6WCSCM
Sixth World Conference on Structural Control and Monitoring

Barcelona, Spain, 15 - 17 July 2014

J. Rodellar, A. Güemes and F. Pozo (Eds.)

Smart control in vibrations of structures

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Abstract

A semi-active control of structural vibrations is presented as an efficient method of damping. We consider structures subjected to a load applied in stationary points or to a moving load. We claim that the periodically switched magnetorheological actuators, controlled dampers, or elastic envelopes filled with granular materials subjected to controlled underpressure, give a more efficient vibration reduction than a permanently activated ones. In our work we show the efficiency of such a control strategy applied to a beams under moving inertial load, cantilevers and rotating shaft. The mathematical analysis allowed us to propose the particular control strategy. The finite element simulation, together with the solution for the control problem, proved that the damping devices should act only for a short period of each cycle of vibration. The control function depends on the type of the structure, excitation and the type of vibrations. The efficiency of the concept was proved in the experimental tests. The considered structures exhibit the reduction of amplitudes at the range 10–40% in the periodically controlled case, in comparison to the constant damping.

I. INTRODUCTION

Nowadays adaptive techniques can improve several operational parameters of the vibrating structures. First, we can decrease the amplitudes of displacements or accelerations in the sensitive points of the system. This can increase the life time of the structural elements, decrease the fatigue and noise and prevent form transmitting the vibrations from the source to environment. As an example we can give the advanced anti-seismic technologies. Second, by taking advantage of the adaptive technologies and real time control we can increase the load carrying capacity of the structures, for example the bridge spans. The load carried by bridges grows and the permissible speed of vehicles increases as well. The strengthening of the bridge structures can be performed dynamically with the additional supports controlled semi-actively. Third, stiffening of the guideways can improve the precision in robotics and mechatronics.

The vibration attenuation can be performed in a several ways. The simplest one is the passive damping treatment. Unfortunately, this method is usually not sufficiently effective, as we lack the possibility of adjusting the system parameters to varoius excitations. The second possibility is the active way. The used actuators require constant energy supply and can apply forces in a prescribed manner. The fundamental disadvantage is the admission of the control out of the phase, which can lead to damaging the structure. The energy consumption also plays an important role. The third approach is the semi-active control, that combines advantages of both of the previous types of the control. It is performed by the controlled dampers or frictional elements, that temporarily change the dynamic properties of the structures and allow the phase trajectory to be shifted towards the curve of a lower energy on the phase plane. The magnetorheological or electrorheological fluids can be successfully used in the control devices. The advantage of the semi-active approach is evident when we consider the energy consumed by the damping devices. The semi-active control of the structural vibrations is considered as the best and most perspective method of damping.

The literature background on the issues related to the topic concerned in this paper is very rich. We will focus on the chosen technical problem, which is the increase of the intensity of the damping in vibrating dynamic systems. The influence of the parametric control on vibrations is well known from academic textbooks. The control strategy is also elaborated in the case of the fundamental systems. Unfortunately, the real structures rarely correspond to their theoretical or numerical models, especially control devices like actuators, dampers, valves, etc. do not act perfectly as their models tend to suggest. Experimental verification always confirms limited efficiency of each of the approach. The direct control theory and the theory of the inverse problems has stimulated the development of health monitoring of structures: identification of the dynamic load [1] and simultaneous identification of the load and any damage caused to a structure [2].

The problem of reducing beam vibrations using active control methods is also widely considered in [3]. The analysis in the frequency domain allowed the authors to reduce the extreme amplitudes. The actively controlled string system was considered in [4]. A good example of the control of vibrations under a load is described in [5], which presented a method for computing the response induced by a load traveling over a 1D elastic continuum supported by a set of semi-active viscous dampers. The adaptive open loop control strategy was proposed. The damping functions were taken to be piecewise constant. The control strategy was suboptimal, but it outperformed the passive case. The numerical results were presented for the cases of the string and the Bernoulli–Euler beam. In [6] two elastic beams were coupled by a set of the controlled

*This work was supported by the National Centre for Research and Development (NCBiR)

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dampers. The relative velocity of the spans provided an opportunity for the efficient control by the means of the adaptive suspension. As a result, bang-bang type of control was taken into account. The controlled system outperformed the passive solutions over a wide range of considered conditions. Other interesting results concerning the structural control are presented in [7], [8], [9].

In the paper we consider three types of structures subjected to a various load applied in stationary points or a moving load. We claim that the periodically switched magnetorheological actuators, controlled dampers, or granular materials compacted with underpressure, result in a more efficient vibration reduction than a permanently acting ones. In the work we show the efficiency of such a control applied to the rotating shaft, beams under moving massless and inertial load, and cantilever beams. The mathematical analysis allowed us to propose the particular control strategy. The finite element simulation together with the solution of the control problem proved that the dampers should act only in a short period of the highest displacements of the structure. The same was proven in the experimental tests. We exhibited the amplitude reduction at the range 10-40% of the displacements over those with a constant control with maximum power, depending on the type of the formulated mechanical problem.

The presented extensive research is addressed to both, researchers and practicing engineers.

II. Unconventional Materials in Damping of Vibrations

There are numerous techniques that are more efficient in vibration attenuation than the simple, passive material damping. Depending on the rate of dissipation we can enhance the efficiency of the passive methods with the parametric modification of the governing differential equations. In such a way various physical quantities like velocity or acceleration can be decreased significantly [10], [11]. The composite sandwich sheets with introduced layers of smart materials like piezo-actuators, magneto- and electrorheological fluids or other controllable elastomers, are the most popular semi-active, modern solutions for the attenuation of the unwanted structural vibration. We also investigate the possibility to use the controlled granular materials as a light weight, low cost, interesting alternative for the classical damping solutions dedicated for the layered structures.

A. Magnetorheological fluids and MR dampers

Since the discovery of the magnetorheological (MR) effect by Rabinow in 1948 [12], [13], these materials have been developed into a family with MR fluids, foams, greases, gels and elastomers. Generally, the MR materials are ferromagnetic, micrometer-sized carbonyl iron particles suspended in the carrier medium. The most common of this group are the MR fluids, with ferrous particles suspended in a silicone oil. When exposed to a magnetic field, the particles polarize along the magnetic flux lines in a chain-like structures, and the fluid changes its state from the free-flowing to a semi-solid which is called the active one. The yield stress and apparent viscosity of the fluid increases as a consequence of the particles rearrangement. The transition between the states is a fully reversible process and takes only several milliseconds to complete. The possibility of controlling the yield strength of the MR fluids by the means of the magnetic field, predestines them to be utilized in devices like dashpots, rotary brakes, clutches, bearings, stabilizers, polishing systems and other. The mechanical construction of the MR device and the properties of the fluid itself result in a controllable dynamic response of the system. For example magnetorheological fluid dampers have a semi-controllable damping force output that is dependent on the current input to the damper, as well as the relative velocity. They have been been successfully used in the automotive industry in an intelligent car suspensions, for the stabilization of high buildings vulnerable to seismic activity or semi-active control of bridge structures. Despite the fact of growing number of practical applications, producing high-quality MR fluid with desired characteristic is still hard to achieve. Different types of commercially available fluids share similar limitations like susceptibility to settling and wearing of the ferrous iron particles. Moreover, high content of magnetic particles comprised in carrier fluid is an inevitable condition for enhanced magnetorheological performance leading to the constitution of high-weight system. This stimulates to search for modified types of fluids with improved properties and different then classical properties. The idea of a magneto-rheological fluid damper for rotor applications was presented for example in [14], [15], [16], [17].

B. Layered structures with magnetorheological elastomers

The experimental work presenting new production methods of smart elastomers [18] is also carried out. In the literature, there are attempts of application of new materials in engineering problems. One of them is the development and the simulation evaluation of the isolator for seat vibration, controlled in order to improve the comfort and safety of the passengers [19]. The use of the layered structures with the controllable materials is a natural way to meet the requirements imposed by the innovative industries. Series of theoretical papers on the smart sandwich structures deal with the studies on parametric instability regions, natural frequencies, and the loss factors for different values of the electric or magnetic fields [20], [21], [22]. In [23] the possibility of using electrorheological materials for the control of layered structures was demonstrated. A series of papers related to the finite element method has also been published. They concern the determination of the dynamic characteristics of particular sandwich structures. The influence of a magnetic field on the shear modulus [24],

[25], the natural frequencies and the loss factor [26], [27], [28] has been examined. Also experimental work related to the assessment of the dynamic parameters of layered structures using electrorheological and magnetorheological materials with varying electric or magnetic field [29], [30], [31], [32] has been carried out.

The dynamics of the layered structures has been the subject of the study for many years. The pioneering work [33] refers to the transverse vibration of an infinitely long beam with a damping layer. In [34], [35], the longitudinal free vibrations of a finite three-layer beam with a viscoelastic core were examined. The transversal oscillations of a sandwich beam of finite length, excited by an external force, was considered in [36]. The vast majority of following studies has made use of the mathematical basis provided by the solutions presented above, identifying the loss factor or the stability of the systems [37]. In [38] a non-uniform shear stress variation across the thickness of each layer was assumed. An analytical model that takes into account the compressional vibration of the layered beam was treated in [39]. The attempts to describe the sandwich beam with the simple models were made in [40]. In order to take into account the large amplitude vibrations of sandwich structures, the nonlinear modeling has also been carried out [41], [42].

The aim of the explored concept is to combine two components in a single entity: a layered structure with a core of variable dynamic properties and the advantages the adaptive control. Optimal control in the dynamics of a structure is quite common. Most of the existing solutions are based on the linear quadratic regulation (LQR), but these solutions are not satisfactory. The absence of the adequate theoretical solutions has resulted in a small number of attempts to use the controlled magnetorheological elastomers (MRE) in real technical applications. However, in the field of the dynamics of the the structure, the control of the system parameters over time is often confused with a one-time selection.

C. *Granular materials*

Generally, granular damping methods has been widely studied in the literature over the years. Most of the literature-present solutions lack the possibility of adjusting the damping parameters of the system, as the granular materials are utilized in a fully passive manner. In granular materials the common method for reducing the vibration is based on the dissipative nature of the particle collisions and is a derivative of a single-mass impact damper. It is a relatively simple concept where the particles of a small size are placed in a container that is attached to the structure. The movement of the loose grains inside the enclosure causes the dissipation of the part of the energy through the non-conservative collisions among the grains and the container. This mechanism is widely applied in the particle impact dampers [43], [44] or bean-bag dampers [45], [46] and was further adopted for beams by placing the container at the tip of the oscillating cantilever [47], [48], [49] or for the beams under the centrifugal loads [50].

Instead of placing the granules inside the artificially attached container, the bulk material can be filling the specially prepared cavities inside the beam. In [51] authors described the structural vibration damping capabilities of the loose, lightweight particles, filling longitudinal and transverse canals in the plates. In [52] authors investigated the vibration damping of the beams with elastomeric beads tightly packed in the core, while in [53] the damping behavior of laminated honeycomb cantilevers with fine solder balls placed in the cells was studied. The attenuation was achieved by the exchange of momentum through the repeated collisions between the balls and the face sheets.

The problem addressed in this article deals with damping of beam vibrations by means of the granular medium, although it is notably different from the solutions in the publications mentioned above, and uses different principles. In our case, the granules are no longer loosely packed in the container, since their movement is restricted by the hermetic, elastic envelope. When the encapsulated grains are subjected to the underpressure, the contact forces among the grains begin to increase due to the compression of the material. The granular material, tightly surrounded by the envelope, transits from a fluid-like to a solid-like phase, known as the jammed-state. The particle interactions in the jammed state can be weaken or intensified, depending on the level of compression which is adjusted by the underpressure. Due to the the shear deformation of the granular member, the beam exhibits damping behaviour.

III. MATHEMATICAL AND NUMERICAL MODELS

First we assume a simple mechanical model of the physical problem that contains the actuator or damper controlled with an unknown function. In more general cases the model can contain the series of dampers or a uniformly distributed medium with modified nonlinear, constitutive properties. The semi-analytical solution of the governing differential equations is obtained at the first stage. Since it can not be fully tested mathematically, we limit the solution to one or two terms of a series. It allows us to determine the control of the damper that results in the minimum of the control function. The searched control function must not be smooth. For practical reason it is reduced to the stepping function or the on/off type of function. It allows us to determine the theoretical efficiency of the control strategy. Then the numerical model is elaborated with the discrete methods. We consider the realistic structure. The optimization process with the simulation results results in the definitive control function that is verified experimentally.

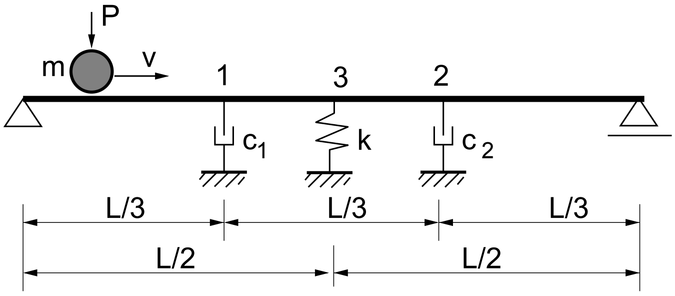


Fig. 1. The scheme of the test stand with three measured points.

A. Beam under a moving inertial load

Let us consider a simply supported Euler beam of the length L under a concentrated mass m, accompanied by a point force P traveling at a variable velocity $\mathsf{v}(t)$. The moving load is accelerated to a fix velocity, than travels through a part of the beam and then brakes before the end support. Examined problem was shown in Figure 1. Vertical displacement of the beam is denoted by w. Coordinates x_1 and x_2 describe the positions of the viscous supports. At $x = x_3$ we place the spring that decrease the static displacement of the flexible beam. Differential equation of motion can be written in the following form

$$\begin{aligned} EI\frac{\partial^4 w(x,t)}{\partial\, x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial\, t^2} + \delta(x-x_3)\, k w(x,t) + \sum_{i=1}^{2}\delta(x-x_i)\, c_i(t)\frac{\partial w(x,t)}{\partial\, t} = \\ = \delta(x-f(t))P - \delta(x-f(t))m\frac{\mathrm{d}^2 w(f(t),t)}{\mathrm{d}t^2}\ . \end{aligned} \tag{1}$$

We assume zero displacements and velocities as the initial conditions. At both ends the displacements are equal to zero. In the Eqn. (1) E, I, ρA, k, $c(t)$ are the Young modulus, inertia moment of the cross section, linear mass density, stiffness and damping coefficients of the supports, respectively. The viscous term enables the control in time. The acceleration of the moving mass describes the Renaudot formula

$$\frac{\mathrm{d}^2 w(f(t),t)}{\mathrm{d}t^2} = \left[\frac{\partial^2 w(x,t)}{\partial t^2} + 2\mathsf{v}\frac{\partial^2 w(x,t)}{\partial x \partial t} + \mathsf{v}^2\frac{\partial^2 w(x,t)}{\partial x^2} + \dot{\mathsf{v}}\frac{\partial w(x,t)}{\partial x}\right]_{x=f(t)}\ , \tag{2}$$

where

$$f(t) = x_0 + \mathsf{v}t + \frac{1}{2}\dot{\mathsf{v}}t^2\ , \tag{3}$$

describes the position of the load. Due to the moving inertial term the analytical solution of Equation (1) is unknown. The sine Fourier transform that naturally fulfills boundary conditions leads to the coupled system of ordinary differential equations of the 2nd order with respect to time. The solution of this system requires numerical integration.

The equation (1) is discretized with the finite element method. Stationary terms can easily be written in a discrete matrix form. According to the Renaudot formula (2), all three matrices of the motion equation, i.e. mass, damping and stiffness, must be modified by contributing mass matrices to a single finite element in every time step:

$$(\mathbf{M}+\mathbf{M}_m)\ddot{\mathbf{w}}^{i+1} \;+\; (\mathbf{C}+\mathbf{C}_m)\dot{\mathbf{w}}^{i+1} \;+\; (\mathbf{K}+\mathbf{K}_m)\mathbf{w}^{i+1} = \mathbf{F}^{i+1} + \mathbf{e}_m^i\ . \tag{4}$$

i and $i+1$ denote two successive whiles. $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrix, respectively. $\mathbf{F}$ is the external load vector and $\mathbf{e}$ is the vector of nodal forces contributed by a concentrated mass, while $\mathbf{w}$ is the vector of displacements. In one node we have both the vertical displacements and rotations of nodes. Index m indicates matrices that contribute the influence of the moving mass. The term contributing the moving mass requires more complex analysis. We can address the reader to [54]. The discretization of the term contributing the influence of the moving mass in the case of a variable velocity gives three matrices, which, after multiplication by accelerations, velocities and displacements contribute inertia, Coriolis, and centrifugal forces, respectively.

The virtual energy of the moving mass placed on a finite element of the length $b = \Delta x$ is written by multiplying the force by the virtual displacement $w^*(x)$

$$\Pi_m = \int_0^b w^*(x)\delta(x-f(t))\, m\, \frac{\mathrm{d}^2 w(f(t),t)}{\mathrm{d}t^2}\mathrm{d}x\ . \tag{5}$$

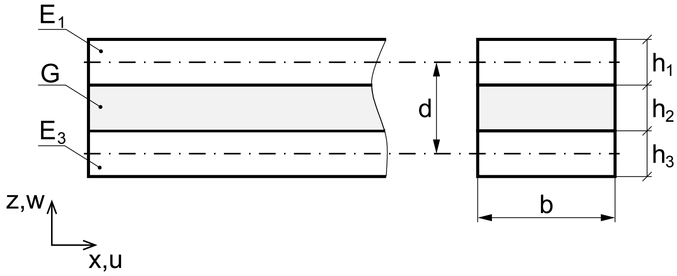


Fig. 2. Three-layered sandwich beam.

The discrete model of vibrations of a beam subjected to moving force has a form of a sequence of static problems. Each one is described by a system of algebraic equations. A moving point mass is contributed by adding the matrices $\mathbf{M}_m$, $\mathbf{C}_m$, and $\mathbf{K}_m$, to respective degrees of freedom in the global matrix of coefficients, and the vector $\mathbf{e}_m$, to the right hand side vector. We notice that the influence of the moving mass is proportional to the velocity v. Moreover, the acceleration $\dot{\mathsf{v}}$ influences the matrix $\mathbf{K}_m$. All the above matrices have simple forms since they are based on linear shape functions. They are valid for both the Bernoulli-Euler beam and the Timoshenko beam. In our analysis the thin beam will be modeled as the first one. More detailed description of the moving mass problem is given in [55], [56].

B. Vibrating cantilever beam

Theoretical analysis can not be performed for arbitrary structures with equal simplicity. For an analytical solution we choose the simply supported beam as one of the most representative structures. The governing set of differential equations for the vibrating sandwich beam was derived in [36]. The necessary assumptions and simplifications of the analytical model are briefly described below. Let us consider a three-layered sandwich beam. Its cross-sectional geometry has a characteristic width b and the thicknesses of each layer is h_1, h_2, and h_3 (Figure 2). Longitudinal displacements u in the x direction and transverse displacements w in the z direction of the beam were taken into account. The face-plates are purely elastic, with Young modulus E_1 and E_3, respectively. The core is linearly viscoelastic and defined by shear modulus G. The mathematical model is obtained under some physically simplifying assumptions. The shear strains in the outer layers and the stresses in the longitudinal direction in the core were neglected. Moreover, the transversal strains in each layer were neglected as well, so the displacements w of the entire cross-section of the beam are constant.

In order to determine the relationship between the longitudinal displacements u_1 and u_3, and the relationship between their derivatives with respect to x, the condition of zero resultant axial force in the whole section was used. Finally, we obtain

$$\frac{\partial^4 w}{\partial x^4} - gY\frac{\partial^2 w}{\partial x^2} + g\frac{db}{D_t}E_3h_3\frac{\partial u_3}{\partial x} = \frac{p}{D_t}\,, \tag{6}$$

$$\frac{\partial^2 u_3}{\partial x^2} - \frac{g}{b}u_3 = -gY\frac{D_t}{E_3h_3b^2d}\frac{\partial w}{\partial x}\,, \tag{7}$$

where

$$g = \frac{G}{h_2}\left(\frac{1}{E_1h_1} + \frac{1}{E_3h_3}\right)\,, \tag{8}$$

$$Y = \frac{d^2b}{D_t}\frac{E_1h_1E_3h_3}{E_1h_1 + E_3h_3}\,. \tag{9}$$

We assume initial deflection and zero initial velocity. The shear modulus G is controlled. A more detailed discussion of the presented analytical model can be found in [36]. This mathematical formulation will be used for the simply supported three-layered beam with a controllable core.

C. Rotating structures

We consider the hyperbolic differential equation (10) which describes the motion of the rotating shaft (Figure 3). The second equation (11) describes the motion of the damper. They are coupled at the point B:

$$-GI\frac{\partial^2\varphi}{\partial x^2} + \rho I\frac{\partial^2\varphi}{\partial t^2} + \delta(x-x_B)c\left(\frac{\partial\varphi}{\partial t} - \frac{\partial\vartheta}{\partial t}\right) = \delta(x-x_A)\,f(t)\,, \tag{10}$$

$$I_d\frac{\mathrm{d}^2\vartheta}{\mathrm{d}t^2} + c\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}t} - \frac{\mathrm{d}\varphi}{\mathrm{d}t}\right) = 0\,. \tag{11}$$

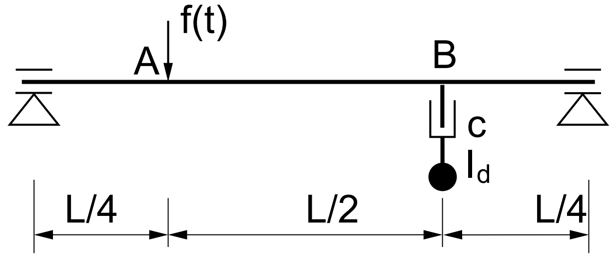


Fig. 3. Scheme of the problem for theoretical analysis.

We assume the following boundary conditions:

$$\varphi'(0,t) = 0, \quad \varphi'(L,t) = 0 \ . \tag{12}$$

Here, $\varphi(x,t)$ is the angular displacement in time t of the point x of the shaft, ϑ is the angular displacement of the rotating disk of the damper with inertia I_d, and c is the damping coefficient. Two Dirac delta functions select arguments for the point where the force is applied and the point for the damper. $f(t)$ is an arbitrary external load function. Further we assume it as a harmonic function $F\sin(\omega t)$.

The closed solution of the above equation is complicated. We take two ways to approximate the solution: an analytical solution with a one term expansion of the Fourier series, and a semi-analytical solution with an n-term expansion. The first solution allows us to examine features of the solution and test its sensitivity to the parameters. The second one allows of a quantitative investigation.

Now we will solve the above system of differential equations with the angular displacements φ as the unknown functions. We can apply the cosine Fourier transformation

$$\Phi_j(t) = \int_0^L \varphi(x,t)\cos\frac{j\pi x}{L}\mathrm{d}x\,, \tag{13}$$

where

$$\varphi(x,t) = \frac{1}{L}\Phi_0(t) + \frac{2}{L}\sum_{j=1}^{\infty}\Phi_j(t)\cos\frac{j\pi x}{L}\,, \tag{14}$$

which fulfills the boundary conditions. For the reason of further analytical complexity we limit the solution to the first term of the series (14). Finally, according to (14), the displacement including only the constant term can be written

$$\varphi(t) = \frac{1}{L}\Phi_0(t)\,. \tag{15}$$

The solution is periodic.

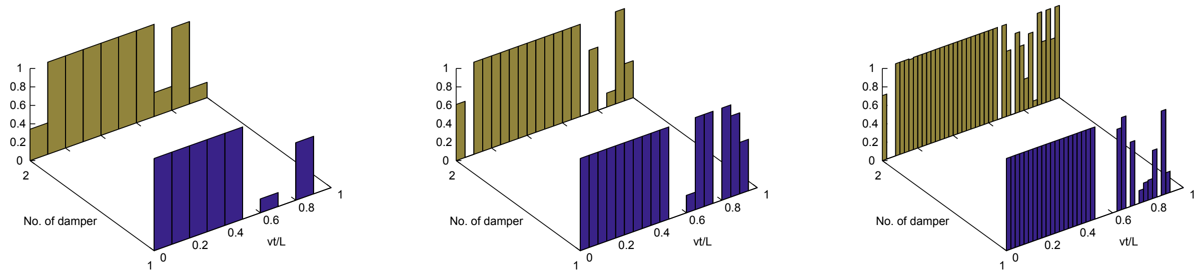
IV. CONTROL STRATEGY

In this section we will focus our investigations on the control functions that minimize objective function J efficiently. The control of the system can be performed in several ways to achieve prescribed goal. From the engineering point of view, the following control functions can be essential:

- limited displacement of selected points of the structure,
- limited stress in chosen points of a structure,
- low accelerations in selected points or under the load.

The basic parameters of the system, i.e. the bending stiffness of the beam and the damping coefficient range, can be chosen during the design stage as constant values. They are usually chosen at the design and optimization stage. They allow us to provide the required load carrying capacity under the dynamic load. However, the damping coefficients that can vary, allows to increase the performance of the system.

Let us first consider a vibrating beam in $\Omega = \{x : 0 \le x \le L\}$, with boundary conditions in $\partial\Omega = \{0, L\}$, $w(x) = 0$, $w(x)'' = 0$, subjected to a gravity load $P(x,t) = \delta(x - \mathsf{v}t)\,mg$, with the concentrated inertia of the moving mass $\delta(x-\mathsf{v}t)\,m\ddot{w}(x,t)_{|x=\mathsf{v}t}$. The beam is supported with the damping material $c(x,t)$. The state of such a system is constituted by vertical displacements $w(x,u,t)$, and the control input $u(x,t)$ as damping coefficients of dampers. The objective of the control is to distribute the damping of each damper over time to achieve the desired vibrations, here denoted by $w_d(x,t)$.



a b c

Fig. 4. Control functions in the case of partition into a) 10, b) 20, and c) 40 time intervals — minimizing displacements at the second measure point.

We assume a finite time horizon T. The optimization problem can be written in the following form.

$$\text{Minimize} \quad J = \frac{1}{2}\int_0^T \int_\Omega \left[w(x,u,t) - w_d(x,t)\right]^2 \, \mathrm{d}x \, \mathrm{d}t + \\ + \frac{\alpha}{2}\int_0^T \int_\Omega \left[u(x,t)\right]^2 \, \mathrm{d}x \, \mathrm{d}t \,, \tag{16}$$

subject to the constraints

$$\mathcal{D}w = \frac{\mathrm{d}^2 w(\xi(t),t)}{\mathrm{d}t^2} + P(x,t) \ , \tag{17}$$

$$w(x,t)_{|t=0} = w_0(x) \quad \text{on} \quad \Omega \,, \tag{18}$$

$$\frac{\partial w(x,t)}{\partial t}_{|t=0} = \dot{w}_0(x) \quad \text{on} \quad \Omega \,,$$

$$w^{(k)}(x,t) = w_k(x), \;\; k = 1,2,...,$$

$$u \in U \,.$$

$\mathcal{D}$ is the spatial differential operator, (k), k=1,2,... are derivatives of the respective orders, $\xi(t)$ is the position of the load, and U is the control space. For (17) in the case of a beam subjected to a travelling inertial load we use (1). For the layered cantilever beam we use (6–7) and for rotating shaft problem — (10–11). The solution of the above problem minimizes displacements of a linear system of differential equations in a quadratic form. The problem is a linear quadratic hyperbolic control problem with distributed control. However, we must emphasize the equation (1) has variable coefficients. The treatment of these type of problems is difficult, due to the weak smoothing property of the associated solutions.

The numerical algorithms developed for optimization problems with partial differential equations are designed for convex problems where the objective function is for example quadratic. In such cases problems have unique solutions. Moreover, quadratic functions enable us to derive simple formulas for gradients by introducing the adjoint state. In our case the objective function has local minima and the global search is required. Since gradient tools are ineffective and the number of design variables is low, we can successfully use random methods. We will intentionally apply low number of of time intervals n, since the controlled damping devices in real applications can not be switched instantaneously and certain delay in action is typical. We will also consider higher n and compare time trajectories of considered design variables, i.e. damping.

The minimization process gives us the pair of control functions as depicted in Figure 4. The comparison of the displacements of the 2nd measured point (Figure 1) in terms of the damping coefficients is presented in Figure 5. We notice that higher damping enables more efficient reduction of amplitudes. Unfortunately, MR fluid disables high jump of dissipative properties. We must assume the range of damping 100–500 Ns/m. We can state here that the higher number of the dampers improves the efficiency of the damping strategy more than in the case of two dampers.

The same strategy of the control of the displacements in time was considered for the cantilever beam connected at its tip with the magnetorheological elastomer. Only the 1st mode of vibration was excited. The rectangular on/off control was performed, switching the magnetic flux density between 0 and 700 mT in the moments as presented in Figure 6. Small number of decision variables results in sufficiently accurate normalized control function (Figure 6a). Increasing precision improves shapes of slopes in the diagram. All the values practically vary between extreme values, i.e. zero and one. Our

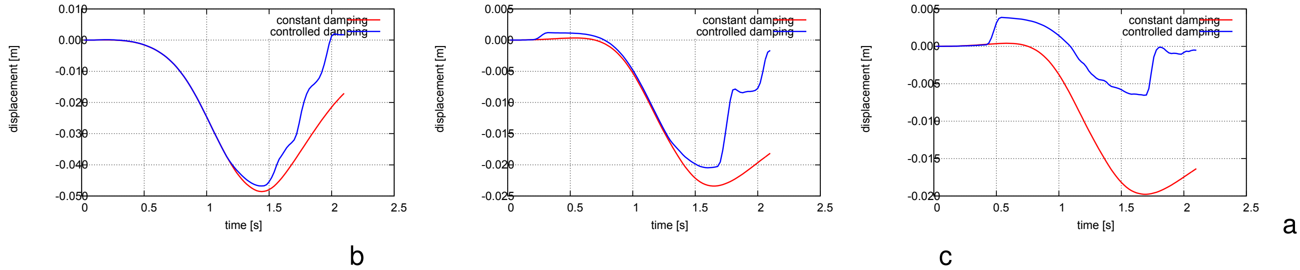


Fig. 5. Displacements in time of the point 2 for different damping coefficients: a) c=500 Ns/m, b) c=2000 Ns/m, c) c=5000 Ns/m.

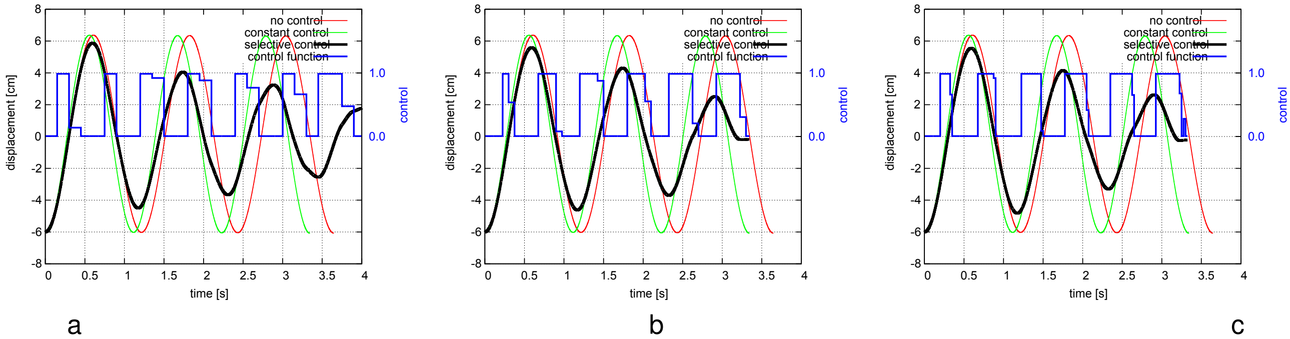


Fig. 6. The control function in the cantilever beam damped with MR elastomer, computed with partition of the time horizon into a) 40, b) 80, and c) 240 time intervals.

control requires activation at the time of the extreme displacement and switching off in the static equilibrium state, i.e. after 1/4 of the vibration period. The action of activating the magnetorheological elastomer is carried on during half of the total time. For comparison, diagrams in Figure 6 depict vibrations without the control and with the permanent control. It is obvious that the response of the structure vibrating with constant-in-time low or high shear stiffness of the inner layer and excited with the same initial deflection differs only in period of vibrations.

Let us consider a rotating shaft. Successive time refinements into 10, 20, and 40 subdomains reduces the objective function to 439, 401, and 377, respectively (Figure 7). The damping function takes on an almost harmonic shape with double the frequency of the displacement solution.

The preliminary analytical results were compared with the numerical analysis. The finite element model was used. The shaft was divided into 60 segments. The external load was applied to the node along 1/4 of its length and the damper was fixed at 3/4 of the length. Furthermore, the right-hand end was elastically supported with a relatively small spring k_φ=20 Nm. We use the bang-bang control of the damping. The action of the damper is demonstrated in Figure 8. The significantly high amplitude of the shear strain for the undamped case decreases when the damper is activated constantly. The controlled switching of the damping in time reduces the deformations significantly from 20% up to 50%. The applied control coincides with that obtained with the analytical model (Figure 7).

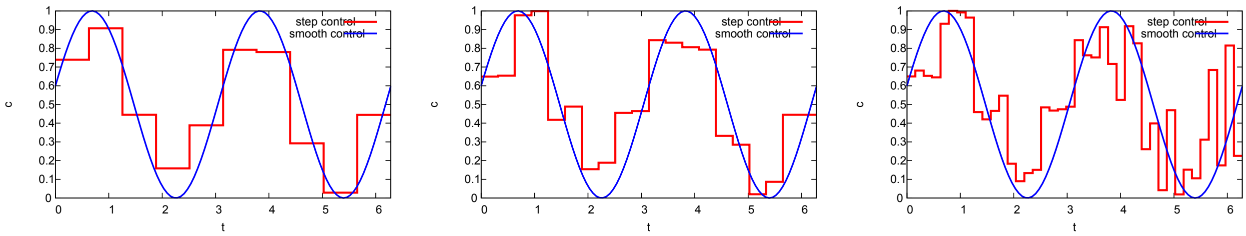


Fig. 7. The control of the damper in refined subdivisions of the period of vibrations.

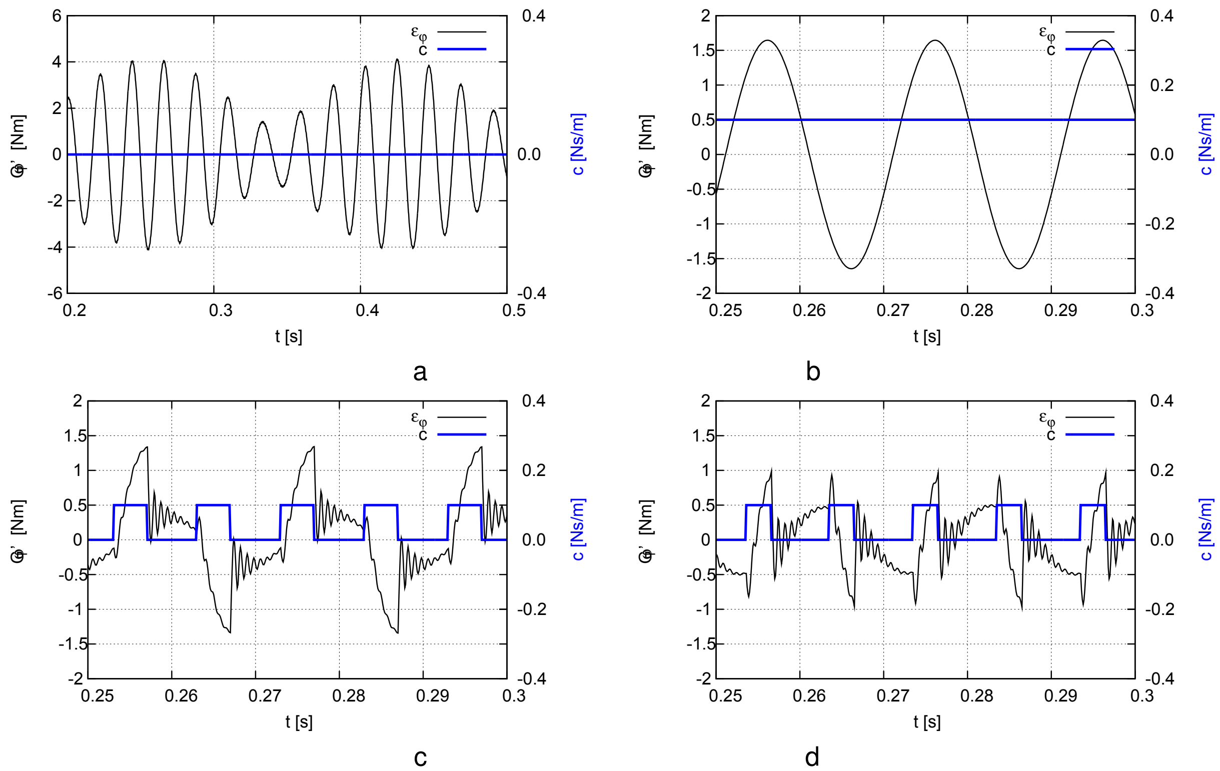


Fig. 8. Torque in the case of: a – no damping, b – continual damping, c, d – selective damping.

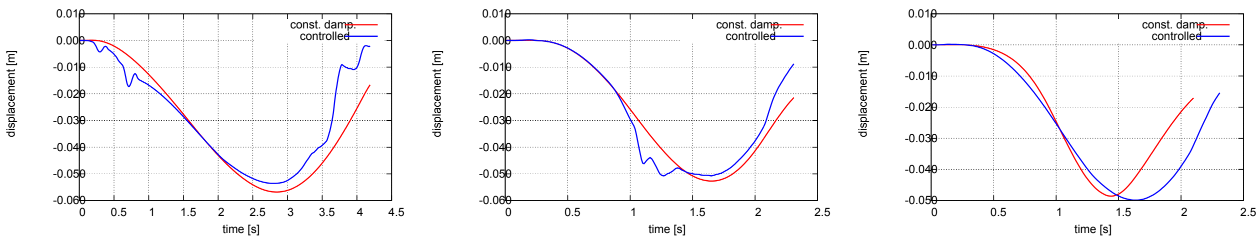


Fig. 9. Comparison of displacements of point 2 in time, for velocity v=1, 2, and 3 m/s.

V. RESULTS

The unwanted vibrations of structures can be efficiently reduced with the controlled damping devices. The controlled action significantly attenuates vibrations when compared to the constant activation of the damping. The mathematical analysis performed with the simplified models proved that the optimum effect is ensured if the damping function has a special form, different for each problem. The simply supported beam supplied with dampers, carrying the moving inertial load requires special control, with null intervals at the beginning and in the final period of the process (Figure 4). The cantilever sandwich beam treated with damper placed at a tip requires double frequency rectangular control function (Figure 6). In the case of the rotating shaft it is a double frequency harmonic function (Figure 7). Numerical simulations proved that such a stepped control function shape results in a reduction by about 20%-40% of the displacements over those with a constant control with maximum power, depending on the type of the defined mechanical problem.

A. Vibrations of a beam under a moving load

Let us compare the efficiency of the simulated controlled damping in the case of various velocities v. Figure 9 depicts displacements of the point No. 2 under the load moving at the velocity v=1 m/s, acceleration a=7 m/s^2 and v=3 m/s, a=4 m/s^2. We notice that at the lower velocity the efficiency of displacement reduction is of the same range as in the case of higher speed. The optimization procedure locates our solutions in local minima that have almost the same value of the objective function.

TABLE I

TECHNICAL PARAMETERS OF THE TEST STAND.

maximum torque of the driving motor	21 Nm
velocity range of the moving mass	0–4 m/s (with m=5 kg)
max. acceleration and deceleration of the moving mass	7 m/s^2
range value of the moving mass (min–max)	0.7–10 kg
guideway length	4.05 m
damping (min-max)	100–650 Ns/m
stiffness of one supporting spring	1000 N/m

Fig. 10. View of the test stand.

The test stand was designed to verify the developed control strategy. Construction of the test stand consists of four main parts: the executive, the drive, the supporting structure and the control–measurement equipment. The executive part is the moving load achieved by a trolley traveling along the guideway supported on the two, equally spaced, controlled magnetorheological, rotary dampers and the line springs. Moreover the guideway is simply supported. Carriage is accelerated to the setpoint speed at which it passes the greater part of the beam, and then brakes in a suitable time before reaching the final support. The dampers have a small rotational inertia, which does not significantly affect the outcome of the experiment. The system is driven by a stepper motor with the drive belt and the gearwheels that propel the trolley. The supporting structure of the device is sufficiently rigid multi-sections aluminum truss frame. Measurement and control system consists of the motor control driver, the MR damper control application and the data archiving software which records acceleration and the displacement. Detailed data regarding the test stand parameters are given in Table I.

In the middle of its length, the test rig beam is supported by the spring of the stiffness 1 N/mm attached to the truss frame. These springs enables returning the beam to the initial position after passing the moving mass. The system of a single beam is supported by the set of rotary magnetorheological dampers (brakes) mounted on the truss. The MR dampers are connected with the control unit by a current amplifier. The amplifier generates the current control signal, which alters the magnetic filed and thus changes the damping capacity of the MR device. The moments of switching the dampers on and off is controlled by the computer control algorithm in the LabView environment. The LabView environment has been used to communicate with the measurement equipment and the control actuators.

Accordingly slender guideway is consistent with the Euler beam theory and provides a relatively low speed of wave propagating in the beam. The modular construction of the supporting structure allows the extend the trolley route at ease. Figure 10 performs the assembled test stand.

First we will verify the accuracy of the measurements and compare them with the numerical simulations. The damping coefficient tends to be the the most difficult parameter to be determined. Magnetorheological fluid is not exactly the viscous fluid. Its viscosity is perturbed by the ferrous particles that change the velocity–force relation and contribute friction. Preliminary tests of our MR dampers exhibit hysteresis that depends on the current supplied. Figure 11 depicts the velocity-force relation for the velocity values ± 100 mm/s. For no current applied the viscous parameter c is about 100 Ns/m. The current I=0.5 A gives the damping c=370 Ns/m around the point of the low velocity. For current I=1 A, the damping reaches c=650 Ns/m. The damping in a wider range of the velocity is generally lower. In further tests the current I=0.75 A

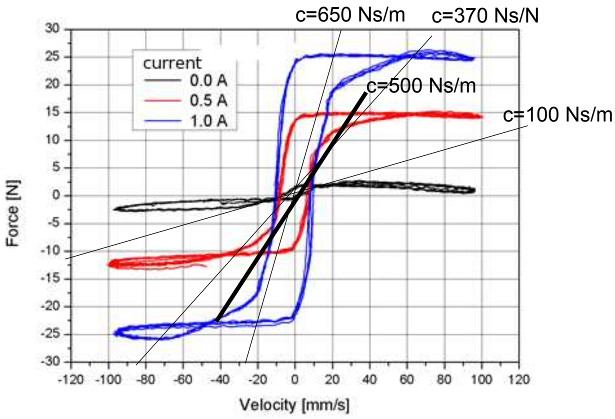


Fig. 11. The velocity-force relation in the MR damper.

was applied and the average damping reached 500 Ns/m.

The comparison between the simulated efficiency of the control and experimental data was made. Figure 12 depicts the displacements in time at three points for the case of mass traveling at the velocity v=3 m/s. The pair of control functions is depicted in Figure 12d. The 10% improvement of the results was noticed. The control functions have short breaks, sufficient for easy rearrangement of the beam near the dampers. The velocity of v=2 m/s results in decreased efficiency of the control.

B. Vibrating cantilever sandwich beam

The experiments were carried out in order to find out whether the overall damping ratio of the structure could be increased. by placing the magnetorheological elastomer at the tip of the beam. The theoretically obtained control strategy for the smart core was verified and evaluated on the fabricated beam. The laboratory stand intended for the research of free vibrations of beams consists of a fixture frame, supported firmly by a steady base plate. A massive mount, acting as a mechanical vice attached to the frame, allows suspending the tested beam vertically in a clamped-free configuration. In order to set the initial displacement of the beam, a holding band was connected to the free tip of the beam. The band was strained to give an initial transverse displacement of 0.06 m. The data acquisition starts when the holding band is released and the beam starts to oscillate around the equilibrium point. The component of the displacement of the amplitude was the basic, directly measured variable. The displacement was measured at three points with dedicated laser sensors, with resolution up to 8 μm and 10 kHz sampling frequency. The measurement system featured functions for compensating the inaccuracy of measurements up to 15° of inclination angle. A 16-bit data acquisition card connected to a computer was used to record the measurement results. The Programmable Logic Controller (PLC) with relay outputs allows to directly program the cycles of turning the actuators on and off, depending on the control strategy. A photo of the real, deflected specimen and measurement system is presented in Figure 13.

The first 60 seconds of vibrations were acquired. This gave us enough information about the process. These results showed that if the smart material is embedded between the face layers, the overall damping of the beam increases. Figure 14 illustrates the first segment of 30 seconds of vibration. The presented plots show how the magnetic field affects the amplitude of the displacement of the beam's tip for an initial deflection of 0.06 m, in three different cases: MRE not activated, MRE turned on constantly, and MRE activated for selected moments. The case of free vibrations of a beam with the non-activated smart core is treated as the reference measurement. In this case, the only damping mechanisms were related to the shear deformation of the non-activated MRE.

All three curves in Figure 14 exhibit damping. The elastomer, although merged locally in the sandwich beam, causes a significant decrease of amplitudes both in constant and periodic magnetic action. The experiment differs in this case from our theoretical analysis. However, the efficiency of the control with a small elastic inclusion, related to the entire length of the beam is effective. Longer observations allowed us to estimate the rate of damping. After 60 s, the amplitude of displacement for 0 mT is 12 mm, which is 20% of the initial deflection. If MRE was activated constantly, the amplitude after 60 s of vibrations decreased to 4.2 mm, which is 7% of the initial value. In the controlled case the amplitude dropped to 2.6 mm, i.e. to 4% of the initial value.

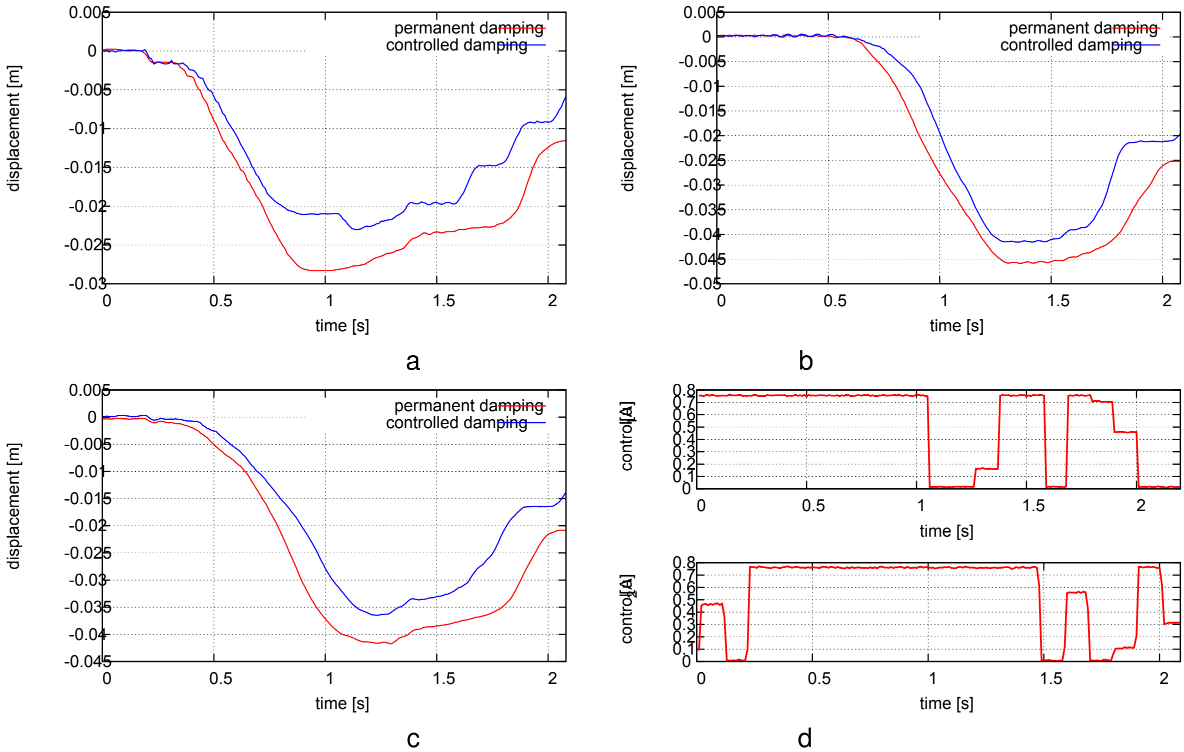


Fig. 12. Displacements in time in the 1st (a), 2nd (b), 3rd (c) point, and the control functions (d), in the case of controlled damping and permanent damping at v=3 m/s, a=4 m/s^2.

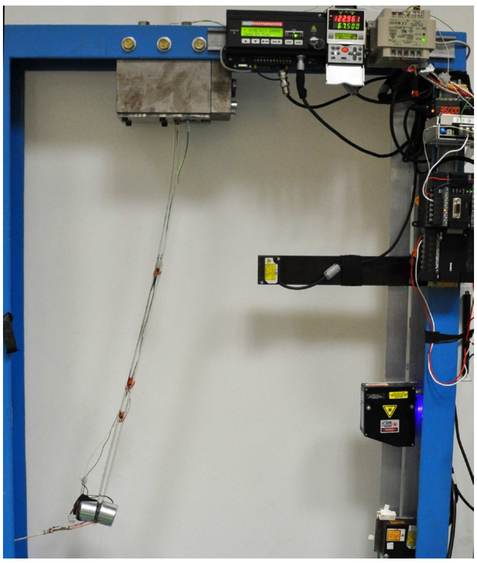


Fig. 13. Photography of the deflected sandwich beam with embedded magnetorheological elastomer.

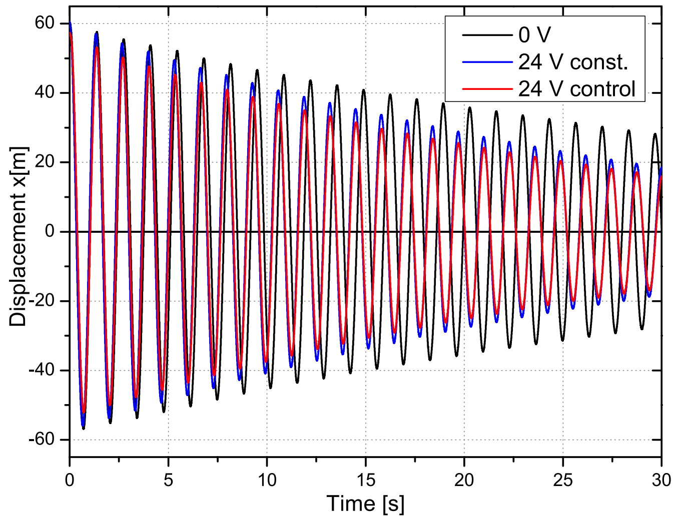


Fig. 14. Displacement in time for different states of MRE damping element caused by the magnetic field.

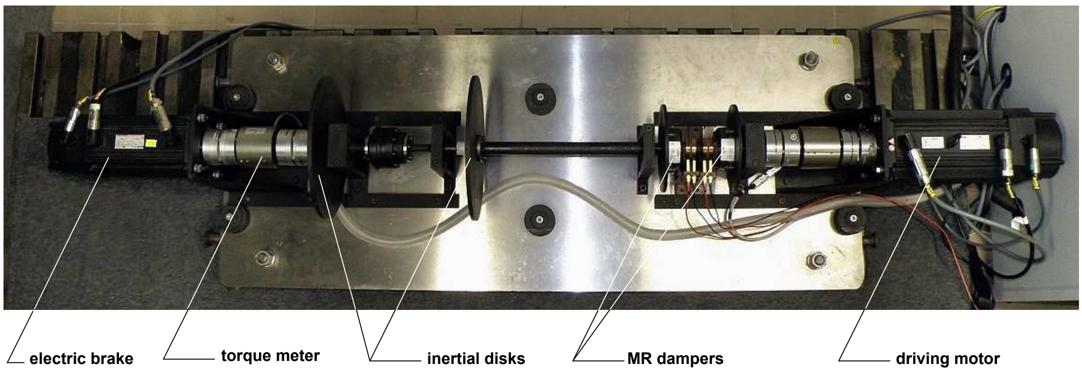


Fig. 15. The scheme of the test stand.

C. Rotating shaft

Further experimental verification of the theory was performed on the test stand. It differs from our previous model, since it has not a uniform cross section area and has a point masses. We will try to apply our control technique to this real structure. In the laboratory drive system presented in Figure 15 the power is transmitted from the servo-asynchronous motor to the driven machine tool in the form of an electric brake. The drive system, which is made up of a multi-segmented shaft, is supported by bearings. It contains an electromagnetic overload coupling, two multi-disk elastic couplings with built in torque meters, two rotary dampers with magneto-rheological fluid, and a measurement control system. Moreover, this drive system is equipped with two inertial disks with adjustable mass moments of inertia and the possibility of axial positioning. This enables us to tune the drive train to the proper natural frequencies. The control voltage is applied to the magneto-rheological damper with sliders. The external magnetic field acts on the fluid inside the damper. Consequently, the characteristics of the magneto-rheological fluid are changed, in a way that controls the torsional vibrations. Since the average rotational speeds of the ring and of the shaft are similar, only small wearing effects can be expected and vibrations can be suppressed without significantly influencing the rigid body motion of the drive system.

The left hand end of the shaft was excited by a driving motor with a sine torque. Its amplitude was set to 1 Nm. The frequency as set within the range 30–60 Hz. The right hand end was terminated with an electric brake. At this point, the boundary condition $\varphi(l, t) = 0$ was used. This condition can be changed in other tests and, for example, a constant rotational velocity can be imposed. However, we can always shift our results as a rigid body motion. Figure 16 depicts torque in time experimentally registered with the constant and periodic damping action of the device, excited with 45 Hz and 50 Hz. The advantages of the proposed controlled system are clearly depicted.

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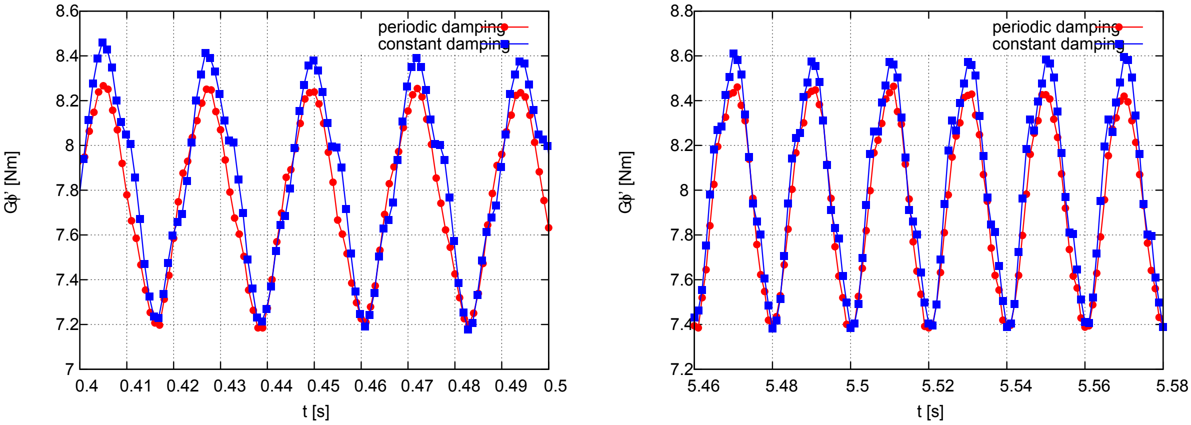

Fig. 16. Torque in time experimentally registered with constant and periodic damping in the damper, excited with 45 Hz and 50 Hz.

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