\epsilon = \frac{1}{2}\epsilon^T\,\mathbf{Q}_\epsilon\,\epsilon$, where $\mathbf{Q}_\epsilon = \mathrm{diag}(k_1 + k_2, m_1 + m_2)$. It can be shown that

$$\begin{aligned}\dot{V}_\epsilon =& (k_2 - k_1)(\bar{x}_1 + \bar{x}_3)\epsilon_2 + \\ &+ \left[k_1\left(\frac{m_2}{m_1} - 1\right)\bar{x}_1 + k_2\left(\frac{m_1}{m_2} - 1\right)\bar{x}_3\right]\epsilon_2 + \\ &- c\left(\frac{1}{m_1} + \frac{1}{m_1}\right)(m_1 + m_2)\epsilon_2^2 + \\ &- u\left(\frac{1}{m_1} + \frac{1}{m_1}\right)(m_1 + m_2)\epsilon_1\epsilon_2\,. \end{aligned} \tag{6}$$

From (6) we observe that the controller tends to synchronize the oscillators with the maximum rate of convergence. This is undesired effect, since, as we stated before, the best rates of decrease of the energy V are provided when the states are desynchronized. The key in resolving this antagonistic issue lies within the structure of terms in the first and the second lines in (6). By taking $k_1 \neq k_2$ the first line term oscillates between positive and negative values. If, for instance, we assume $k_2 > k_1$ and the initial condition such that $(\bar{x}_1(0)+\bar{x}_3(0))(\bar{x}_2(0)-\bar{x}_4(0)) > 0$, then under $k_2 - k_1$ large enough, we have $\dot{V}_\epsilon > 0$ for some time period. In that period the system energy V decreases very quickly. The latter change of the sign of $(\bar{x}_1(0) + \bar{x}_3(0))(\bar{x}_2(0) - \bar{x}_4(0))$ does not result in significant lose of the performance. Analogous conclusion can be stated for the second line term when $m_1 \neq m_2$.

4. Numerical validation

Simulations are carried on under the following setting. Two couplers are symmetrically crossed (the crossing provides slower synchronization) and located between the beams at the ending section. We consider four variants: variant 1–both couplers are not controlled (referred as the passive case), variant 2 (variant 3)–only one of the couplers is under control, variant 4–both couplers are controlled. Both beams are given with identical parameters and are initially excited with identical velocity. Displacement of the beam ending point and the metric corresponding to the potential energy are compared in Fig. 2 and Fig. 3, respectively. Each of the controlled variants clearly outperforms the passive case (see also Tab. 1).

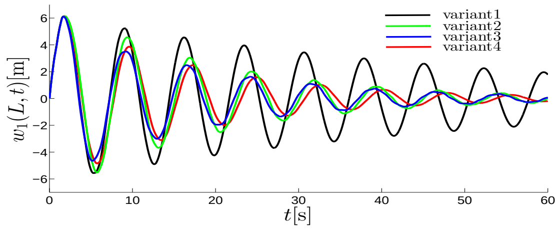

Figure 2: Displacement of the ending point of the beam 1

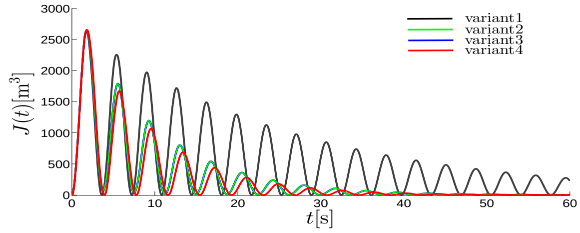

Figure 3: Evolution of the metric corresponding to the potential energy

Table 1: Comparison of the total energy metric (values are normalized to the variant 1)

variant 1	variant 2	variant 3	variant 4
1.000	0.466	0.466	0.430

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