Kinematic approach for dynamic contact problems — the geometrical soft way method

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Abstract

A way of taking into account the geometrical constraints in evolution problems of solid systems that limit the possibility of motion by the history of variation of the velocity field is developed in the paper. A formulation that can be adapted particularly to a numerous problems of solid systems subjected to dynamics effects with large deformations, large displacements, large rotations is described. The computational cost of one iteration in the method proposed is the same as in other classical methods because of the frontal approach. However, the iterative process is much more efficient. If an adaptive space and time meshing were chosen it could become a less cost method. A numerical example of the contact analysis in which both the spatial and temporal partition was adapted according to the evolution of the geometry proved the efficiency of the approach.

1 Introduction

Several possibilities of taking into account the unilateral contact conditions are described in the literature [1, 2]. However, numerical experiments show that the computation time is not equivalent for each approach chosen for a particular problem.

The choice of geometrical constraints as restrictions imposed on the variation of the velocity field enables to gain a great ratio in computational time.

First a velocity formulation is presented. It can be adapted particularly to a wide range of problems such as rigid mechanisms [3] or deformable solid systems submitted to dynamic effects [4]. It must be emphasized that the choice of the velocity formulation is not incompatible with the description of large deformations, large displacements, large rotations. Moreover, the expressions are less intricate and the contact evolution between deformable solids is easier to formulate since it is a tangent space formulation. The fundamental general formulation (instantaneous updated lagrangian formulation) is suitable for the discretization by the space–time finite element method [5, 6, 7, 8, 9, 10], which avoids to dissociate spatial discretization and time discretization. The virtual work principle is formulated by the integration in time of the virtual power principle. The velocity field is considered as the principal variable. Elements proposed have a linear dependence in time of the real interpolation functions. The appropriate choice of the virtual velocity field is discussed.

The problem of points entering into contact is formulated with the use of introduced special elements. These elements have extra time nodal points that are eliminated by the normal contact conditions. The interpolation function are piece–time dependent. The tangential conditions are formulated by friction laws with the same type of formulation that the rheological laws. The tensor theory in dynamics of surface allows to join the space behavior and the surface behavior. Certain conditions of compatibility have to be fulfilled but a phenomenological way is open to choose a more adapted law to a specific problem with dry or lubricated friction. The viscoplastic friction law is used as a more realistic behavior, applied particularly to the forming problems.

The solution algorithm uses a fixed point method that leads to a gain of computation time in regard to the Newton–Raphson method. In [4] the Newton–Raphson method was used to a similar mechanical problem. In the proposed paper the solving by a fixed point method gives good results because of the less number of arithmetical operations and the good conditioning of the system of equations. The algorithm uses a frontal method that leads to a problem of the same size as in the classical finite element method. Therefore, the usual classical software can be simply implemented.

Numerical examples related to different application fields are presented.

2 Velocity formulation

The velocity is considered as the main variable. Generally, the virtual power principle can be formulated by:

$$\mathcal{P}_i^* + \mathcal{P}_e^* - \mathcal{P}_a^* = 0 \tag{1}$$

where for a deformable solid:

$$\mathcal{P}_i^* = -\int_\Omega \boldsymbol{\sigma} : \mathbf{D}^* \mathrm{d}\Omega \tag{2}$$

$$\mathcal{P}_e^* = \int_\Omega \rho\, \mathbf{f} \cdot \mathbf{v}^* \mathrm{d}\Omega + \int_{\partial\Omega} \mathbf{F} \cdot \mathbf{v}^* \mathrm{d}(\partial\Omega) \tag{3}$$

$$\mathcal{P}_a^* = -\int_\Omega \rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\, \mathbf{v} \mathrm{d}\Omega \tag{4}$$

$\mathbf{D}$ — the Euler strain rate tensor, $\boldsymbol{\sigma}$ — the Cauchy stress tensor, ρ is the mass density, $\mathbf{f}$ — external body forces, $\mathbf{F}$ — external surface forces. The virtual work $\mathcal{W}^*$ during the interval within the interval $[t_0;\, t_1]$ is defined by the integral:

$$\mathcal{W}^* = \int_{t_0}^{t_1} \mathcal{P}^* \mathrm{d}t \tag{5}$$

Therefore the virtual work is calculated by the integration over the space and time domain $\{\Omega \times [t_0, t_1]\}$:

$$\mathcal{W}^* = \int_{t_0}^{t_1} \int_\Omega \mathcal{F}\, v^* \mathrm{d}\Omega \mathrm{d}t \tag{6}$$

where $\mathcal{F}$ is a generalized force and v^* is a virtual velocity. The virtual velocity field $\mathbf{v}^*$ chosen, is formulated as a product of a spatial and time functions

$$\mathbf{v}^* = \mathbf{f}(\mathbf{x}(t))\, \mathbf{g}(t) \quad , \tag{7}$$

where $\mathbf{f}(\mathbf{x}(t))$ is a spatial distribution, which in a general case can vary in time, since the geometry depends on time and $\mathbf{g}(t)$ can be either a constant distribution within the step of time or one or several Dirac peaks. In the first case we recognize the momentum theorem and in the second case we recognize the equation of motion.

After the integration (5) the equations are implicitly described by the velocity field:

$$\phi(\mathbf{v}, \bar{\boldsymbol{\Omega}}) = 0 \tag{8}$$

Boundary conditions include contact dissipations and the variation of the domain and its boundary $\bar{\boldsymbol{\Omega}}$ is the function of variation of the velocity field. $\bar{\boldsymbol{\Omega}}$ is estimated by

$$\mathbf{x} = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v} \mathrm{d}t \tag{9}$$

In the case of large rotations the objectivity of the rheological law is guaranteed by an objective derivative in its expression. The commonly used derivative are either the Jauman derivative or the Truesdell derivative.

However, the expression in a fixed axis is not easy to evaluate. It was chosen here to determine the variation of stress tensor in the relative coordinates of Jauman. For instance, the elastic behavior is well formulated by hypoelasticity, where the relation between strain rate tensor and the derivative of the Cauchy stress tensor with regards to time is linear when the components are considered in the corotational axis [11]. Truesdell derivative is well adapted when the Piola stress tensor is considered. The components can be considered in corotational axis too.

3 Dynamic contact conditions

The solid S_j is considered as a reference one. The external normal (effective or predicted) $\mathbf{n}_{ji}$ is associated at the effective or predicted contact point. Afterwards the base of the tangent plan defines two other reference axes.

Below we will use indices i and j to note the number of the body. n is a normal direction and T is a tangential one.

The relative normal displacement of a point of the solid S_i with regard to a solid S_j is:

$$u_{n_{ij}} = \mathbf{u}_{ij} \cdot \mathbf{n}_{ji} \tag{10}$$

$$\mathbf{u}_{ij}(t) = \int_{t_0}^{t} \mathbf{v}_{ij} \mathrm{d}t \tag{11}$$

Let us notice that $u_{ij}(t_0) = 0$, $\mathbf{v}_{ij}$ is the relative velocity between the solid S_i and the reference axis associated with S_j is compounded by a normal and tangential component:

$$\mathbf{v}_{ij} = \mathbf{v}_{n_{ij}} + \mathbf{v}_{T_{ij}} \tag{12}$$

The normal force from the solid S_j to the solid S_i is define by

$$F_{n_{ij}} = \mathbf{F}_{ji} \cdot \mathbf{n}_{ji} \tag{13}$$

the tangential contact force $\mathbf{F}_{T_{ji}}$:

$$\mathbf{F}_{T_{ji}} = \mathbf{F}_{ji} - F_{n_{ij}} \mathbf{n}_{ji} \tag{14}$$

A friction behavior law is considered here as:

$$\mathbf{F}_{T_{ji}} = \mathbf{H}(\mathbf{v}_{T_{ij}}) \tag{15}$$

As an example, the Norton–Hoff law can be formulated by:

$$\mathbf{F}_{T_{ji}} = -\alpha \|\mathbf{v}_{T_{ij}}\|^{p-1} \mathbf{v}_{T_{ij}} \tag{16}$$

The Signorini conditions are satisfied at time t_0 and t_1:

$$u_{n_{ij}} - d_0 \leq 0 \tag{17}$$

where d_0 is a distance between i and j.

$$F_{n_{ji}} \leq 0 \tag{18}$$

$$F_{n_{ji}}(u_{n_{ij}} - d_0) = 0 \tag{19}$$

Furthermore, at any time t we have the power of the normal force within the time step:

$$F_{n_{ji}} v_{n_{ij}} = 0 \tag{20}$$

where $v_{n_{ij}}$ is the normal velocity. Therefore the work is equal to zero:

$$\int_{t_0}^{t_1} F_{n_{ji}} v_{n_{ij}} \mathrm{d}t = 0 \tag{21}$$

This expression is associated to the duality of the normal force and the normal relative velocity. Therefore the virtual velocity field is chosen as compatible with these contact conditions.

We can integrate (21) by parts:

$$\Big[F_{n_{ij}}(u_{n_{ij}} - d_0)\Big]_{t_0}^{t_1} - \int_{t_0}^{t_1} \frac{\mathrm{d}F_{n_{ij}}}{\mathrm{d}t} \Big(u_{n_{ij}} - d_0\Big) \mathrm{d}t = 0 \quad . \tag{22}$$

Therefore, with the Signorini conditions for the interval $[t_0;\ t_1]$ we have:

$$\int_{t_0}^{t_1} \frac{\mathrm{d}F_{n_{ij}}}{\mathrm{d}t} \Big(u_{n_{ij}} - d_0\Big) \mathrm{d}t = 0 \tag{23}$$

4 Space and time discretization

The instantaneous updated Lagrangian description is used in the formulation. The instantaneous configuration is the configuration for any definition of tensorial variable. The new velocity field is the main variable which is iteratively calculated. At each new approximation an improved approximation of the geometric configuration is estimated [4]. A coupled space and time integration scheme is contributed as a new element.

When the motion of any point of the space domain is regular during the step of time, we can interpolate the velocity field by linear interpolation in time. The linear interpolation is chosen with regard to the fact that when a point comes into a contact the velocity becomes a discontinuous function and the acceleration is not defined. Then it suffices for most of problems to assume such a kind of interpolation. A higher degree of interpolation could be chosen if another strategy of contact solving was used by the mesh refining in time.

We must emphasize that the test examples presented in the paper allow to check the ability of the proposed modeling. However, the method close to the one used here has already been used for more complex problem, i.e. viscoplastic deformation problem [12].

The choice of the interpolation in space is defined mainly by the type of the material behavior. When the motion of a boundary point becomes non smooth since it comes into a contact during the step of time, it is necessary to divide the time interval into several steps of time. In the domain of a reference space–time element (Fig. 1) we consider a space–time interpolation function defined by a product of a spatial interpolation function and time interpolation function (Fig. 3):

$$P_{ij}(x, \tau) = N_i(x) g_j(\tau) \tag{24}$$

$$\mathbf{v}(x, \tau) = \sum P_{ij}(x, \tau) \mathbf{V}(x_i, \tau_{ij}) \tag{25}$$

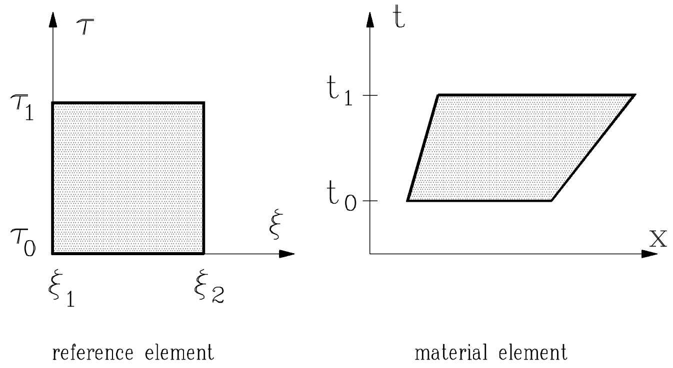

Figure 1: Space-time element

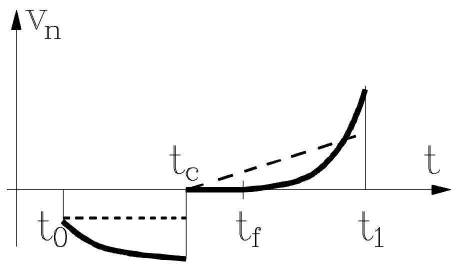

Figure 2: Variation of the normal relative velocity within a step of time and linear interpolation by pieces.

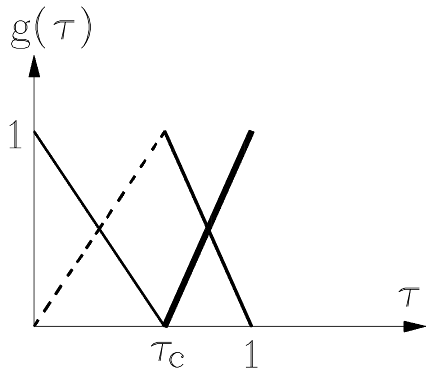

Figure 3: Time interpolation functions

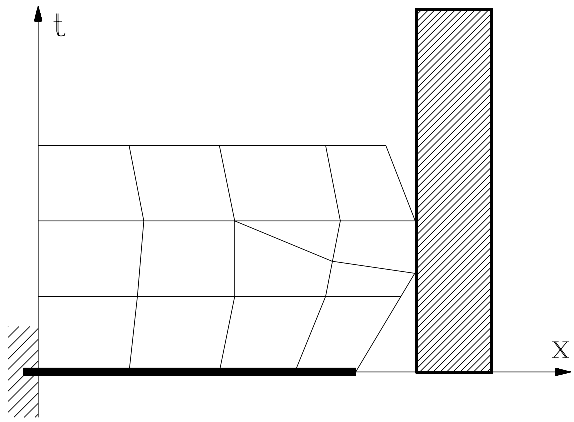

Figure 4: Space–time mesh evolution

$\tau = (t - t_0)/h$, where $h = t_1 - t_0$.

If the solid S_i passes to a contact at time t_c within the step of time h, two space and time elements are considered. The step of time is assumed to be sufficiently short. In the first element the initial value of the velocity v_0 is known. At time t_c^- it is possible to assume the same velocity or an explicit linear interpolation function from the previous step of time.

In the second time step the initial value of the velocity at time t_c^+ is zero because of the contact. The velocity is discontinue at time t_c (Fig. 2). The velocity at time t_1 is unknown. For instance in figure 4 we present a bar submitted to a dynamic elongation. In the second time step the right point of the bar passes into contact with a wall. A space–time remeshing is performed for the elements neighboring with the contact area. The figure 4 shows only the principle. The depth of the space remeshing in practice is not too large. Here, the time influence of the modification of contact status is estimated by the rate d/l (where d is the size of the spacial element and l the length of the bar).

5 Numerical results

Two examples were solved to verify the efficiency of the presented approach. In the first example the elastic bar of the length L=1, Young modulus E=1 and mass density ρ=1, hits the rigid wall with a speed v=1. The spatial mesh is composed of two elements. The elements were divided in the contact zone into trapezoidal and triangular element as it is depicted in Fig. 4. Fig. 5 presents the position in time of both ends on the bar.

The second example was calculated with the same mesh as in the previous case. However, here the end of the bar was fixed while for the frontal nodal point the initial velocity v=1 was assumed. In Fig. 6 displacements in time of two remaining points

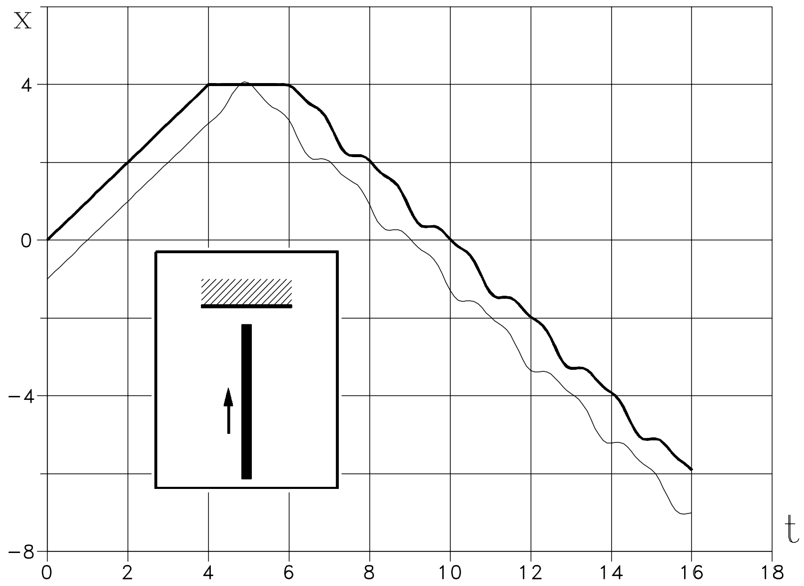

Figure 5: Collision of an elastic bar with rigid wall.

were depicted (point of the contact – thick line, inner point – thin line). A case with the unilateral restriction additionally imposed to a nodal point was compared with the case with only a point fixed.

6 Conclusions

The formulation of contact conditions is efficient considering the computational time. There are two reasons of if: the kinematic approach and the space–time interpolation which takes into account the modification of the status of a boundary point during the step of time. The way of computer calculation is also very convenient for thermo–mechanical problems.

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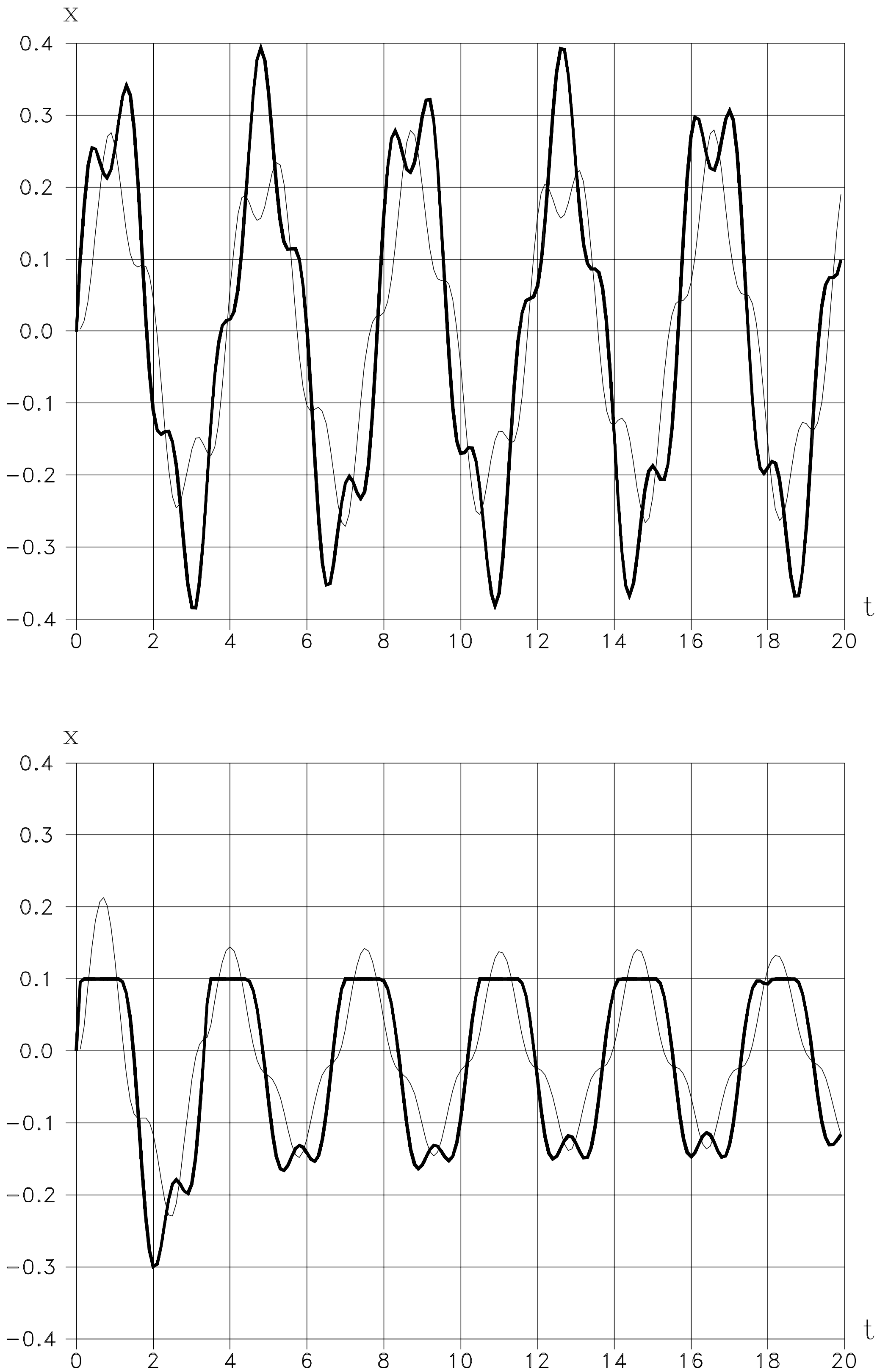

Figure 6: Vibration of a bar without contact (upper figure) and with contact (lower figure).

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