PROPAGATION OF PERTURBANCES GENERATED IN CLASSIC TRACK AND TRACK WITH Y-TYPE SLEEPERS *

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Abstract

The railway track with classic and Y- shaped sleepers or slab track is composed of two rails assumed as infinitely long and joined with sleepers by visco-elastic pads. There are numerous assumptions results in different simplifications in railway track modelling. Two-dimensional periodic model of the track consists of two parallel infinite Timoshenko beams (rails) coupled with equally spaced sleepers on the viscoelastic foundation.

Nowadays the interest of engineers is focused on the slab track and track with Y-shaped sleepers. The fundamental qualitative difference between the track with classic or "Y" sleepers is related to local longitudinal symmetric or antymetric features of railway track The sleeper spacing influences the periodicity of foundation elasticity coefficient, mass density (rotational inertia) and shear effective rigidity. The track with classic concrete sleepers is influenced much more by rotational inertia and shear deflections than the track with "Y" sleepers. The increase of elastic wave velocity in track with Y-shaped sleepers and more uniform load distribution will be proved by the analysis and simulations.

The analytical and numerical analysis allows us to evaluate the track properties in a range of moderate and high speed of the train. However, the correct approach is not simple, since the structure of the track interacts with wheels, wheelsets and vehicles, which constitute complex inertial load. We can notice that the amplitude growth in selected ranges of the velocity strongly depends on the track type.

Keywords: track dynamics, vibrations, moving load, Y-type track.

1 Introduction

Nowadays high speed trains and increasing load carrying capacity are the main reasons of damages of track and noise emission. The noise reduction must be done on both stages: elimination of sources and protection of humans. Main sources of vibrations are: periodicity

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of track (sleepers), dynamic coupling and interaction between wheelsets and vehicles moving along the track, wear of wheels and rail (corrugations), friction (stick and sleep zones) between the rail and the wheel, breaks, creepage of wheels on curves.

Dynamic phenomena in railway transportation can be divided into two groups: 1) vibrations of the track and vehicle systems, 2) wave phenomena (wave propagation in the track, oscillation in the rail/wheel contact with friction, wave coupling of travelling multi point load). In the first group we consider dynamic properties of the track, i.e. rails, sleepers, elastic pads and the ballast, rigidity of wheelsets, vehicles and the train. The phenomena of the second group occur intensively in the case of high speed trains. The research in several centres is carried out intensively. However, in engineering practice the attention and understanding of wave phenomena is low. Wave phenomena can not be neglected since they result in increased wear of wheels and rails, noise and even accidents. We must recall a group of scientific publications which deal with this problem [1, 2, 3].

The vibration frequency and intensity strongly depends of the track type: classical sleepers, Y-type sleepers, systems with continuous or periodic supports of rails. Measurements and numerical simulations proved lower noise emission of the track with modified sleepers. The Y-type track with steel sleepers is especially efficient and allows to reduce considerably the noise emission. Up to now there are several hundred kilometres of experimental tracks in the world (majority in Germany, some segments in Poland). However, material parameters (sort of ballast, elastic pads, foundation etc.) strongly influence track properties. There exist ranges of parameters (for example speed or track, rigidity) for which waves are transmitted with increasing intensity, aside of bands which exhibit decay properties.

We present the dynamic analysis of Y-type track under moving smooth or oscillatory load. The Y-type track is compared with the classical one. The amplitude growth under the moving load and in certain distance in front of the contact points is the main quantity investigated in the paper.

2 Influence of sleepers and pads features on dynamics of railway track

The conventional and reinforced railway track is composed of two rails mounted to the sleepers by means of visco-elastic pads. There are various assumptions leading to the different simplification in railway track modelling. The two-dimensional periodic model of track consists of two parallel infinite beams (rails) coupled by means of visco-elastic foundation (or equally spaced sleepers). The qualitative difference in track modelling with classic or Y-type of sleepers is connected with local longitudinal symmetric or antymetric features of railway track. The dynamical analysis of above tracks models as periodic structures can be based on Floquet's theorem. The Timoshenko beam or plate model placed on an elastic or visco-elastic foundation can also be used to describe the vertical or lateral motion of the slab. In such a case sleeper spacing influences the periodicity of elastic foundation coefficient, mass density (rotational inertia) and shear effective rigidity. The track with classic concrete sleepers is influenced stronger by rotational inertia and shear deflections than the track with Y sleepers. The increase of elastic wave velocity in track with "Y sleepers" and more uniform load distribution will be proved in analysis and simulations.

Let us consider the Timoshenko beam. The motion is described by the following set of

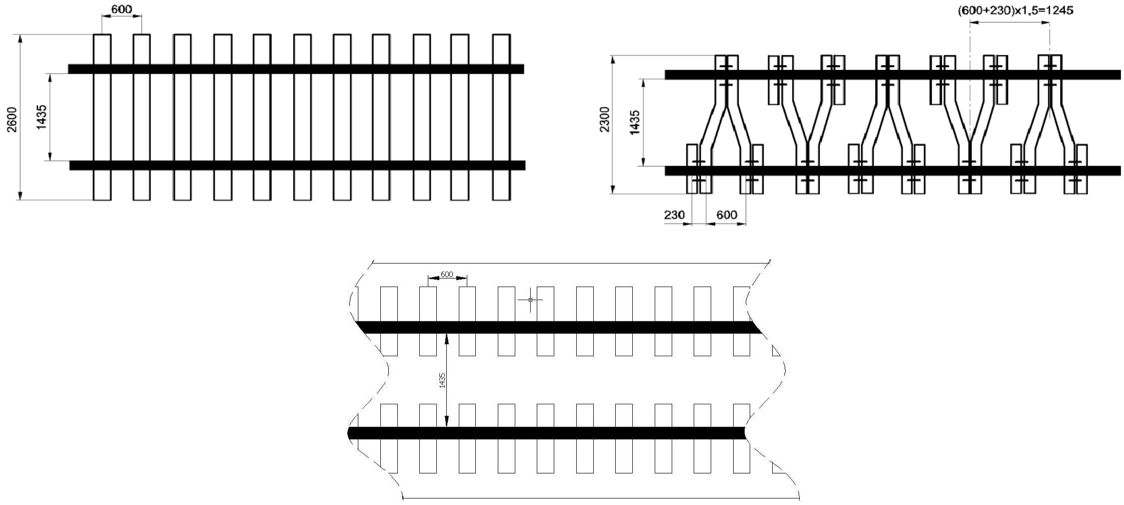

Figure 1: Classical type of the track (left), "Y" type track (right) and slab track (bottom).

partial differential equations:

$$\frac{\partial}{\partial x}\left(K\left(\frac{\partial w}{\partial x}-f\right)\right)-mA\frac{\partial^2 w}{\partial x^2}-cw=p(x,t) \tag{1}$$
$$EI\frac{\partial^2 f}{\partial x^2}+K\left(\frac{\partial w}{\partial x}-f\right)-mI\frac{\partial^2 f}{\partial t^2}=0$$

where: EI - flexural rigidity, K - shear coefficient, G - shear modulus of elasticity, A - cross-sectional area with moment of inertia I, w - displacement, f - angle of the beam rotation, c -coefficient of elastic foundation and m - mass density.

In the simple case of the technical equation of beam motion (Bernoulli-Euler beam), we have:

$$EI\frac{\partial^4 w_i}{\partial x^4}-T\frac{\partial^2 w_i}{\partial x^2}+mA\frac{\partial^2 w_i}{\partial t^2}+cw_i=cq,\quad (i=1,2) \tag{2}$$

The classical track shown in Fig. 1a is defined by following parameters: E=2.1·10^{11} N/m^2, I=3.052·10^{-5}m^4, m=60.31 kg/m, c=2.6·10^8 N/m, sleeper distance L=0.6 m, sleeper mass M = 145 kg and T - tensile force. The plate motion can be described by the following equations:
– vertical displacements

$$D\frac{\partial^4 W}{\partial x^4}-T_0\frac{\partial^2 W}{\partial x^2}+\rho lh\frac{\partial^2 W}{\partial t^2}+c_0lW+c(W-w_1-w_2)=0 \tag{3}$$

– rotatory displacements

$$EI_\omega\frac{\partial^4 \varphi}{\partial x^4}-GI_{0x}\frac{\partial^2 \varphi}{\partial x^2}+\rho I_0\frac{\partial^2 \varphi}{\partial t^2}+2c_0l/3\varphi+2c/l_0(w_1-w_2)=0 \tag{4}$$

I_ω, I_{0x}, and I_0 are inertia moments for the case of the C-shaped plate cross section. They are reduced to a set of ordinary moments in the case of a rectangular cross section of the plate. D is the plate stiffness, l is the width of the plate, h – the plate height, l_0 – the rail spacing, c – the elastic pad stiffness, c_0 – the foundation elasticity coefficient.

The equation of sleeper motion for the case of symmetric (in phase) rail vibrations is as follows:

$$M\ddot{q} + B\dot{q} + Cq = 2b(\dot{w} - \dot{q}) + 2c(w - q), \quad w = \frac{1}{2}(w_1 + w_2) \tag{5}$$

Above b=6.3·10^4 Ns/m, visco-elastic foundation C=1.8·10^8 N/m, B = 8.2·10^4 Ns/m.

In the case of antymetric rails vibration we have following equation of sleepers motion

$$J\ddot{p} + B_0\dot{p} + C_0 p = 2b/l_0(\dot{w}_1 - \dot{w}_2) + 2c/l_0(w_1 - w_2) \tag{6}$$

Looking for the solution in the following form of travelling waves

$$w = W_0 \exp ik(x - vt), \qquad q = Q_0 \exp ik(x - vt) \tag{7}$$

the elastic wave speed dependent on the wave number k is given by the formula:

$$v = f(m, M, EI, c, C, T, k) \tag{8}$$

In the case of rails motion described by the Timoshenko beam which parameters fulfil inequality (9) minimum of the elastic wave speed in rails is smaller then shear wave speed ($V_{cr} < V_G$)

$$F(V_G, V_E) = E - KG(KGA^2 - Ic) < 0 \tag{9}$$

The critical speed and elastic wave velocities in the track for two cases of sleepers modelling can be obtained using graphical or numerical methods. In the case of in phase vibrating rails, described by the equations 2 and 5 and in the case of out of face motion (Eqs. (1) and (5)) the displacement-pressure ratio f_1 and f_2 (pressure between sleepers and rails) versus the wave velocities for an elastic case is shown in Fig. 2. It is visible that the point of resonance in the case of in phase motion we obtain at grater velocity than in the case out of phase motion. The superposition of the effects is visible in Fig. 2 [4]. Crossing points P_1, P_2, P_3 determine the wave velocities in the track. The minimum speed of elastic waves in the track v is determined by point P_1. The value of the speed is smaller than the half of the share wave velocity V_G. Determination of velocities of elastic waves (P_1, P_2, P_3) in the track enables us to estimate maximal speed of the train. The response analysis of the track subjected to the moving and oscillating wheelset motion is possible in the analytical way by using the Floquet's technique (for example [5]).

Dynamic perturbations can be generated by corrugations [6] or impacts of waved rolling surfaces [7, 8]. In the case of traveling load the coupling of dynamic effects is described [5].

The classic and reinforced railway track is composed of two infinite rails separated from the sleepers by visco-elastic pads. There are numerous simplifications in railway track modelling. The rails are modelled as infinite Timoshenko beams, sleepers by lumped masses or elastic bodies and ballast as visco-elastic foundation. The fundamental qualitative difference between the track with classic or Y sleepers is related to local longitudinal symmetric or antymetric features of railway track. The sleeper spacing influences the periodicity of elastic foundation coefficient, mass density (rotational inertia) and shear effective rigidity. The track with classical concrete sleepers is influenced much more by rotational inertia and shear deflections than the track with Y sleepers. The increase of elastic wave velocity in track with Y sleepers and more uniform load distribution will be proved by the analysis and simulations.

In a classical track sleepers are placed in a distance of 60 cm, leaving the rail between them unsupported. The distance between wheelsets is equal to the multiplication of it,

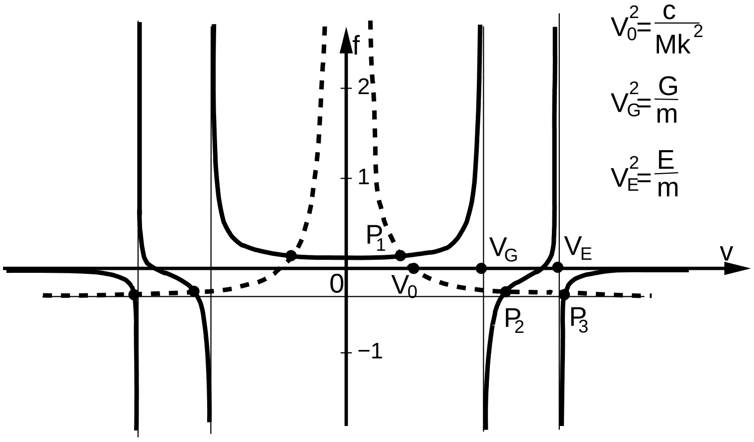

Figure 2: Wave velocities in conventional track for the case when both rails and sleepers vibrate in phase.

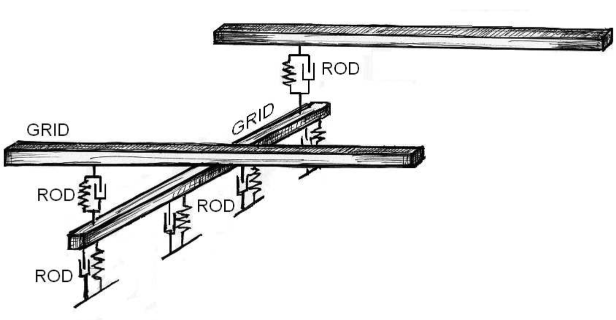

Figure 3: Scheme of the computational model of the track.

usually 3 m. It results in lower rotatory motion of the vehicle. However, the vertical motion of the vehicle is significant, since all the wheels are affected by the same alteration of the stiffness of rails.

"Y" sleepers are made of steel. The principal idea is to increase the transverse stiffness and to enlarge the damping of the track by incorporating the ballast between the sleepers. Experimental investigations of the track exhibit lower noise level. Numerical simulations show reduced vertical amplitudes. The Y-type track is designed for medium travelling speed, however, in the higher speed range it also exhibits good dynamic properties.

Numerical track model was composed of grid and bar finite elements. Both rails and sleepers were modelled as a grid separated by visco-elastic pads assumed as bar elements (Fig. 3). The Winkler type foundation was modelled by visco-elastic springs. The total length of the track used in simulation was 20 m. Both ends were fixed. Higher damping allowed us to reduce the influence of boundary conditions. The vehicle was built as a mass and spring 3-dimensional system combined with frame elements. The distance between wheelsets was equal to 250 cm. The coupling of displacements in right and left contact points was performed both by wheelsets and the vehicle frame and was stronger than in the case of the coupling between leading and hind wheelsets.

Advantages of Y-type sleepers are significant for practical use. They are characteristic of lower amplitude level and in the same time lower acoustic emission. The wear (for example

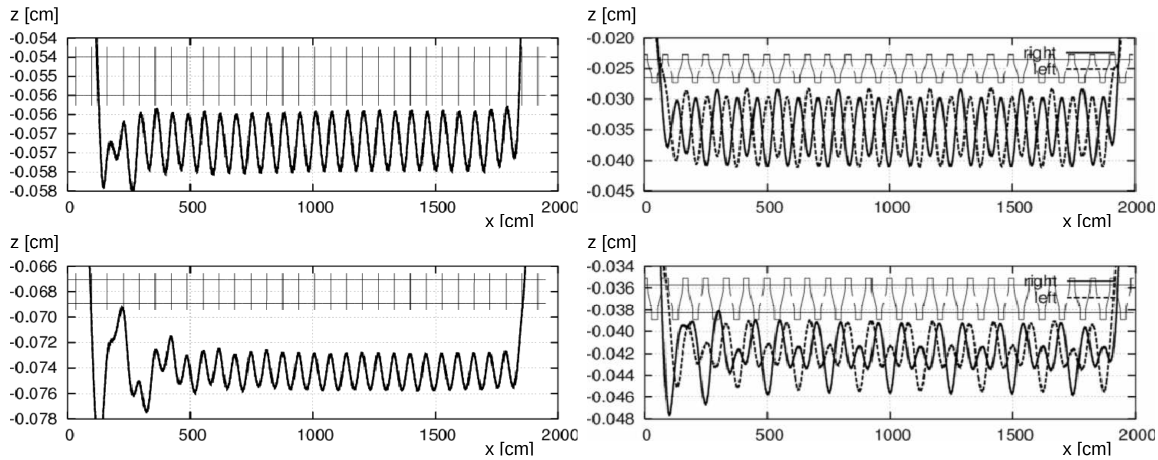

Figure 4: Vertical displacements of contact points in the case of smooth rolling, with the speed 30 m/s (upper) and 50 m/s (lower).

corrugations) should be generated more slowly since the contact force does not oscillate as strongly as in the case of the classic track.

Two characteristic cases were selected. The rolling of a vehicle with regular, perfect wheels was the first one. The vehicle with polygonized, corrugated wheels, which subjected the tract with oscillating forces was the second one. The initial stage of rolling on the rails was an excitation of the system. The response of the wheelset/track system depends on the velocity. In higher range of the velocity, in the case of perfect wheels, we can notice significant influence of sleepers type (Fig. 4).

The second case of the problem is depicted in Fig. 5-8. We notice higher level of oscillations of Y-type sleepers. However, the average level of displacements is considerably lower. Moreover, with the increasing velocity of the train the amplitudes in both cases start to be similar.

In the second test the contact point was additionally subjected to eccentric wheels load. Such a case usually occurs in practice. It can be considered as a periodic load which acts to a wheelset together with the periodicity of the track structure at the speed 40 m/s. The simulation proved significantly lower vibration level of the track with Y-type sleepers than in the classic case of the track (Fig. 6). Vibration measured in a specified distance in front of the wheelset is important in the case of coupling of interactions of successive vehicles travelling over straight or waved rail. The comparison of both tracks in the whole period of simulation is depicted in Fig. 8. We can see the waved time-space surface in the case of the classic track. The interesting phenomenon of waves travelling towards the source can be observed in this example. The similar phenomena was described in [9]. The Y-type track exhibits considerably lower level of amplitudes. Additional measuring point was placed on the vehicle frame. At a velocity 34-35 m/s one can notice increased level of higher frequency vibration (Fig. 7). However, the lower amplitude level is observed in the case of Y-type track.

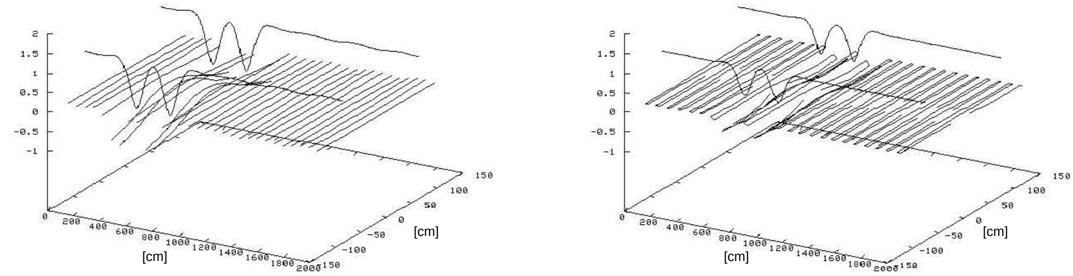

Figure 5: Simulation of the classic and Y-type track under the vehicle (both vertical position and displacements scaled for visualisation purpos)

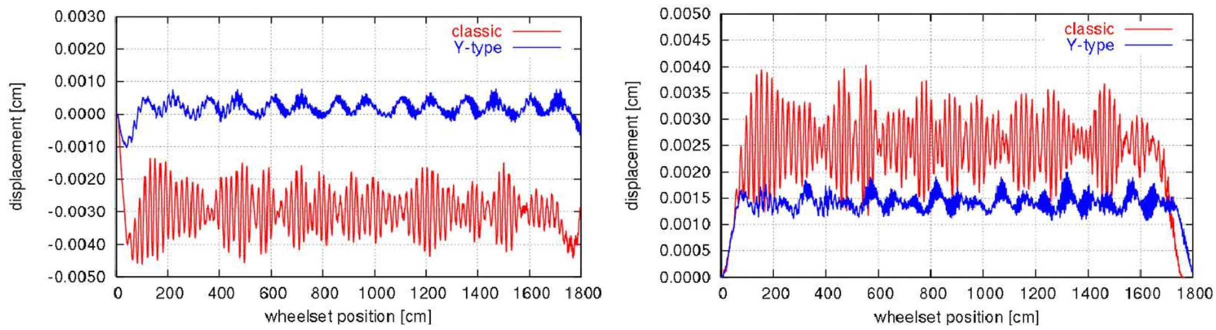

Figure 6: Vertical displacements registered 120 cm (left) and 180 cm (right) in front of the contact points of the buggy for classic and Y-type track at the speed 40 m/s.

3 Travelling wave solution

Continuous modelling of wheels and rails subjected to the moving load is described by partial differential equations of hyperbolic or parabolic type. Looking for the solution in the form of travelling wave we can determine regions of qualitatively similar solutions. The configuration of the regions depends on parameters of the beam, like Young and shear modulus, moment of inertia, mass density and foundation stiffness (E, G, I, m, c). An example of the shear beam is shown in Fig. 9a. The response of wheels and rails subjected to the moving and oscillating contact load in the first regions (small frequency and small velocity of excitation) is illustrated in Fig. 9. The decreasing wave form of solution moving with the speed of excitation source is obtained in the first region only. The travelling wave solution with waves moving to the source of excitation before the load and from the excitation source behind it is shown in Fig. 10. Such a phenomenon is possible in the region 2A only, when the group velocity of wave is greater than the phase velocity. The case of all waves travelling from the excitation source is shown in right hand side of Fig. 10.

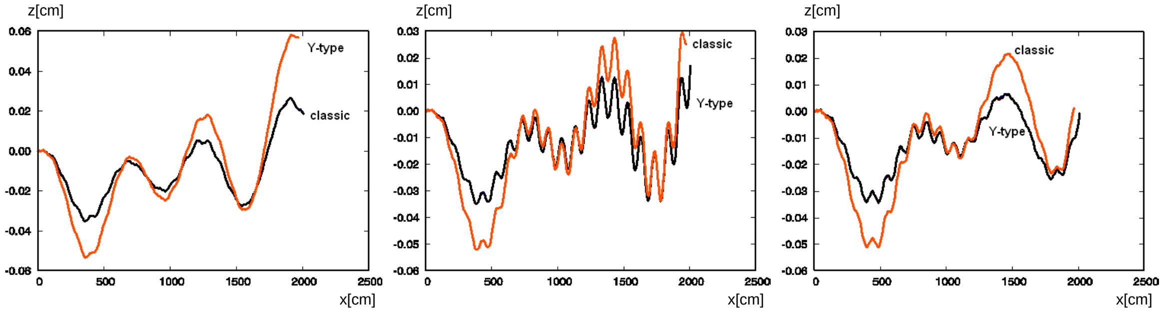

Figure 7: Vertical displacements of the vehicle frame at 30, 34 and 36 m/s.

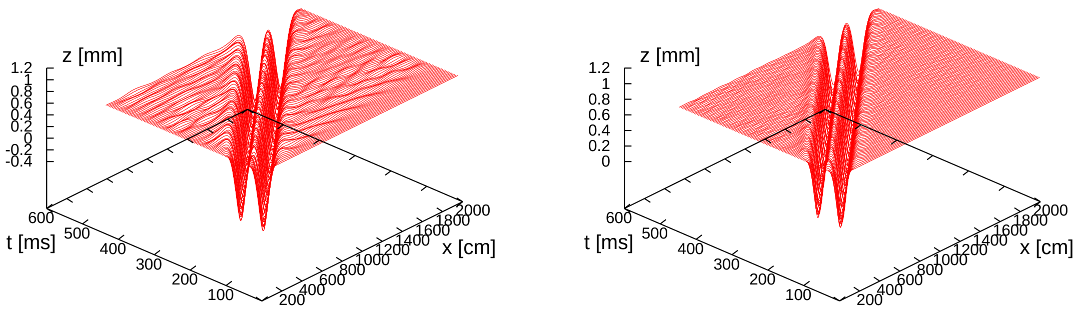

Figure 8: Vertical displacements of a classic track (left) and Y type track (right) in time-space domain subjected by buggy moving with the speed 40 m/s.

4 Conclusions

Analytical investigations give the qualitative relations between the track vibrations, especially bending and shear waves, and the speed of the travelling inertial coupled load. The existence of resonance regions was described in several papers devoted to the dynamics of the beam under moving load (for example [5]). Quantitative analysis in practice can be performed numerically. However, the coincidence of obtained results with real measurements requires precise values of the track parameters. Especially, the system is sensible to the elasticity of the elastic/visco-elastic pad. The rigidity of springs in the vehicle has minor importance. The important question is the modelling of the wheel/rail contact. The analysis with the wheel assumed as a continuous 2 or 3-dimensional disk discretized by finite elements is relatively simple. In the case of vehicle made as a spring-mass system we can not say what exact amount of the wheel mass should attache the rail in vertical motion. However, numerical analysis proved analytical calculations.

Advantages of Y-type sleepers are significant for practical use. They are characteristic of lower amplitude level and in the same way lower acoustic emission. The wear (for example corrugations) should decrease since the contact force does not oscillate as strongly as in the case of the classic track.

The investigation of the boogie interaction with the non-conventional track will be published in the next paper.

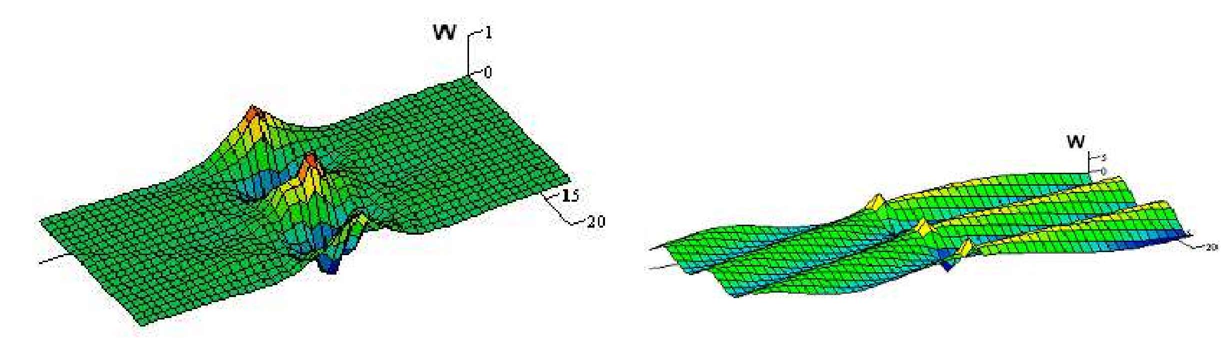

Figure 9: Qualitatively different solutions for moving and oscillating force and with $v = 0.1\ v_{cr}$ and frequency $\Omega = 0.3\,\Omega_{res}$ (left) and $\Omega = 0.95\,\Omega_{res}$ (right), with waves travelling towards the source of excitation.

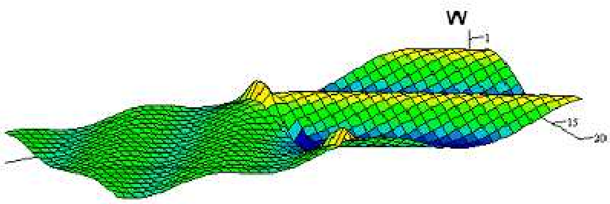

Figure 10: Waves generated by moving oscillatory force in the case of $v = 0.1\,v_{cr}$ and frequency $\Omega = 1.1\,\Omega_{res}$, with waves travelling from the excitation source.

References

[1] K. Knothe, Rail Corrugations. ILR Bericht 56, Berlin 1983.

[2] R. Bogacz and Cz. Bajer. On modelling of contact problems in railway engineering. In Recent Advances in Applied Mechanics, J.T. Katsikadelis, D.E. Beskos and E.E. Gdoutos, Eds., Nat. Techn. Univ. of Athens, Greece, 2000, pp. 77-86.

[3] R. Bogacz, S. Dżuła, Dynamics and stability of a wheelset/track interaction modelled as nonlinear continuous system. Machine Dynamics Problems, 20, 1998, pp. 23-34.

[4] R. Bogacz, S. Nowakowski, K. Popp, On the stability of a Timoshenko beam on an elastic foundation under moving spring mass system, Acta Mechanica, 61, 1986, pp. 117-127.

[5] R. Bogacz, T. Krzyżyński and K. Popp, Application of Floquet's theorem to high-speed train/track dynamics, DSC-vol.56/DE/vol.86, Advanced automotive technologies, ASME Congres 1995, pp. 55-61.

[6] C. Bajer. The space-time approach to rail/wheel contact and corrugations problem. Comp. Ass. Mech. Eng. Sci., 5:267-283, 1998.

[7] R. Bogacz, Z. Kowalska. Simulation of elastic-plastic wheel/rail system. Machine Dynamics Problems. 25 No 3/4 2001,pp.97-106.

[8] R. Bogacz, Z. Kowalska, Computer simulation of the interaction between a wheel and a corrugated rail, Eur. J. Mech. A/Solids 20, 2001, pp. 673-684.

[9] R. Bogacz, M. Kocjan, W. Kurnik, Wave propagation in wheel tyre. Machine Dynamics Problems. 27 No 4, 2003, pp.168-179.