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Reflection and transmission of plane waves between two different fluid saturated porous half spaces

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Abstract. The present study is concerned with the reflection and transmission of plane waves between two different fluid saturated porous half spaces when longitudinal and transversal waves impinge obliquely at the interface. Amplitude ratios of various reflected and transmitted waves are obtained .The variations of amplitude ratios with angle of incidence are depicted graphically. A particular case of reflection at the free surface in fluid saturated porous half spaces has been deduced and discussed. A special case of interest has also been deduced from the present investigation.

Key words: porous, amplitude ratios, reflection, transmission, longitudinal waves.

1. Introduction

The dynamic response of porous media is of great interest in various areas such as geophysics, soil-mechanics, civil engineering, petroleum engineering and environmental engineering. As most of the modem engineering structures are generally made up of multiphase porous continuum, the classical theory, which represents a fluid saturated porous medium as a single phase material, is inadequate to represent the mechanical behavior of such materials especially when the pores are filled with liquid. In this context the solid and liquid phases have different motions. Due to these different motions and different material properties and the complicated geometry of pore structures; the mechanical behavior of a fluid saturated porous medium is very complex and difficult. So from time to time, researchers have tried to overcome this difficulty and considerable work has been done in this regard.

Based on the work of von Terzaghi [1, 2], Biot [3] proposed a general theory of three-dimensional consolidation. Taking the compressibility of the soil into consideration, the water contained in the pores was taken to be incompressible. Biot [4, 5] developed the theory for the propagation of stress waves in porous elastic solids containing a compressible viscous fluid and demonstrated the existence of two types of compressional waves (a fast and a slow wave) along with one shear wave. The Biot's model was broadly accepted and some of his results have been taken as standard references and the basis for subsequent analysis in acoustic, geophysics and other such fields.

Based on the work of Fillunger model [6] (which is further based on the concept of volume fractions combined with surface porosity coefficients), Bowen [7] and de Boer and Ehlers [8, 9] developed and used another interesting theory in which all the constituents of a porous medium are assumed to be incompressible. There are reasonable grounds for the assumption that the constituents of many fluid saturated porous media are incompressible. For example, taking the composition of soil, the solid constituents are incompressible and liquid constituents, which are generally water or oils are also incompressible. Moreover in an empty porous solid as a case of classical theory, the change in the volume is due to the changes in porosity during the propagation of longitudinal waves. The assumption of incompressible constituents does not only meet the properties appearing in many branches of engineering practice, but it also avoids the introduction of many complicated material parameters as considered in the Biot theory. So this model meets the requirements of further scientific developments. Based on this theory de Boer and Ehlers [10] and Recently, Kumar and Hundal [11–15] studied some problems of wave propagation in fluid saturated incompressible porous media.

In the study of propagation of seismic waves in liquid saturated porous solids, problem of reflection of seismic waves is useful both for its seismological interest and geophysical exploration such as hydrocarbons prospecting, ground water prospecting or mineral prospecting. Employing Biot's theory, reflection of waves at free permeable boundary was studied by Deresiewicz [16] and Malla Reddy and Tajuddin [17]. Expression for reflection coefficients were obtained but neither numerical work was carried out nor discussed. Kumar and Deswal [18] studied the reflection of waves in micro-polar liquid-saturated porous solid for a free boundary and calculated only reflection coefficients. Tomar and singh [19] studied the problem of transmission of longitudinal waves through a plane interface between two dissimilar porous elastic solid half spaces. Tomar and Arora [20] investigated the reflection and transmission of elastic waves at an elastic / porous solid saturated by two immiscible fluids. Tajuddin and Hussaini [21] studied the reflection of plane waves at boundaries of a liquid filled poroelastic half- space. However, no attempt

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has been made to study the problem of reflection and transmission of plane waves at the interface of two fluid saturated incompressible porous media.

In the present investigation, we studied the reflection and transmission of plane waves at the interface between two fluid saturated incompressible porous half spaces. The reflection coefficients of reflected waves at the free surface have also been obtained. We also obtained the components of stress in two fluid saturated incompressible porous half spaces with incidence angle of longitudinal and transversal waves.

2. Basic equations

Following de Boer and Ehlers [9] the equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are

$$\nabla.(\eta^S \dot{\boldsymbol{u}}_S + \eta^F \dot{\boldsymbol{u}}_F) = 0, \tag{1}$$

$$\begin{aligned}(\lambda^S + \mu^S)\nabla(\nabla.\boldsymbol{u}_S) + \mu^S \nabla^2 \boldsymbol{u}_S - \\ - \eta^S \nabla p - \rho^S \ddot{\boldsymbol{u}}_S + s_V(\dot{\boldsymbol{u}}_F - \dot{\boldsymbol{u}}_S) = 0,\end{aligned} \tag{2}$$

$$\eta^F \nabla p + \rho^F \ddot{\boldsymbol{u}}_F + s_V(\dot{\boldsymbol{u}}_F - \dot{\boldsymbol{u}}_S) = 0, \tag{3}$$

$$\boldsymbol{T_E^S} = 2\mu^S \boldsymbol{E_S} + \lambda^S \boldsymbol{E_S}.\boldsymbol{I})\mathbf{I}, \tag{4}$$

$$\boldsymbol{E_S} = \frac{1}{2}(\text{grad}\boldsymbol{u}_S + \text{grad}^T \boldsymbol{u}_S), \tag{5}$$

where $\boldsymbol{u}_i$, $\dot{\boldsymbol{u}}_i$, $\ddot{\boldsymbol{u}}_i$, $i = F, S$ denote the displacement, velocities and acceleration of fluid and solid phases respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^s and ρ^F are the densities of the solid and fluid respectively. $\boldsymbol{T_E^S}$ is the stress in the solid phase and $\boldsymbol{E_S}$ is the linearized langrangian strain tensor. λ^s and μ^s are the macroscopic Lame's parameters of the porous solid and η^s and η^F are the volume fractions satisfying $\eta^s + \eta^F = 1$.

The case of isotropic permeability, the tensor $\boldsymbol{S}_V$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers [9] as

$$\boldsymbol{S}_V = \frac{(\eta^F)^2 \gamma^{FR}}{K^F}\mathbf{I},$$

where γ^{FR} is the effective specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

3. Formulation of the problem

We consider two fluid saturated incompressible porous half spaces being in contact with each other at the plane surface which we designate as plane $z = 0$ of rectangular Cartesian coordinate system OXYZ. The Z-axis is taken downward pointing into the medium. For two dimensional problem,we assume the displacement vector $\boldsymbol{u}_i$ $(i = F, S)$ as

$$\boldsymbol{u}_i = (u^i, 0, w^i) \text{ where } i = F, S. \tag{6}$$

Using Eqs. (6) in Eqs. (1)–(3) we obtain the following equations for fluid saturated incompressible porous medium as:

$$\begin{aligned}(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \\ - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_V \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0,\end{aligned} \tag{7}$$

$$\begin{aligned}(\lambda^S + \mu^S)\frac{\partial \theta^s}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \\ - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_V \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^s}{\partial t}\right] = 0,\end{aligned} \tag{8}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_V \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{9}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_V \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^s}{\partial t}\right] = 0, \tag{10}$$

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial t}\right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial t}\right] = 0, \tag{11}$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z}.$$

We define the dimensionless quantities defined as:

$$x' = \frac{\omega^*}{C_1}x, \qquad z' = \frac{\omega^*}{C_1}z, \qquad t' = \omega^* t,$$

$$u'^S = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}u^S,$$

$$w'^s = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}w^S,$$

$$u'^F = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}u^F,$$

$$w'^F = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega *}{C_1}w^F,$$

$$p' = \frac{p}{E}, \qquad T'_{31} = \frac{T_{31}}{E}, \qquad T'_{33} = \frac{T_{33}}{E}. \tag{12}$$

In these relations E is the Young's modulus of the solid phase, ω^* is a constant having the dimensions of frequency, C_1 is the velocity of a longitudinal wave propagating in a fluid saturated incompressible porous medium and is given by

$$C_1 = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}}. \tag{13}$$

If pore is absent or gas is filled in the pores then ρ^F is very small as compare to ρ^S and can be neglected so the relation reduce to

$$C_0 = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \tag{14}$$

This gives the velocity of the longitudinal wave propagating in an incompressible empty porous solid where the change in volume is due to the change in porosity and well known result of the classical theory of elasticity. In an incompressible non porous solid $\eta^F \to 0$, then (13) becomes $C_1 = 0$ and physically acceptable as longitudinal wave cannot propagate in an incompressible medium.

Making use of non dimensional quantities given by (12) in Eqs. (7)–(11) we obtain the following equations:

$$(1-\delta^2)\frac{\partial\theta^S}{\partial x}+\delta^2\nabla^2u^S-\eta^S\frac{\partial p}{\partial x}-\delta_1^2\frac{\partial^2u^S}{\partial t^2}+ + \delta_2\left(\frac{\partial u^F}{\partial t}-\frac{\partial u^S}{\partial t}\right)=0, \tag{15}$$

$$(1-\delta^2)\frac{\partial\theta^S}{\partial z}+\delta^2\nabla^2w^S-\eta^S\frac{\partial p}{\partial z}-\delta_1^2\frac{\partial^2w^S}{\partial t^2}+ + \delta_2\left(\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right)=0, \tag{16}$$

$$\eta^F\frac{\partial p}{\partial x}+\frac{\rho^F}{\rho^S}\delta_1^2\frac{\partial^2u^F}{\partial t^2}+\delta_2\left(\frac{\partial u^F}{\partial t}-\frac{\partial u^S}{\partial t}\right)=0, \tag{17}$$

$$\eta^F\frac{\partial p}{\partial z}+\frac{\rho^F}{\rho^S}\delta_1^2\frac{\partial^2w^F}{\partial t^2}+\delta_2\left(\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right)=0, \tag{18}$$

$$\eta^S\left(\frac{\partial^2u^S}{\partial x\partial t}+\frac{\partial^2w^S}{\partial z\partial t}\right)+\eta^F\left(\frac{\partial^2u^F}{\partial x\partial t}+\frac{\partial^2w^F}{\partial z\partial t}\right)=0, \tag{19}$$

where

$$\delta_1=\frac{C_1}{C_O},\qquad \delta=\frac{\beta_0}{C_O},\qquad \beta_O=\sqrt{\frac{\mu^S}{\rho^S}},$$

$$\delta_2=\frac{S_VC_1^2}{w^*\rho^SC_O^2}.$$

The displacement components u^i and w^i are related to the non dimensional potential ϕ^i and ψ^i as

$$u^i=\frac{\partial\phi^i}{\partial x}+\frac{\partial\psi^i}{\partial z}, \qquad w^i=\frac{\partial\phi^i}{\partial z}-\frac{\partial\psi^i}{\partial x},\qquad i=F,S. \tag{20}$$

With the help of (20) we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F, p as:

$$\nabla^2\phi^S-\frac{\partial^2\phi^S}{\partial t^2}-\frac{\delta_2}{(\eta^F)^2}\frac{\partial\phi^S}{\partial t}=0, \tag{21}$$

$$\phi^F=-\frac{\eta^S}{\eta^F}\ \phi^S, \tag{22}$$

$$\delta^2\nabla^2\psi^S-\delta_1^2\frac{\partial^2\psi^S}{\partial t^2}+\delta_2\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \tag{23}$$

$$\delta_1^2\frac{\rho^F}{\rho^S}\frac{\partial^2\psi^F}{\partial t^2}+\delta_2\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \tag{24}$$

$$(\eta^F)^2p-\eta^S\delta_1^2\frac{\rho^F}{\rho^S}\frac{\partial^2\phi^S}{\partial t^2}-\delta_2\frac{\partial\phi^S}{\partial t}=0, \tag{25}$$

4. Reflection and transmission of the waves

We consider a plane wave propagating through a medium M_1 which we designate as the region $Z>0$ and incident at the plane $z=0$ and making an angle θ_0 with normal to the surface. Corresponding to each incident wave (longitudinal/transversal wave) we get two reflected waves in the medium M_1 and two transmitted waves in medium M_2. We write all the variables without bar in the region $Z>0$ (medium M_1) and attach a bar to denote the variables in the region $Z<0$ (medium M_2) as shown in Fig. 1.

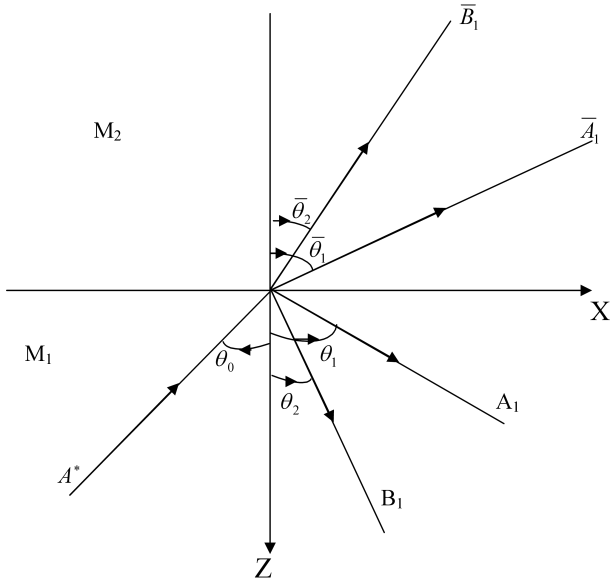

Fig. 1. Geometry of the problem

We assume the solution of the system of Eqs. (21)–(25) in the form

$$(\phi^S,\phi^F,\psi^S,\psi^F,p)=(\phi_1^S,\phi_1^F,\psi_1^S,\psi_1^F,p_1) \exp[i\,k(x\sin\theta-z\cos\theta)-i\omega t], \tag{26}$$

where k is the wave number and ω is the complex circular frequency.

Making the use of (26) in Eqs. (21)–(25) we obtain two quadric equations in V given by

$$V^2+AV+B=0, \tag{27}$$

$$V^2+CV+D=0, \tag{28}$$

where

$$A=\frac{i\,\delta_2}{k\,(\eta^F)^2},\qquad B=-1,$$

$$C=\frac{\delta_2}{k\delta_1^2}\left[i+\frac{\delta_2\rho^S}{\delta_1^2\rho^F\omega+i\delta_2\rho^S}\right],$$

$$D=-\frac{\delta^2}{\delta_1^2},$$

where $V=\dfrac{\omega}{k}$ is the velocity of the wave.

The velocities of the longitudinal waves are the roots of Eq. (27) are:

$$V_1=\frac{-A\pm\sqrt{A^2-4B}}{2}$$

and the velocities of the transversal waves are the roots of Eq. (28) are:

$$V_2=\frac{-C\pm\sqrt{C^2-4D}}{2},$$

where the upper sign correspond to the incident wave and lower sign correspond to the reflected wave.

5. Boundary conditions

Boundary conditions at the interface $z = 0$ are

$$(T_{33}^S - p)_{M_1} = (T_{33}^S - p)_{M_2}, \quad (29)$$

i.e. total normal stress of medium M_1 is equal to total normal stress of medium M_2.

$$(T_{31}^S)_{M_1} = (T_{31}^S)_{M_2}, \quad (30)$$

i.e. total tangential stress of medium M_1 is equal to total normal stress of medium M_2.

$$(u^S)_{M_1} = (u^S)_{M_2}, \quad (31)$$

$$(w^S)_{M_1} = (w^S)_{M_2}, \quad (32)$$

In view of (26) we assume the values of ϕ^S, ϕ^F, ψ^S, ψ^F and p satisfying the boundary conditions for medium M_1 and M_2 as:

Medium M_1:

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\} [A_{01} \exp\{ik_1(x\sin\theta_0 - z\cos\theta_0) - i\omega_1 t\} + A_1 \exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) - i\omega_1 t\}], \quad (33)$$

$$\{\psi^S, \psi^F\} = \{1, m_3\} [B_{01} \exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) - i\omega_2 t\} + B_1 \exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) - i\omega_2 t\}]. \quad (34)$$

Medium M_2:

$$\{\overline{\phi}^S, \overline{\phi}^F, \overline{p}\} = \{1, \overline{m}_1, \overline{m}_2\} [\overline{A}_1 \exp\{i\overline{k}_1(x\sin\overline{\theta}_1 - z\cos\overline{\theta}_1) - i\overline{\omega}_1 t\}], \quad (35)$$

$$\{\psi^S, \psi^F\} = \{1, \overline{m}_3\} [\overline{B}_1 \exp\{i\overline{k}_2(x\sin\overline{\theta}_2 - z\cos\overline{\theta}_2) - i\overline{\omega}_2 t\}], \quad (36)$$

where

$$m_1 = -\frac{\eta^S}{\eta^F},$$

$$m_2 = -\left[\frac{\eta^S\delta_1^2\rho^F\omega^2 + i\omega\delta_2\rho^S}{(\eta^F)^2\rho^S}\right],$$

$$m_3 = \frac{i\delta_2\rho^S}{\delta_1^2\rho^F\omega + i\delta_2\rho^S},$$

$$\overline{m}_1 = -\frac{\overline{\eta}^S}{\overline{\eta}^F},$$

$$\overline{m}_2 = -\left[\frac{\overline{\eta}^S\overline{\delta}_1^2\overline{\rho}^F\omega^2 + i\omega\delta_2\overline{\rho}^S}{(\ \overline{\eta}^F\ )^2\,\overline{\rho}^S}\right],$$

$$\overline{m}_3 = \frac{i\overline{\delta}_2\overline{\rho}^S}{\overline{\delta}_1^2\overline{\rho}^F\omega + i\overline{\delta}_2\overline{\rho}^S}.$$

and A_{01}, B_{01} are amplitudes of the incident longitudinal and transversal waves respectively. A_1, B_1 and $\overline{A}_1$, $\overline{B}_1$ are amplitudes of the corresponding reflected and transmitted waves respectively.

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\overline{\theta}_1}{\overline{V}_1} = \frac{\sin\overline{\theta}_2}{\overline{V}_2}, \quad (37)$$

where

$$k_1V_1 = k_2V_2 = \overline{k}_1\overline{V}_1 = \overline{k}_2\overline{V}_2 = \omega \text{ at } z = 0. \quad (38)$$

For longitudinal wave,

$$V_0 = V_1, \qquad \theta_0 = \theta_1. \quad (39)$$

For transversal wave,

$$V_0 = V_2, \qquad \theta_0 = \theta_2. \quad (40)$$

Making the use of potentials given by Eqs. (33)–(36) in boundary conditions (29)–(32) and using the Eqs. (37)–(40), we get a system of four non- homogenous equations which can be written as

$$\sum_{i=1}^{4} a_{ij}Z_j = Y_i, \quad (j = 1, 2, 3, 4), \quad (41)$$

where

$$a_{11} = -(r_1\sin^2\theta_1 + \cos^2\theta_1)k_1^2 - m_2,$$
$$a_{12} = -(r_1 - 1)(\sin\theta_2\cos\theta_2)k_2^2,$$
$$a_{13} = (r_3\sin^2\overline{\theta}_1 + r_4\cos^2\overline{\theta}_1)\overline{k}_1^2 + \overline{m}_2,$$
$$a_{14} = (r_4 - r_3)(\sin\overline{\theta}_2\cos\overline{\theta}_2)\overline{k}_2^2,$$
$$a_{21} = (-2r_2\sin\theta_1\cos\theta_1)k_1^2,$$
$$a_{22} = r_2(\sin^2\theta_2 - \cos^2\theta_2)k_2^2,$$
$$a_{23} = (-2r_5\sin\overline{\theta}_1\cos\overline{\theta}_1)\overline{k}_1^2,$$
$$a_{24} = r_5(\cos^2\overline{\theta}_2 - \sin^2\overline{\theta}_2)\overline{k}_2^2,$$
$$a_{31} = (ik_1\sin\theta_1),$$
$$a_{32} = (ik_2\cos\theta_2),$$
$$a_{33} = -(i\overline{k}_1\sin\overline{\theta}_1),$$
$$a_{34} = (i\overline{k}_2\cos\overline{\theta}_2),$$
$$a_{41} = (ik_1\cos\theta_1),$$
$$a_{42} = (-ik_2\sin\theta_2),$$
$$a_{43} = (i\overline{k}_1\cos\overline{\theta}_1),$$
$$a_{44} = (i\overline{k}_2\sin\overline{\theta}_2),$$

and

$$r_1 = \frac{\lambda^S}{\lambda^S + 2\mu^S}, \qquad r_2 = \frac{\mu^S}{\lambda^S + 2\mu^S},$$
$$r_3 = \frac{\overline{\lambda}^S}{\lambda^S + 2\mu^S}, \qquad r_4 = \frac{\overline{\lambda}^S + \overline{\mu}^S}{\lambda^S + 2\mu^S},$$
$$r_5 = \frac{\overline{\mu}^S}{\lambda^S + 2\mu^S},$$

$$Z_1 = \frac{A_1}{A^*}, \quad Z_2 = \frac{B_1}{A^*}, \quad Z_3 = \frac{\overline{A}_1}{A^*}, \quad Z_4 = \frac{\overline{B}_1}{A^*}. \quad (42)$$

(i) For incident longitudinal wave:

$$A^* = A_{01}, \qquad B_{01} = 0, \qquad Y_1 = -a_{11},$$
$$Y_2 = a_{21}, \qquad Y_3 = -a_{31}, \qquad Y_4 = a_{41}.$$

(ii) For incident transversal wave:

$$A^* = B_0, \qquad A_{01} = 0, \qquad Y_1 = a_{12},$$
$$Y_2 = -a_{22}, \qquad Y_3 = a_{32}, \qquad Y_4 = -a_{42},$$

where Z_1, Z_2, are amplitude ratio's of reflected longitudinal wave making an angle θ_1 and transversal wave making an angle θ_2 and Z_3, Z_4 are amplitudes ratio's of the transmitted longitudinal wave making an angle $\overline{\theta}_1$ and transmitted transversal wave making an angle $\overline{\theta}_2$ with the normal to the surface.

6. Particular cases

Case 1. Reflection and transmission at the interface between fluid saturated porous half space and elastic half space. If ρ^F is very small as compare to ρ^S and can be neglected then medium M_2 reduces to elastic half space and we obtain a system of four non- homogenous equations which can be written as given by (39) with the changed values of a_{mn} as:

$$a_{13} = (r_6 \sin^2 \overline{\theta}_1 + r_7 \cos^2 \overline{\theta}_1)\overline{k}_1^2,$$
$$a_{14} = (r_7 - r_6)(\sin \overline{\theta}_2 \cos \overline{\theta}_2)\overline{k}_2^2,$$
$$a_{23} = (-2r_8 \sin \overline{\theta}_1 \cos \overline{\theta}_1)\overline{k}_1^2,$$
$$a_{24} = r_8(\cos^2 \overline{\theta}_2 - \sin^2 \overline{\theta}_2)\overline{k}_2^2,$$

and

$$r_6 = \frac{\overline{\lambda}^e}{\lambda^S + 2\mu^S}, \qquad r_7 = \frac{\overline{\lambda}^e + \overline{\mu}^e}{\lambda^S + 2\mu^S},$$
$$r_8 = \frac{\overline{\mu}^e}{\lambda^S + 2\mu^S}.$$

Case 2.Reflection from free surface. We consider a plane wave (longitudinal/transversal wave) propagating through the fluid saturated incompressible porous half space ($Z > 0$) and incident at the free surface $Z = 0$ with direction of propagation making an angle θ_0 with Z axis. Corresponding to each incident wave we get two reflected waves. Boundary conditions in this case are reduced to

$$T_{33}^S - p = 0, \tag{43}$$
$$T_{31}^S = 0 \tag{44}$$

and we obtain a system of two non- homogenous equations which can be written as

$$\sum_{i=1}^{2} a_{ij} Z_j = Y_i, \quad (j = 1, 2), \tag{45}$$

where a_{11}, a_{12}, a_{21}, a_{22} are given by Eqs. (39).

7. Numerical results and discussion

With the view of illustrating the theoretical results and for numerical discussion we take a model for which the values of the various physical parameters are taken from de Boer and Ehlers [10] as follows:

$$\eta^S = .67, \qquad \eta^F = .33,$$
$$\rho^S = 1.34 \text{ Mg/m}^3, \qquad \rho^F = .33 \text{ Mg/m}^3,$$
$$\lambda^S = 5.5833 \text{ MN/m}^2, \qquad K^F = .01 \text{ m/s},$$
$$\gamma^{FR} = 10.00 \text{ KN/m}^3, \qquad \mu^S = 8.3750 \text{ N/m}^2,$$
$$\overline{\eta}^S = .6, \qquad \overline{\eta}^F = .4,$$
$$\overline{\rho}^S = 2.0 \text{ Mg/m}^3, \qquad \overline{\rho}^F = 0.01 \text{ Mg/m}^3,$$
$$\overline{\lambda}^S = 4.2368 \text{ MN/m}^2, \qquad \overline{K}^F = .02 \text{ m/s},$$
$$\overline{\gamma}^{FR} = 9.00 \text{ KN/m}^3, \qquad \overline{\mu}^S = 3.3272 \text{ MN/m}^2,$$
$$\overline{\rho}^e = 2.65 \text{ Mg/m}^3, \qquad \overline{\lambda}^e = 2.238 \text{ MN/m}^2,$$
$$\overline{\mu}^e = 2.238 \text{ MN/m}^2.$$

A computer programme has been developed and amplitude ratios of various reflected and transmitted waves has been computed. The variations of amplitude ratios with angle of incidence for longitudinal wave (LW) and transversal wave (TW) has been shown by solid lines and small dashed lines respectively. Solid lines without central symbols correspond to the case of fluid saturated porous half space (FS) whereas Solid lines with central symbols correspond to the case of elastic half space (ES).

Figure 2 depicts the behavior of variations of amplitude ratios $|Z_1|$ for LW and TW for FS and ES. Initially values of amplitude ratios in case of LW are more than TW and they are less in the remaining range for both FS and ES. For LW values of amplitude ratios in case of ES are greater than FS in the range $0^\circ \leq \theta^\circ \leq 70^\circ$ and are close to each other in the remaining range whereas for TW, values of amplitude ratios for ES are less than FS in the range $0^\circ \leq \theta^\circ \leq 55^\circ$ and then reverse behavior is observed in the remaining range. Values of amplitude ratios $|Z_2|$ for FS and ES are close to each for both LW and TW.

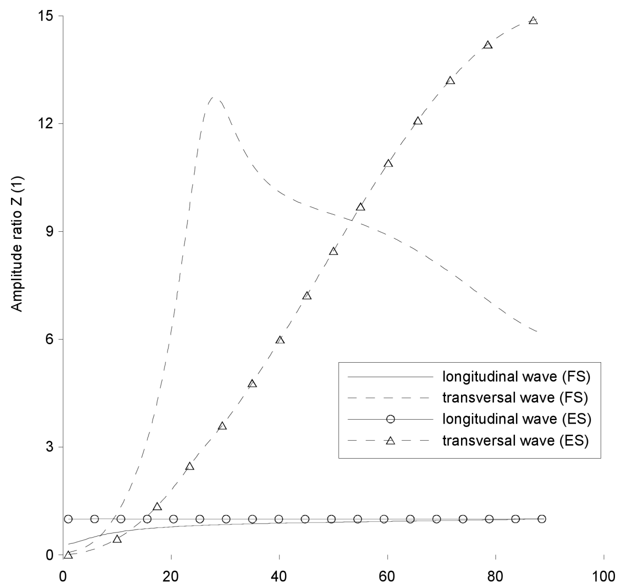

Fig. 2. Variations of the amplitude ratio Z(1) with incident angle of longitudinal and transversal wave

Values of amplitude ratios $\|Z_2|$ in case of LW decrease sharply in the range $0^\circ \leq \theta^\circ \leq 5^\circ$ and decrease slowly in the remaining range for both FS and ES whereas for TW, its values in crease sharply in the range $0^\circ \le \theta^\circ \le 25^\circ$ and decrease in the range $25^\circ \le \theta^\circ \le 30^\circ$ and increase in the remaining range for both FS and ES. These variations are shown in Fig. 3.

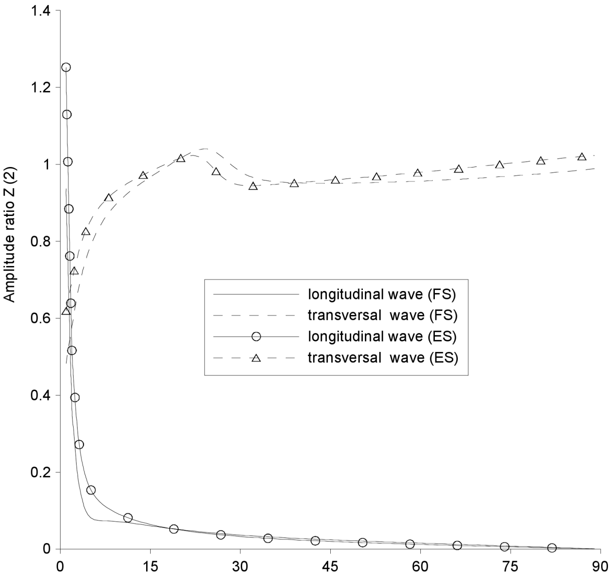

Fig. 3. Variations of the amplitude ratio Z(2) with incident angle of longitudinal and transversal wave

It is observed from Fig. 4 that values of amplitude ratios $|Z_3|$ for LW decrease in the range $0^\circ \le \theta^\circ \le 50^\circ$ and then approach to a constant value in the remaining range for both FS and ES, whereas for TW, its values increase sharply in the range $0^\circ \le \theta^\circ \le 25^\circ$, and decrease in the range $25^\circ \le \theta^\circ \le 40^\circ$ and again increase in the remaining range for both FS and ES.

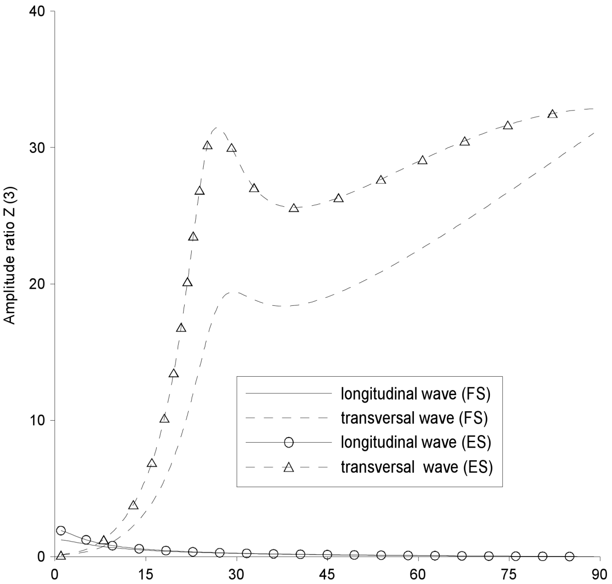

Fig. 4. Variations of the amplitude ratio Z(3) with incident angle of longitudinal and transversal wave

It is observed from Fig. 5 that values of amplitude ratios $|Z_4|$ in case of ES are more than FS in the range $0^\circ \le \theta^\circ \le 25^\circ$, $85^\circ \le \theta^\circ \le 90^\circ$ and less in the remaining range for TW whereas for LW its values are close to each other near the value zero for both FS and ES.

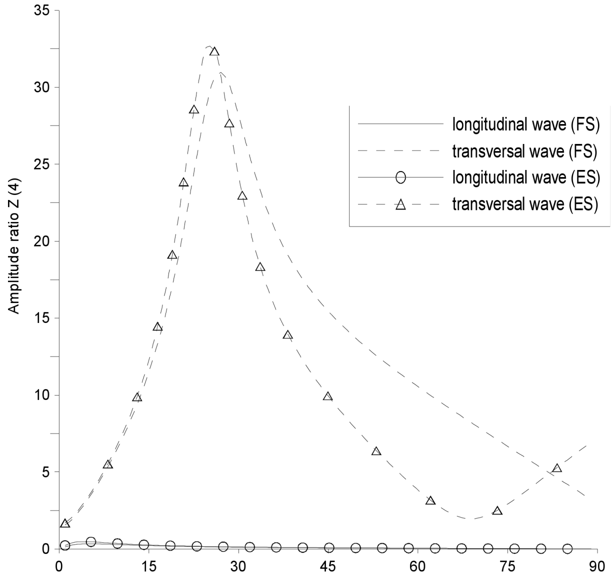

Fig. 5. Variations of the amplitude ratio Z(4) with incident angle of longitudinal and transversal wave

Figure 6 depicts the behavior of variations of amplitude ratios $|Z_1|$ for longitudinal and transversal waves for FS and ES incident at the free surface. values of amplitude ratios in case of LW are more than TW in the range $0^\circ \le \theta^\circ \le 55^\circ$ and are less in the remaining range for FS whereas for ES values of amplitude ratios start with small initial increase for both TW and LW then oscillate in the range $17^\circ \le \theta^\circ \le 19^\circ$ and $40^\circ \le \theta^\circ \le 42^\circ$ for TW and in the range $30^\circ \le \theta^\circ \le 35^{circ}$ for LW and decrease slowly in the remaining range for both TW and LW.

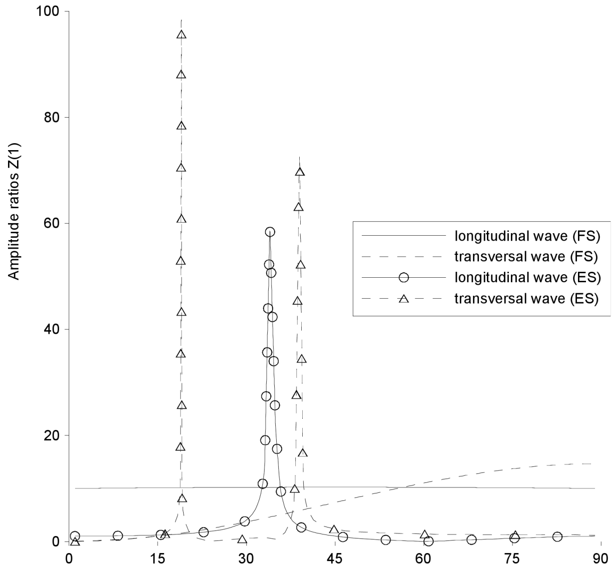

Fig. 6. Variations of the amplitude ratios Z(1) with incident angle of longitude and transversal wave at the free surface $Z = 0$

Figure 7 depicts the behavior of variations of amplitude ratios $|Z_2|$ for longitudinal and transversal waves for FS and ES incident at the free surface. Values of amplitude ratios for LW are more than TW for FS for the whole range. For ES values of amplitude ratio start with small initial increase for both TW and LW and they oscillate in the range $17° \leq \theta° \leq 19°$ and $40° \leq \theta° \leq 42°$ for TW and in the range $30° \leq \theta° \leq 35°$ for LW and decrease slowly in the remaining range for both TW and LW.

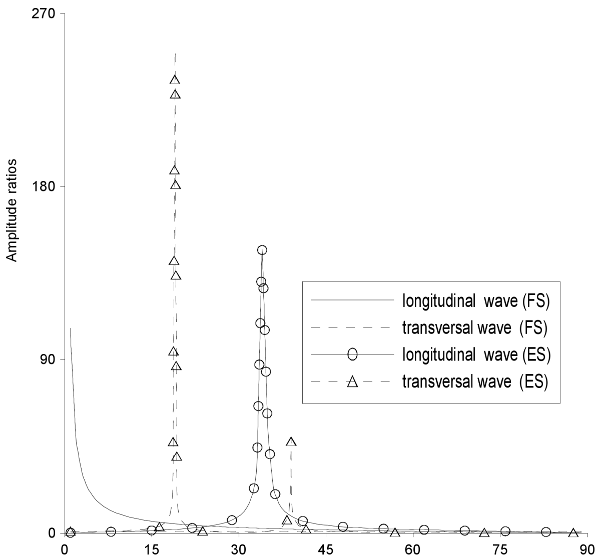

Fig. 7. Variations of the amplitude ratios Z(2) with incident angle of longitude and transversal wave at the free surface $Z = 0$

Figure 8 depicts the variations of normal stress T_{33} in medium M_1 with angle of incidence for longitudinal and transversal waves. Values of T_{33} for TW are more than LW for the whole range.

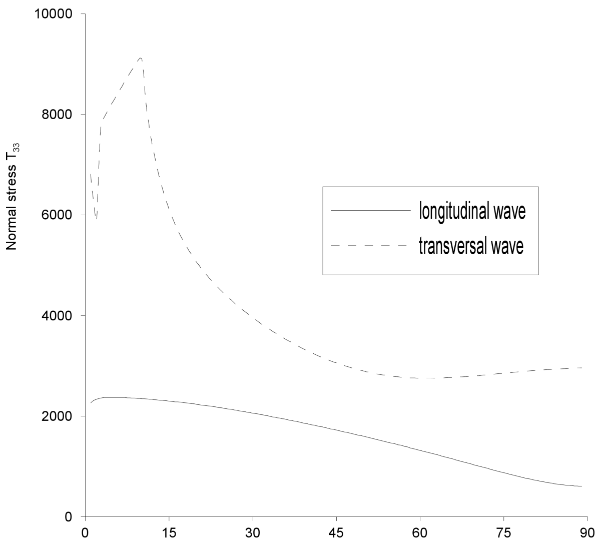

Fig. 8. Variations of the normal stress T_33 in medium M_1 with incident angle of longitude and transversal wave

Figure 9 shows the variations of tangential stress T_{31} in medium M_1 with angle of incidence for longitudinal and transversal waves. Values of T_{31} for LW are more than TW in the range $0° \leq \theta° \leq 3°$ and $10° \leq \theta° \leq 23°$ and are less in the remaining range.

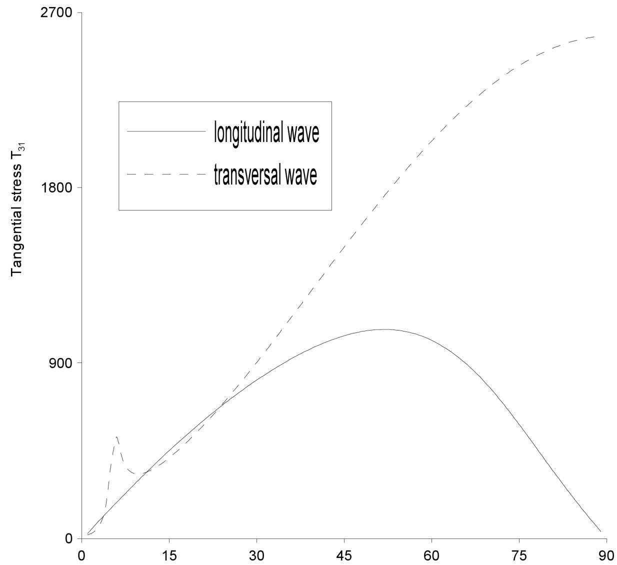

Fig. 9. Variations of the tangential stress T_31 in medium M_1 with incident angle of longitude and transversal wave

Figure 10 depicts the variations of normal stress T_{33} in medium M_2 with angle of incidence for longitudinal and transversal waves. Its values for LW increase sharply in the range $0° \leq \theta° \leq 15°$ and then decrease in the remaining range whereas for TW its values increase in the range $0° \leq \theta° \leq 20°$ and decrease in the remaining range.

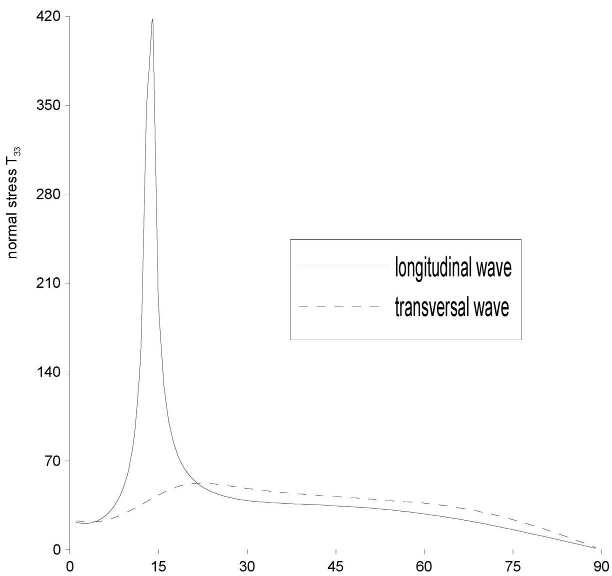

Fig. 10. Variations of the normal stress T_33 in medium M_2 with incident angle of longitude and transversal wave

Trend of variation of tangential stress T_{31} in medium M_2 with angle of incidence for longitudinal and transversal waves is observed in Fig. 1. Values of T_{31} initially increase sharply in the range $0° \leq \theta° \leq 15°$, and decrease in the remaining range whereas for LW its values increase in the whole range.

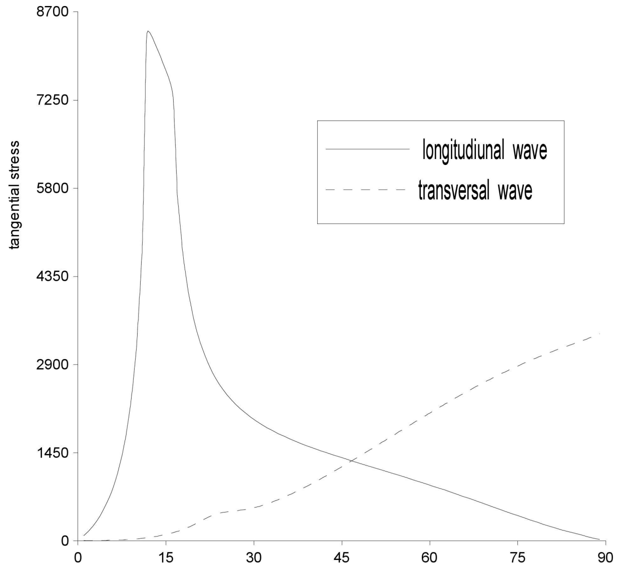

Fig. 11. Variations of the tangential stress T_31 in medium M_2 with incident angle of longitude and transversal wave

8. Conclusions

Detailed numerical calculations have been presented for longitudinal and transversal waves incident at the surface of model considered. Appreciable effect of porosity has been observed on the amplitude ratios and stress components. The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

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