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PASSIVE CONTROL OF A BEAM SUBJECT TO TRAVELLING INERTIAL LOAD

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The subject of the study is the minimization of transverse vibration of elastic guideway excited by high-speed travelling inertial load. Two types of systems of passive control are considered, the post truss system and the dynamic vibration absorbers. In the first one, the beam is supported by posts fixed to the beam span and prestressed by tendons connected to the ends of beam. The considered system corresponds to the experimental test stand. It is modelled by a simply supported Bernoulli-Euler beam, where the material passive damping is represented by means of the Voigt model. In the system the control is performed by point transverse forces and bending moments applied to the beam span.
The significant reduction of the amplitude of vibrations is achieved for a guideway supported by truss system consisting of two posts. The obtained results are compared with the data obtained for active control of the considered guideway.

1. Introduction

The guideway systems subjected to high-speed travelling inertial loads are often used as transport structures in civil engineering and for robotic devices or handling equipment in the light industry. The problem of elastic structures excited by moving loads was studied since 1848 when Stokes and Willis, motivated by railway damages, published their first results. The up-to-date development of computation techniques leads to a revival of this study. New passive systems of protection against extereme vibration and, on the other hand, the real-time active control of vibrating structures, are continually under study for a new lightweight and adaptive structures.

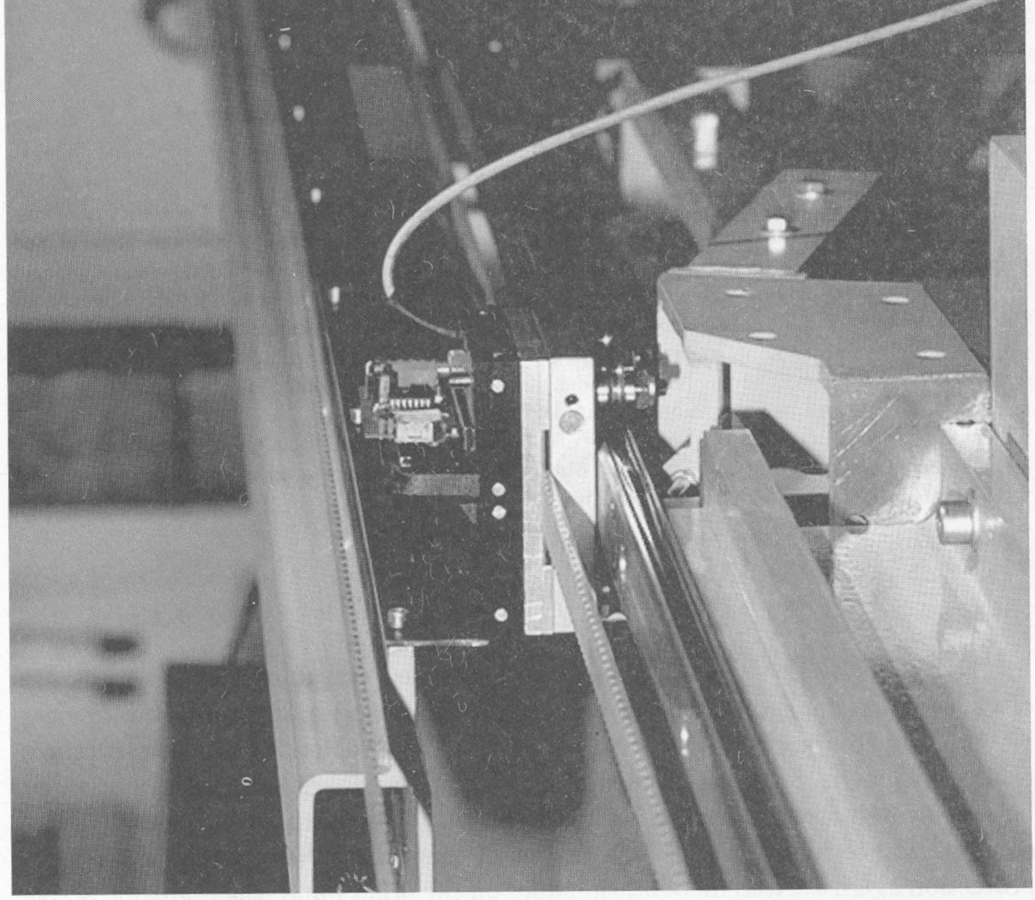

FIG. 1. Experimental test-stand.

In this study, a guideway system is investigated to compare the properties of passive and active systems. We consider a transverse vibration of a simply supported, visco-elastic beam used as a guideway. The excitation is given by a mass moving along the beam, see Figs. 1 and 2. The beam is supported by a truss system composed of posts located between ends of the span. The post system consists of king-post or two posts. Electric actuators which are applied to the system stretch the tendons and through the posts introduce as the transverse forces or/and bending moment to the beam span. In active control, the active forces are introduced by actuators. On the other hand, the truss system with the optimal posts localization and tendon prestressing represents one of the passive systems considered herein. The second type of passive system is considered by means of dynamic vibration absorbers (DVA) as a set of additional masses flexibly attached to the main strucure. The optimal position of DVA in the beam span as well as its mass and tuning are taken as design variables in the optimization process.

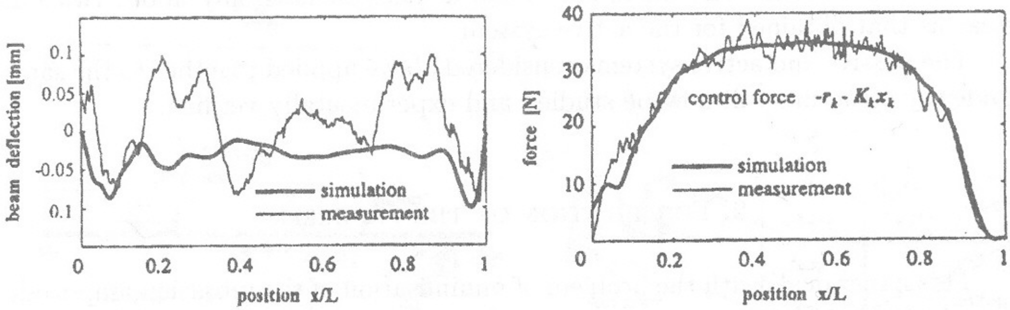

FIG. 2. Active control. Deflection of the beam at speed of the mass 5 m/s (left) and total force in the midspan of the beam (right).

The minimisation of both the maximal deflection of the beam and the deflection under the moving mass is the aim of the present investigations. The results obtained for passive systems are compared with those obtained for the same system actively controlled.

The theoretical and experimental study of active control of the considered structure was done for example by FRISCHGESELL *et al.* [1 – 4]. The commercial linear guideway with one post located in the middle of the beam span, shown in Fig. 1, was used in the experiment. The driving force of the horizontal motion of the mass is generated by a servomotor and a synchronous belt drive. Due to the post fixed to the beam span, the system enables us to control the predominant first two vibration modes of the beam. In the active control, the investigations are performed by means of the open and closed-loop approach. The best results are achieved for the closed-loop technique. Two different closed-loop controllers are used. The first one is based on the pole allocation method. The second one is an optimal controller, which takes into account the system disturbances due to the weight of the moving mass and of the beam. This considerations were curried out with the use of finite element method. Theoretical and experimental results for the best displacement minimisation are depicted in Fig. 2. It is shown that the deflections in the numerical simulation and in the measurements reaches the minima in the starting and final positions, respectively.

The deflection in the final position is 0.1 mm. Regarding the accuracy of the measurement system of about 0.15 mm, the simulation and measurements coincide. Moreover, the total transverse force in the numerical simulation corresponds well with the total force applied to the test stand in the experiments (the right-hand plot).

In the following we formulate the problems for both the beam with prestressing truss system and the beam with dynamic vibration absorber. It follows from the results that the passive systems adopted herein gives considerably reduces the beam deflection. The maximum beam displacement is only about twice as great as that obtained for the active system.

The passive and active systems considered above applied together to the same guideway structure will now be studied and experimentally verified.

2. Formulation of the problem

This paper deals with the problem of minimisation of the vibration amplitude of a visco-elastic guideway exited to transverse vibration by the mass m moving with the constant velocity v. The scheme of the guideway and the mechanical models are shown in Fig. 3.

Two purposes have to be achieved in the study. The first one is the minimisation of the current beam deflection under the mass w, which is related to the possibly straight, horizontal motion of the mass along the beam. It can be defined by the performance index J.

$$J = v \int_0^{l/v} w^2(vt,t)dt. \tag{2.1}$$

The second question is the minimisation of the maximal beam deflection $w_{\max}$ for any point of the beam. It is described by the condition.

$$\min w_{\max}(x). \tag{2.2}$$

The experimental guideway is supported at its ends by movable pivot bearings and is modelled as a simply supported Bernoulli-Euler beam, where the material passive damping is represented by means of the Voigt model. The post truss system is composed of king-post or two posts as shown in Fig. 3a and 3c. For small vibration, the interaction of the beam with the king post truss system is realised by the post masses m_i, $i = 1, 2$ for two posts or $i = r$ for the king-post, which are attached to the beam mid-span and supported by a massless springs of constant stiffnesses: transverse k_i and rotational c_i, representing the elasticity of the tendons. The moment of inertia of masses m_i is denoted by I_i.

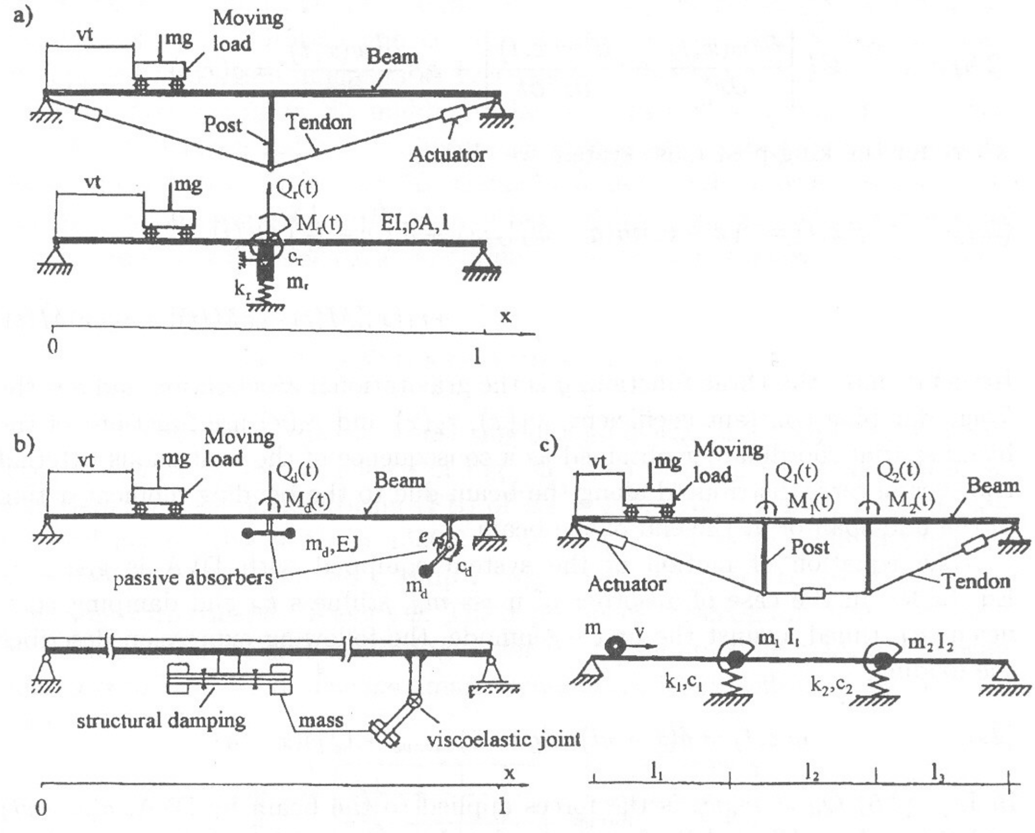

FIG. 3. The structure under consideration and mechanical models for active control (a) and passive control (b), (c).

The control forces generated by electric actuators are imposed on the tendons, and through the post to the beam span. The generated transverse control force $Q(t)$ and bending moment $M(t)$, enables us to control the first two predominant vibration modes of the beam. In a passive system, they are forces of optimal constant value.

Let us turn to the passive control by means of the dynamic vibration absorber (DVA). Transverse and rotational absorbers considered herein are shown in Fig. 3b. Additional masses of DVA, flexibly attached to the main system, vibrate out of phase with respect to the main structure and result in elimination of vibration of the main structure. The DVA influences on the guideway beam by forces $Q_d(t)$ and $M_d(t)$ (Fig. 3b).

Investigations are performed by means of a continuous as well as a discrete model. In the continuous mechanical model, the solution is assumed in the following form:

$$EI\left[\frac{\partial^4 w(x,t)}{\partial x^4} + e\frac{\partial^5 w(x,t)}{\partial x^4 \partial t}\right] + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = q(x,t), \tag{2.3}$$

where for the king-post truss system we obtain

$$q(x,t) = \delta(x - vt)m(g - \ddot{w})\,|_{x=vt} + \delta\left(x - \frac{1}{2}\right)Q_r(t) + r_1(x)[M(t) + e\dot{M}(t)] + r_2(x)\ddot{M}(t). \tag{2.4}$$

Here δ denotes the Dirac function, g is the gravitational acceleration and e is the Voigt damping constant coefficient. $r_1(x)$, $r_2(x)$ and $r_3(x)$ are functions of the beam spatial coordinate x obtained as a consequence of the continuous external transversal force distributed along the beam due to the bending moment acting in the mid-span or at the end of the beam.

The equation of motion of the system equipped with DVA is given by Eq. (2.3). In the case of absorber of mass m_d, stiffness k_d and damping coefficient e_d, tuned against the first eigenmode, the following expression describes the loading:

$$q(x,t) = \delta(x - vt)m(g - \ddot{w})\,|_{x=vt} + Q_d\delta(x - a). \tag{2.5}$$

In Eq. (2.5) $Q_d = m_d\ddot{w}_d$ is the forces applied to the beam by DVA, $\delta(x - vt)$ and $\delta(x - a)$ are Dirac-delta functions related to the position of the moving load and absorber, whilst a is the position of the absorber, w_d is the amplitude of vibration of DVA masses. The equation of forces at $x = a$ is as follows:

$$m_d\ddot{w}_d + e_d\dot{w}_d + k_d w_d = e_d\dot{w} + k_d w, \tag{2.6}$$

where $w = w(a,t)$. In the optimization we assume mass, tuning (stiffness) and localisation of the absorbers as design variables. The problem is solved by standard optimization procedures. This research will be complemented by DVA with controllable and adjustable parameters.

In the finite element method the lumped mass m_i and stiffnesses k_i, c_i of the transverse and rotational degree of freedom, respectively, lead to additional components in the mass and stiffness matrices. The equation of motion is given as a system of ordinary differential equations with generalized coordinates $q(t)$ containing deflection $w(x,t)$ and its first spatial derivatives:

$$\mathbf{M}\ddot{q}(t) + \mathbf{D}\dot{q}(t) + \mathbf{K}q(t) = f(t). \tag{2.7}$$

$\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ denote the mass, damping and stiffness matrices respectively, $f(t)$ denotes external excitation and control. Components of this vector represent the applied forces and moments at every node of the beam, i.e. force $Q(t)$ and moment $M(t)$, acting in the middle of the beam as well as the moments acting at the end of beam induced by linear electromagnetic drivers. Components of the matrices $\mathbf{M}(vt)$, $\mathbf{D}(vt)$ and $\mathbf{K}(vt)$ can be found in [1]. In order to obtain the dynamic system response in the form of beam deflections, Eq. (2.3) is integrated step by step using the average acceleration method or the transfer matrix method.

3. Results and experimental verification

Let us consider a structure consisting of a guideway of length $l = 2.5$ m with king-post truss system as shown in Fig. 3a. Two types of the beam are under consideration, the flexible beam (FB) of $EI = 173.2$ Nm^2, $\rho A = 1.0$ kg/m, and the stiff beam (SB) of $EI = 1550.2$ N/m^2, $\rho A = 3.0$ kg/m. The preliminary analysis was carried out on a simply supported SB subjected to inertial load travelling with constant velocity v. The dependence of maximum beam deflection on the mass velocity is shown in Fig. 4, for the ratio of moving mass to the beam mass $\mu = m/\rho Al = 1.0$. The maximum values of the beam deflection are observed for $v = 4.5 - 6$ m/s.

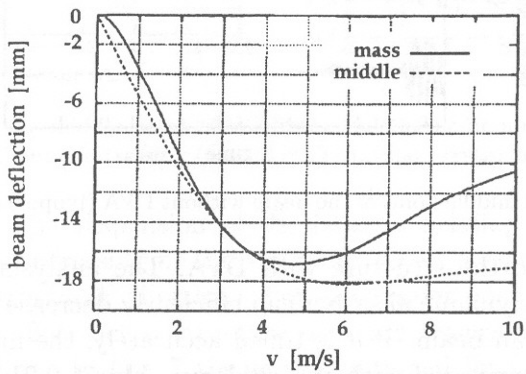

FIG. 4. Maximum deflection of the middle point of the beam for different mass velocities.

Let us turn to the influence of the stiffness of the guideway connected with the king-post truss system. Theoretical results are shown in Fig. 5 for SB and FB, where the velocity parameter α is considered with respect to the first system eigenfrequency ω_1 and is assumed $\alpha = \pi v = 0.5$. The value $\alpha = 0.5$ corresponds to the critical beam speed in the Krylov sense. It is seen that for the passive system, the beam deflection curve indicates a higher influence of the king-post truss system on the FB than in the case of SB. The influence of the Coriolis and centrifugal forces is found to be higher in the case of a more flexible beam. Details of this part of the analysis are presented in [3].

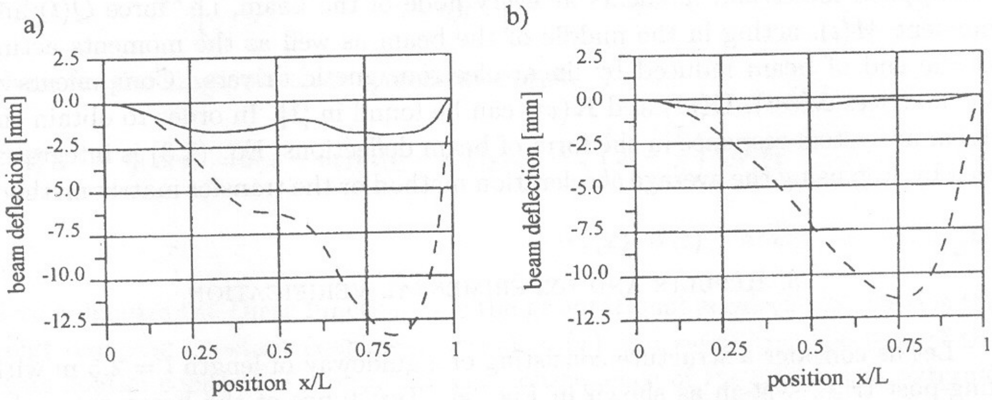

FIG. 5. Deflection under the moving mass of flexible a) and stiff b) beams for passive (- - - - -) and active (——) approaches.

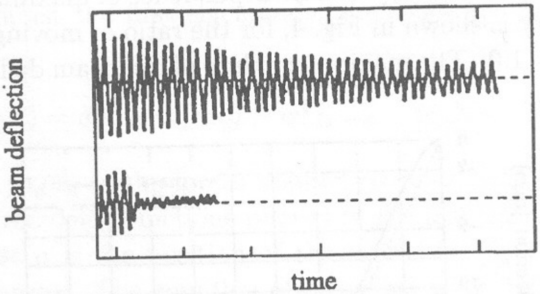

FIG. 6. Response of the middle point of the beam without DVA (lypper) and with DVA (lower).

Let us return to the structure with DVA. The analysis of passive control shows that a tuned dynamic absorber can efficiently decrease the response of the considered single span beam. If it is tuned accurately, the mass of the absorber can be very small compared with the guideway, about 0.01 of the beam mass. The DVA is activated and acts effectively after a few cycles of vibration, since then the amplitude considerably diminishes. This is shown in the Fig. 6. For that reason, when the velocity of the mass is higher than the fundamental resonance frequency of the beam alone, the DVA does not reduce the maximum displacement of the beam. In such cases, application of DVA reveals its advantages in protection of the structure against fatigue and in improving its usability. However, in the case of oscillating force or the load moving slowly with respect to the beam fundamental resonance frequency, the reduction of maximum amplitude of vibration is possible. Such a result is presented in [5].

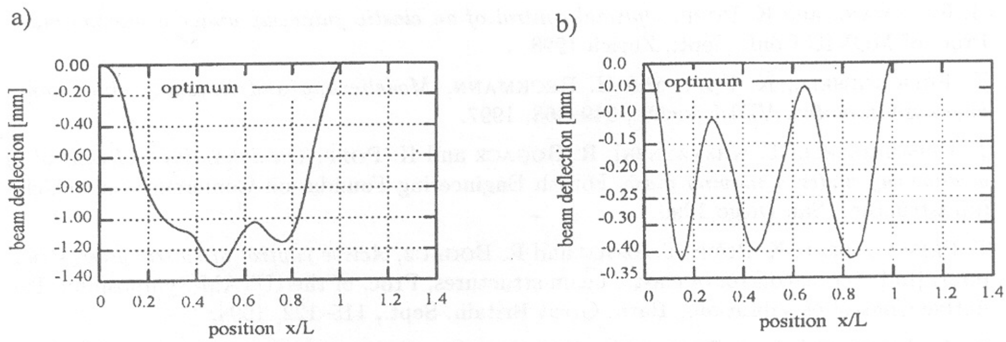

FIG. 7. Deflection of the point under the travelling mass of the beam with prestressing system consisting of two posts with inertia related to rotational (a) and transverse (b) degrees of freedom.

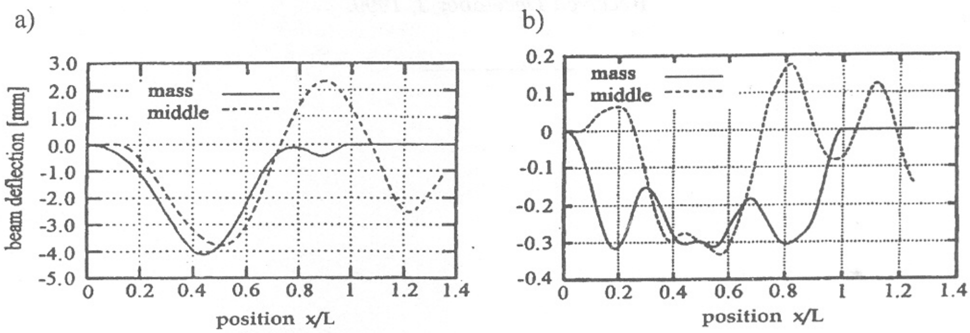

FIG. 8. Deflection of the middle point of the beam and the point under the travelling mass without prestressing (a) and with optimal prestressing (b).

Let us turn to the suspension of the beam consisting of two posts. The study on optimal prestressing of the structure shows a significant reduction of the amplitude of structural vibrations. Let us consider posts which are attached to the beam span. For such a structure only the transverse inertia should be included in modelling. In Fig. 7 the results obtained for suspension system of pinned posts are compared with those obtained for the theoretical model in which the rotation inertia is included only. From this comparison we conclude the predominant influence of transverse inertia for which the significant decrease of amplitude to 0.35 mm is achieved.

The minimum value of deflection for the model with fixed posts is about 0.3 mm. The graphs are shown in Fig. 8.

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