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Positive fractional 2D continuous-discrete linear systems

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Abstract. A new class of positive fractional 2D continuous-discrete linear systems is introduced. The solution to the equations describing by the new class of systems is derived. Necessary and sufficient conditions for the positivity of the fractional 2D continuous-discrete linear systems are established.

Key words: positive, fractional, linear, continuous-discrete, system, solution.

1. Introduction

Recently a dynamical development of the fractional and specially positive fractional linear systems theory has been observed. A dynamical system is called positive if and only if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. Variety of models having positive linear behavior can be found in engineering, management sciences, economics, social sciences, biology and medicine, etc.. An overview of state of the art in positive and fractional systems theory is given in monographs [1–5]. New class of 2D continuous-discrete linear systems has been introduced in [6, 7] and of positive fractional 2D hybrid linear systems in [3, 8]. Stability and robust stability of the general 2D model of a class of continuous-discrete linear systems has been analyzed in [9–11]. Stability, controllability and observability of 2D continuous-discrete linear systems have been investigated in [2, 6, 12–14]. Solvability of 2D hybrid linear systems has been addressed in [15, 16].

In this paper a new class of positive fractional 2D continuous-discrete linear systems is introduced. The solution to the state equations describing the systems is derived and the necessary and sufficient conditions for the positivity of the system is established.

The paper is organized as follows. In Sec. 2 the solution to the equations describing the new class of fractional 2D continuous-discrete linear systems is derived. Necessary and sufficient conditions for the positivity of the fractional system are established in Sec. 3. Concluding remarks are given in Sec. 4.

The following notation is used in this paper. The set of real $n\times m$ matrices is denoted by $\Re^{n\times m}$ and the set of $n\times m$ real matrices with nonnegative entries will be denoted $\Re_+^{n\times m}$ ($\Re_+^n = \Re_+^{n\times 1}$). The $n\times n$ identity matrix will be denoted by I_n.

2. Model and its solution

In this paper the following Caputo definition of the fractional derivative for $0<\alpha<1$ is used [3]

$$ {}_0D_t^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\dot f(\tau)}{(t-\tau)^\alpha}d\tau, \qquad \dot f(\tau) = \frac{df(\tau)}{d\tau}, \tag{1}$$

where

$$\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt, \qquad \mathrm{Re}(x) > 0 \tag{2}$$

is the Euler gamma function.

Consider a class of fractional 2D continuous-discrete linear systems described by the equations

$$\frac{d^\alpha x(t,i+1)}{dt^\alpha} = A_0x(t,i) + A_1\frac{d^\alpha x(t,i)}{dt^\alpha} + A_2x(t,i+1) + Bu(t,i), \qquad 0<\alpha<1, \tag{3a}$$

$$y(t,i) = Cx(t,i) + Du(t,i), \qquad t\in\Re, \quad i\in Z_+ = \{0,1,...\}, \tag{3b}$$

where $x(t,i)\in\Re^n$, $u(t,i)\in\Re^m$, $y(t,i)\in\Re^p$ are the state, input and output vectors and $A_k\in\Re^{n\times n}$, $k=0,1,2$; $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$, $D\in\Re^{p\times m}$.

Boundary conditions for (3) have the form

$$x(t,0)\in\Re^n, \quad t\in\Re, \quad x(0,i)\in\Re^n, \quad i\in Z_+. \tag{4}$$

It is assumed that the continuous variable t (time) and the discrete variable i are independent. Using the Laplace transform and z-transform we shall derive the solution $x(t,i)\in\Re^n$ of (3) satisfying the boundary conditions (4).

Let $X(s)$ and $X(z)$ be the Laplace transform and z-transform of $x(t)$ and $x(i)$, respectively defined by

$$X(s) = \mathcal{L}[x(t)] = \int_0^\infty x(t)e^{-st}dt, \tag{5a}$$

$$X(z) = \mathcal{Z}[x(i)] = \sum_{i=0}^\infty x(i)z^{-i} \tag{5b}$$

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and

$$X(s,z)=\mathcal{Z}\left[\mathcal{L}[x(t,i)]\right], \quad U(s,z)=\mathcal{Z}\left[\mathcal{L}[u(t,i)]\right]. \tag{5c}$$

Applying the Laplace transform and z-transform to the Eq. (3) and taking into account that [3]

$$\mathcal{L}\left[\frac{d^\alpha x(t,i)}{dt^\alpha}\right]=s^\alpha X(s,i)-s^{\alpha-1}x(0,i), \quad 0<\alpha<1, \tag{6}$$

$$\mathcal{Z}\left[x(t,i+1)\right]=zX(s,i)-zx(t,0) \tag{7}$$

we obtain

$$\begin{gathered}[I_n s^\alpha z-A_0-A_1 s^\alpha-A_2 z]X(s,z)=\\ =[I_n s^\alpha z-A_2 z]X(s,0)+[I_n s^{\alpha-1}z-A_1 s^{\alpha-1}]X(0,z)-\\ -I_n s^{\alpha-1}zx(0,0)+BU(s,z),\end{gathered} \tag{8}$$

or

$$\begin{gathered}[I_n-A_0 s^{-\alpha}z^{-1}-A_1 z^{-1}-A_2 s^{-\alpha}]X(s,z)=\\ =[I_n-A_2 s^{-\alpha}]X(s,0)+\\ +[I_n s^{-1}-A_1 s^{-1}z^{-1}]X(0,z)-I_n s^{-1}x(0,0)+\\ +Bs^{-\alpha}z^{-1}U(s,z).\end{gathered} \tag{9}$$

From (9) we have

$$\begin{gathered}X(s,z)=[I_n-A_0 s^{-\alpha}z^{-1}-A_1 z^{-1}-A_2 s^{-\alpha}]^{-1}\cdot\\ \cdot\{[I_n-A_2 s^{-\alpha}]X(s,0)+\\ +[I_n s^{-1}-A_1 s^{-1}z^{-1}]X(0,z)-\\ -I_n s^{-1}x(0,0)+Bs^{-\alpha}z^{-1}U(s,z)\}.\end{gathered} \tag{10}$$

Let

$$\begin{gathered}[I_n-A_0 s^{-\alpha}z^{-1}-A_1 z^{-1}-A_2 s^{-\alpha}]^{-1}=\\ =\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}T_{k,l}s^{-k\alpha}z^{-l}.\end{gathered} \tag{11}$$

Taking into account that

$$\begin{gathered}[I_n-A_0 s^{-\alpha}z^{-1}-A_1 z^{-1}-A_2 s^{-\alpha}]\cdot\\ \cdot\left(\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}T_{k,l}s^{-k\alpha}z^{-l}\right)=\left(\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}T_{k,l}s^{-k\alpha}z^{-l}\right)\cdot\\ \cdot[I_n-A_0 s^{-\alpha}z^{-1}-A_1 z^{-1}-A_2 s^{-\alpha}]=I_n\end{gathered} \tag{12}$$

and comparing the coefficients at the same power of $s^{-\alpha}$ and z^{-1} we obtain

$$T_{k,l}=\begin{cases} I_n \quad \text{(identity matrix) for } k=l=0,\\ A_0T_{k-1,l-1}+A_1T_{k,l-1}+A_2T_{k-1,l}=\\ \quad =T_{k-1,l-1}A_0+T_{k,l-1}A_1+T_{k-1,l}A_2\\ \quad \text{for } k,l\in Z_+, k+l>0,\\ 0 \quad \text{(zero matrix) for } k<0 \text{ and / or } l<0.\end{cases} \tag{13}$$

Substitution of (11) into (10) yields

$$\begin{gathered}X(s,z)=\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}T_{k,l}s^{-k\alpha}z^{-l}\{[I_n-A_2 s^{-\alpha}]X(s,0)+\\ +[I_n s^{-1}-A_1 s^{-1}z^{-1}]X(0,z)-\\ -I_n s^{-1}x(0,0)+Bs^{-\alpha}z^{-1}U(s,z)\}.\end{gathered} \tag{14}$$

Applying the inverse Laplace transform L^{-1}, the inverse z-transform Z^{-1} and the convolution theorem to (14) we obtain

$$\begin{gathered}x(t,i)=\mathcal{Z}^{-1}\left[\mathcal{L}^{-1}[X(s,z)]\right]=\\ =T_{0,i}x(t,0)+\sum_{k=1}^{\infty}\frac{T_{k,i}}{\Gamma(k\alpha)}\int_0^t(t-\tau)^{k\alpha-1}x(\tau,0)d\tau-\\ -\sum_{k=0}^{\infty}\frac{T_{k,i}A_2}{\Gamma[(k+1)\alpha]}\int_0^t(t-\tau)^{(k+1)\alpha-1}x(\tau,0)d\tau+\\ +\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{k,i-l}t^{k\alpha}}{\Gamma(k\alpha+1)}x(0,l)\\ -\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{k,i-l-1}A_1t^{k\alpha}}{\Gamma(k\alpha+1)}x(0,l)-\\ -\sum_{k=0}^{\infty}\frac{T_{k,i}t^{k\alpha}}{\Gamma(k\alpha+1)}x(0,0)+\\ +\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{k,i-l-1}B}{\Gamma[(k+1)\alpha]}\int_0^t(t-\tau)^{(k+1)\alpha-1}u(\tau,l)d\tau,\end{gathered} \tag{15a}$$

or

$$\begin{gathered}x(t,i)=\mathcal{Z}^{-1}\left[\mathcal{L}^{-1}[X(s,z)]\right]=\\ =T_{0,i}x(t,0)+\sum_{k=1}^{\infty}\frac{T_{k,i}}{\Gamma(k\alpha)}\int_0^t(t-\tau)^{k\alpha-1}x(\tau,0)d\tau-\\ -\sum_{k=0}^{\infty}\frac{T_{k,i}A_2}{\Gamma[(k+1)\alpha]}\int_0^t(t-\tau)^{(k+1)\alpha-1}x(\tau,0)d\tau+\\ +\sum_{k=0}^{\infty}\sum_{l=1}^{\infty}\frac{T_{k,i-l}t^{k\alpha}}{\Gamma(k\alpha+1)}x(0,l)-\\ -\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{k,i-l-1}A_1t^{k\alpha}}{\Gamma(k\alpha+1)}x(0,l)+\\ +\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{T_{k,i-l-1}B}{\Gamma[(k+1)\alpha]}\int_0^t(t-\tau)^{(k+1)\alpha-1}u(\tau,l)d\tau,\end{gathered} \tag{15b}$$

since [8]

$$\mathcal{L}[t^\alpha]=\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}.$$

Therefore, the following theorem has been proved.

Theorem 1. The solution of the Eq. (3a) with boundary conditions (4) has the form (2.15), where the matrices $T_{k,l}$ are defined by (13).

Example 1. Compute the solution $x(t,i)$ of the Eq. (3a) for $\alpha=0.5$ with the matrices

$$\begin{gathered}A_0=\begin{bmatrix}0&0\\0&0\end{bmatrix}, \qquad A_1=\begin{bmatrix}0&1\\0&0\end{bmatrix},\\ A_2=\begin{bmatrix}0&0\\1&0\end{bmatrix}, \qquad B=\begin{bmatrix}1\\1\end{bmatrix}\end{gathered} \tag{16}$$

the input

$$u(t,i) = 1 \quad \text{for} \quad t \geq 0, \quad i \in Z_+ \tag{17}$$

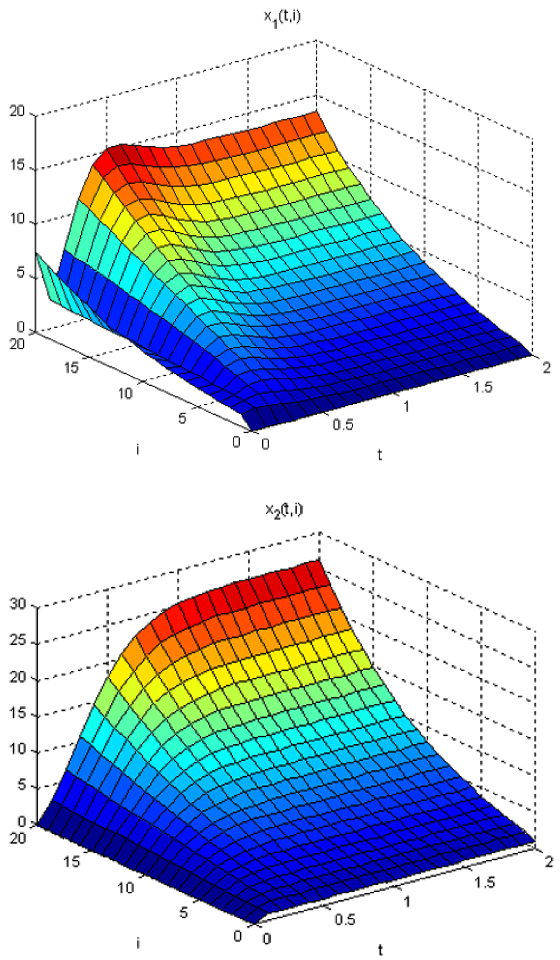
and the boundary conditions

$$x(t,0) = \begin{bmatrix} e^t \\ 0 \end{bmatrix}, \qquad x(0,i) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad i \in Z_+. \tag{18}$$

Using (16) and (13) we obtain

$$T_{k,l} = \begin{cases} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ \text{for } k=1,\ l=0 \text{ and } k=l+1,\ l=1,2,... \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ \text{for } k=0,\ l=1 \text{ and } l=k+1,\ k=1,2,... \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{for} \quad k=l=0,1,2,... \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{otherswise} \end{cases} \tag{19}$$

a)



b)

Fig. 1. State variables of the system from Example 1

Substituting (19), (17) and (18) into (15) we obtain desired solution $x(t,i)$

$$x(t,i) = \begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix} =$$
$$= \left(1 - \frac{t^{i\alpha}}{\Gamma(i\alpha+1)} + \sum_{l=0}^{i-1} \frac{t^{(i-l+1)\alpha}}{\Gamma[(i-l+1)\alpha+1]}\right) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$+ \left(\sum_{l=0}^{i-2} \frac{t^{(i-l-1)\alpha}}{\Gamma[(i-l-1)\alpha+1]} - \frac{t^{(i-1)\alpha}}{\Gamma[(i-1)\alpha+1]} + \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$+ \sum_{l=0}^{i-1} \frac{t^{(i-l)\alpha}}{\Gamma[(i-l)\alpha+1]} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{20}$$

The plots of the state variables (20) are shown in Fig. 1.

3. Positivity of the model

Let $C_{nd}(t)$ be the set of non-decreasing continuous function $f(t)$ satisfying the condition

$$\begin{gathered} f(t) \in C_{nd}(t) \quad \text{if and only if} \\ f(t_1) \geq f(t_2) \quad \text{for all} \\ t_1 \geq t_2, \quad t_1, t_2 \in \Re_+ \end{gathered} \tag{21}$$

Lemma 1. Let $f(t) \in C_{nd}(t)$ and there exist $\frac{d^\alpha x(t,i)}{dt^\alpha}$ for $0 < \alpha < 1$. Then

$$\frac{d^\alpha x(t,i)}{dt^\alpha} \geq 0 \quad \text{for} \quad t \geq 0. \tag{22}$$

Proof. It is well-known that the convolution

$$\int_0^t f_1(t-\tau) f_2(\tau) d\tau$$

of two nonnegative continuous function $f_1(t)$, $f_2(\tau)$ is also nonnegative continuous function. Taking into account this and using (1) we obtain

$$\frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} \dot{f}(\tau) d\tau \geq 0 \quad \text{for} \quad t \geq 0 \tag{23}$$

since $\Gamma(1-\alpha) > 0$, $(t-\tau)^{-\alpha} \geq 0$ for $0 < \alpha < 1$, $0 \leq \tau \leq 1$ and $\dot{f}(t) \geq 0$ for $f(t) \in C_{nd}(t)$. □

It is assumed that $x(t,0)$ in (4) is non-decreasing function of t, i.e.

$$x(t,0) \in C_{nd}^n(t) \quad \text{for} \quad t \geq 0. \tag{24}$$

Definition 1. The system (3) is called (internally) positive if for any boundary conditions

$$\begin{gathered} x(t,0) \in \Re_+^n, \quad x(t,0) \in C_{nd}^n(t) \\ \text{and} \quad x(0,i) \in \Re_+^n, \quad i \in Z_+ \end{gathered} \tag{25}$$

and all inputs vectors $u(t,i) \in R_+^m$, $t \geq 0$, $i \in Z_+$ the state and output vectors satisfy the condition

$$\begin{gathered} x(t,i) \in \Re_+^n, \quad y(t,i) \in \Re_+^p \\ \text{for} \quad t \geq 0, \quad i \in Z_+. \end{gathered} \tag{26}$$

A matrix $A \in \Re^{n\times n}$ is called a Metzler matrix if all its off-diagonal entries are nonnegative.

Theorem 2. The system (3) is positive if and only if:
i) $A_0, A_1 \in \Re_+^{n\times n}$, $A = A_0 + A_1 A_2 \in \Re_+^{n\times n}$, $B \in \Re_+^{n\times m}$, $C \in \Re_+^{p\times n}$, $D \in \Re_+^{p\times m}$,
ii) A_2 is a Metzler matrix.

Proof. The proof will be accomplished by induction. The equation (3a) may be written in the form

$$\frac{d^\alpha x(t,i+1)}{dt^\alpha} = A_2 x(t,i+1) + F(t,i), \quad t \geq 0, \quad i \in Z_+, \tag{27}$$

where

$$F(t,i) = A_0 x(t,i) + A_1 \frac{d^\alpha x(t,i)}{dt^\alpha} + Bu(t,i). \tag{28}$$

From (27) and (28) for $i = 0$ we have

$$\frac{d^\alpha x(t,1)}{dt^\alpha} = A_2 x(t,1) + F(t,0), \tag{29}$$

where

$$F(t,0) = A_0 x(t,0) + A_1 \frac{d^\alpha x(t,0)}{dt^\alpha} + Bu(t,0) \tag{30}$$

is known for given boundary conditions (25).

From (30) it follows that $F(t,0) \in \Re_+^n$ if (25) and $A_0, A_1 \in \Re_+^{n\times n}$, $B \in \Re_+^{n\times m}$ holds. The solution of (29) is given by [3]

$$x(t,1) = \Phi_0(t)x(0,1) + \int_0^t \Phi(t-\tau)F(\tau,0)d\tau \tag{31}$$

where

$$\Phi_0(t) = \sum_{k=0}^{\infty} \frac{A_2^k t^{k\alpha}}{\Gamma[k\alpha+1]}, \quad \Phi(t) = \sum_{k=0}^{\infty} \frac{A_2^k t^{(k+1)\alpha-1}}{\Gamma[(k+1)\alpha]}. \tag{32}$$

It is well-known [8] that

$$\Phi_0(t), \Phi(t) \in \Re_+^{n\times n} \quad \text{for} \quad t \geq 0 \tag{33}$$

if and only if A_2 is a Metzler matrix, i.e. the condition ii) is met. From (3b) for $i = 1$ we have $y(t,1) = Cx(t,1) + Du(t,1)$ for $t \geq 0$ if and only if $C \in \Re_+^{p\times n}$, $D \in \Re_+^{p\times m}$ and $u(t,1) \in R_+^m$, $t \geq 0$. Using (28) for $i = 1$ and (29) we obtain

$$\begin{aligned} F(t,1) &= A_0 x(t,1) + A_1 \frac{d^\alpha x(t,1)}{dt^\alpha} + Bu(t,1) = \\ &= (A_0 + A_1 A_2)x(t,1) + A_1 F(t,0) + Bu(t,1) = \\ &= Ax(t,1) + A_1 F(t,0) + Bu(t,1) \in \Re_+^{n\times n}, \\ &\quad t \geq 0 \end{aligned} \tag{34}$$

since $A \in \Re_+^{n\times n}$, $A_1 F(t,0) \in \Re_+^n$ and $Bu(t,1) \in \Re_+^n$, $t \geq 0$. In a similar way as in [3] by induction it can be shown that $x(t,i) \in \Re_+^n$, $F(t,i-1) \in \Re_+^n$ for $t \geq 0$ and $i \geq 1$ if and only if the condition i) and ii) are satisfied. □

Remark 1. The considerations can be easily extended to positive fractional 2D continuous-discrete linear systems described by the equations

$$\begin{aligned} \frac{d^\alpha x(t,i+1)}{dt^\alpha} &= A_0 x(t,i) + A_1 \frac{d^\alpha x(t,i)}{dt^\alpha} + \\ &+ A_2 x(t,i+1) + B_0 u(t,i) + \\ &+ B_1 \frac{d^\beta u(t,i)}{dt^\beta} + B_2 u(t,i+1), \\ &0 < \alpha < 1, \quad 0 < \beta < 1, \end{aligned} \tag{35a}$$

$$y(t,i) = Cx(t,i) + Du(t,i), \quad t \in \Re, \quad i \in Z_+ = \{0,1,...\}. \tag{35b}$$

In is easy to show that the fractional system is positive if and only if the conditions of Theorem 2 are met and $B_k \in \Re_+^{n\times m}$, $k = 1, 2$.

4. Concluding remarks

A new class of positive fractional 2D continuous-discrete linear systems described by Eqs. (3) has been introduced. Using the Laplace transform and z-transform the solution (15) to Eq. (3a) with the boundary conditions (4) has been derived (Theorem 1). Computation of the solution has been illustrated by a numerical example. Necessary and sufficient conditions for the positivity of the fractional 2D continuous-discrete linear systems have been established (Theorem 2). The considerations can be extended to positive fractional 2D continuous-discrete linear systems where the fractional difference with respect to discrete variable will be also used.

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