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An analysis of the dynamics of a launcher-missile system on a moveable base

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Abstract. The analysis focuses on the dynamics of a hypothetical anti-aircraft system during the missile launch. The results of the computer simulation of motion of the launcher-missile system are represented graphically. The diagrams provide information about the range of kinematic excitations acting on the opto-electro-mechanical coordinator of the target in a self-guided missile.

Key words: missile launcher, system dynamics, vibrations.

1. Introduction

The motion of the launcher-missile system equipped with a gyroscope was considered in a three-dimensional Euclidean space and Earth's gravitational field. The analysis focused on vertical vibrations with low values of generalized displacements of the launcher. The discrete model of the missile-launcher system with four degrees of freedom was described using analytical relations in the form of equations of motion, kinematic relations and three equations of equilibrium [1–3].

In the general case, the launcher-missile system is not symmetric about the longitudinal vertical plane going through the centre of the system mass. The symmetry refers to selected geometric dimensions and properties of the flexible elements. In the general case, the inertial characteristic departs from this symmetry. The launcher turret can rotate with respect to the carrier together with the guide rail and the missile.

The turret rotates in accordance with the azimuth angle, ψ_{pv}, which is the turret yaw angle. The turret and the guide rail mounted on it constitute a rotary kinematic pair. The guide rail can rotate with respect to the turret in accordance with the elevation angle, ϑ_{pv}, which is the guide rail pitch angle. Generally, this leads to an asymmetric distribution of masses. After the turret and the guide rail move to the position of target interception, the turret configuration does not change. The system is reduced to a structural discrete model describing the phenomena that are mechanical excitations in character. The basic motion of the turret is reduced to the basic motion of the carrier. The turret is an object whose inertial characteristic is dependent on the position of a target with respect to the anti-aircraft system. The mass of the turret remains constant, but its moments of inertia and moments of deviation change. After the target is intercepted, the turret inertial characteristic remains unchanged.

The launcher was modeled as two basic masses and four deformable elements, as shown in Fig. 1.

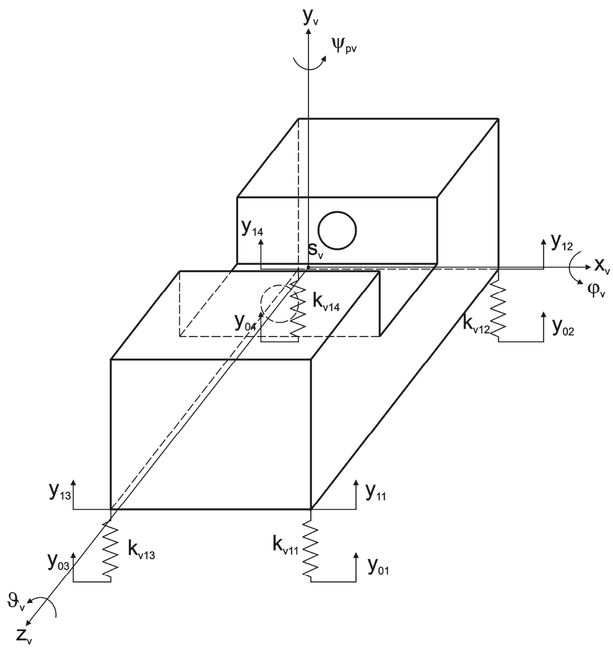

Fig. 1. Physical model of the launcher

The launcher is a perfectly stiff body with mass m_v, moments of inertia I_{vx} and I_{vz} and a moment of deviation I_{vxz}. The launcher is mounted on the vehicle using four passive elastic-attenuating elements with linear parameters $k_{v_{11}}$ and $c_{v_{11}}$, $k_{v_{12}}$ and $c_{v_{12}}$, k_{v13} and $c_{v_{13}}$ and $k_{v_{14}}$ and $c_{v_{14}}$, respectively.

The inertial characteristic of the launcher is dependent on the current position of the components, i.e. the turret and the guide rail. The turret is a perfectly stiff body with mass m_w and main central moments of inertia $I_{w\xi_v'}$, $I_{w\eta_v'}$, $I_{w\zeta_v'}$. The guide rail is also a perfectly stiff body with mass m_{pr} and main central moments of inertia $I_{pr\xi_{pv}}$, $I_{pr\eta_{pv}}$, $I_{pr\zeta_{pv}}$.

If the basic motion of the launcher is not disturbed, then the $0_v x_v y_v z_v$, $S_v x_v y_v z_v$ and $S_v \xi_v \eta_v \zeta_v$ coordinate systems

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coincide at any moment. The turret model as an element of the spatial vibrating system performs a complex motion with respect to the $0_v x_v y_v z_v$ reference system consisting of a straightline motion of the mass centre, S_v, in accordance with a change in the y_v coordinate, a rotary motion about the $S_v z_v$ axis in accordance with a change in the pitch angle ϑ_v and a rotary motion about the $S_v x_v$ axis in accordance with a change in the tilt angle φ_v.

Prior to the launch, the missile is rigidly connected with the guide rail. The mounting prevents the missile from moving along the guide rail. Once the missile motor is activated, the missile moves along the rail.

The guide rail-missile system used in the analysis ensures the collinearity of points on the missile and the guide rail. It is assumed that the longitudinal axis of the missile coincides with the longitudinal axis of the guide rail at any moment and that the missile is a stiff body with an unchangeable characteristic of inertia. The missile is a perfectly stiff body with mass m_p and main central moments of inertia I_{px_p}, I_{py_p}, I_{pz_p}. The geometric characteristic of the guide rail-missile system shown in Figs. 2 and 3 can be used to analyze the dynamics of the controlled system.

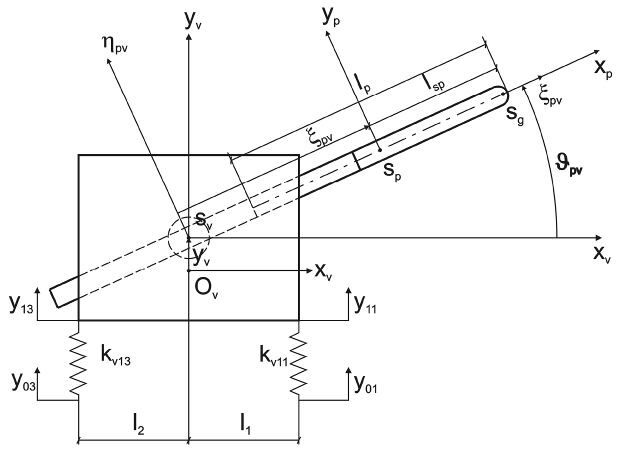

Fig. 2. Side view of the turret-and-missile model

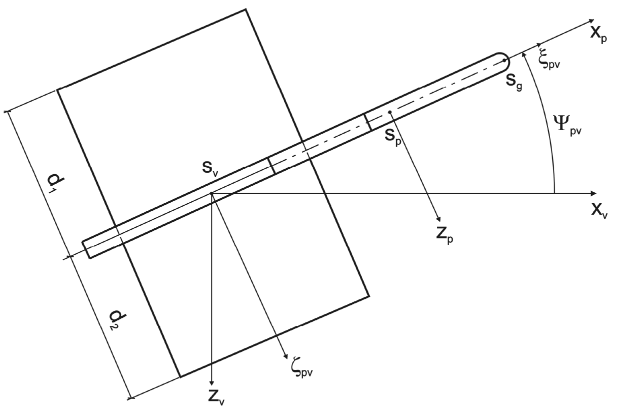

Fig. 3. Top view of the turret-and-missile model

The main view of the turret-missile system model is presented in Fig. 2. It includes the instantaneous position of the missile while it moves along the guide rail. Figure 3 shows the top view of the turret-missile system model. It includes the instantaneous position of the missile while it moves along the guide rail.

The missile model performs a straightline motion of the mass centre S_v with respect to the $S_v \xi_{pv} \eta_{pv} \zeta_{pv}$ reference system according to a change in the ξ_{pv} coordinate.

The model of the launcher-missile system has six degrees of freedom, which results from the structure of the formulated model. The positions of the system at any moment were determined assuming four independent generalized coordinates:

1. y_v – vertical displacement of the turret mass centre, S_v;
2. φ_v – angle of rotation of the turret about the$S_v x_v$ axis;
3. ϑ_v – angle of rotation of the turret about the$S_v z_v$ axis;
4. ξ_{pv} – straightline displacement of the missile mass centre S_p along the $S_v \xi_{pv}$ axis.

2. The mathematical model of the launcher-missile system

The mathematical model of the system was developed basing on the physical model. The system was reduced to a structural discrete model, which required applying differential equations with ordinary derivatives. Four independent generalized coordinates were selected to determine the kinetic and potential energies of the model and the distribution of generalized forces basing on the considerations for the physical model. By using the second order Lagrange equations, it was possible to derive the equations of motion of the analyzed system.

Equations of motion for the turret:

$$
\begin{gathered}
(m_v + m_p)\,\ddot{y}_v + m_p l_\xi \xi_{pv} \ddot{\vartheta}_v - m_p l_\zeta \xi_{pv} \ddot{\varphi}_v + \\
+ m_p \ddot{\xi}_{pv} \left(l_\eta + l_\xi \vartheta_v - l_\zeta \varphi_v\right) + \\
+ 2 m_p \dot{\xi}_{pv} \left(l_\xi \dot{\vartheta}_v - l_\zeta \dot{\varphi}_v\right) + c_{v11} \dot{\lambda}_{v11} + \\
+ c_{v12} \dot{\lambda}_{v12} + k_{v11} \lambda_{v11} + k_{v12} \lambda_{v12} + \\
+ c_{v13} \dot{\lambda}_{v13} + c_{v14} \dot{\lambda}_{v14} + k_{v13} \lambda_{v13} + \\
+ k_{v14} \lambda_{v14} + (m_v + m_p)\, g = 0,
\end{gathered}
\tag{1}
$$

$$
\begin{gathered}
\left[I_{vz} + I_{px_p} l_{x1}^2 + I_{py_p} l_{y1}^2 + I_{pz_p} l_{z1}^2 + m_p \xi_{pv}^2 \left(l_\xi^2 + l_\eta^2\right)\right] \ddot{\vartheta}_v + \\
+ (I_{px_p} l_{x1} l_{x2} + I_{py_p} l_{y1} l_{y2} + I_{pz_p} l_{z1} l_{z2} - \\
- I_{vxz} - m_p \xi_{pv}^2 l_\xi l_\zeta) \ddot{\varphi}_v + \\
+ m_p \xi_{pv} l_\xi \ddot{y}_v + m_p \xi_{pv} \left[\left(l_\xi^2 + l_\eta^2\right) \vartheta_v - l_\xi l_\zeta \varphi_v\right] \ddot{\xi}_{pv} + \\
+ 2 m_p \xi_{pv} \left(l_\xi^2 + l_\eta^2\right) \dot{\vartheta}_v \dot{\xi}_{pv} - 2 m_p \xi_{pv} l_\xi l_\zeta \dot{\varphi}_v \dot{\xi}_{pv} + \\
+ c_{v11} l_1 \dot{\lambda}_{v11} + c_{v12} l_1 \dot{\lambda}_{v12} + k_{v11} l_1 \lambda_{v11} + k_{v12} l_1 \lambda_{v12} + \\
- c_{v13} l_2 \dot{\lambda}_{v13} - c_{v14} l_2 \dot{\lambda}_{v14} - k_{v13} l_2 \lambda_{v13} - \\
- k_{v14} l_2 \lambda_{v14} + m_p g l_\xi \xi_{pv} = 0,
\end{gathered}
\tag{2}
$$

$$\begin{aligned}&\left[I_{vx}+I_{px_p}l_{x2}^2+I_{py_p}l_{y2}^2+I_{pz_p}l_{z2}^2+m_p\xi_{pv}^2\left(l_\eta^2+l_\zeta^2\right)\right]\ddot{\varphi}_v+\\&+(I_{px_p}l_{x1}l_{x2}+I_{py_p}l_{y1}l_{y2}+I_{pz_p}l_{z1}l_{z2}-\\&-I_{vxz}-m_p\xi_{pv}^2l_\xi l_\zeta)\ddot{\vartheta}_v+\\&-m_p\xi_{pv}l_\zeta\ddot{y}_v+m_p\xi_{pv}\left[\left(l_\eta^2+l_\zeta^2\right)\varphi_v-l_\xi l_\zeta\vartheta_v\right]\ddot{\xi}_{pv}+\\&-2m_p\xi_{pv}l_\xi l_\zeta\dot{\vartheta}_v\dot{\xi}_{pv}+2m_p\xi_{pv}\left(l_\eta^2+l_\zeta^2\right)\dot{\varphi}_v\dot{\xi}_{pv}+\\&-c_{v11}d_1\dot{\lambda}_{v11}+c_{v12}d_2\dot{\lambda}_{v12}-k_{v11}d_1\lambda_{v11}+k_{v12}d_2\lambda_{v12}+\\&-c_{v13}d_1\dot{\lambda}_{v13}+c_{v14}d_2\dot{\lambda}_{v14}-k_{v13}d_1\lambda_{v13}+\\&+k_{v14}d_2\lambda_{v14}-m_pgl_\zeta\xi_{pv}=0.\end{aligned} \tag{3}$$

Equations of motion for the missile:

$$\begin{aligned}&m_p[l_\xi^2+l_\eta^2+l_\zeta^2+\left(l_\xi^2+l_\eta^2\right)\vartheta_v^2+\left(l_\eta^2+l_\zeta^2\right)\varphi_v^2-\\&-2l_\xi l_\zeta\vartheta_v\varphi_v]\ddot{\xi}_{pv}+\\&+m_p\xi_{pv}\left[\left(l_\xi^2+l_\eta^2\right)\vartheta_v-l_\xi l_\zeta\varphi_v\right]\ddot{\vartheta}_v+\\&+m_p\xi_{pv}\left[\left(l_\eta^2+l_\zeta^2\right)\varphi_v-l_\xi l_\zeta\vartheta_v\right]\ddot{\varphi}_v+\\&+2m_p\left[\left(l_\xi^2+l_\eta^2\right)\vartheta_v-l_\xi l_\zeta\varphi_v\right]\dot{\xi}_{pv}\dot{\vartheta}_v+\\&+2m_p\left[\left(l_\eta^2+l_\zeta^2\right)\varphi_v-l_\xi l_\zeta\vartheta_v\right]\dot{\xi}_{pv}\dot{\varphi}_v+\\&+m_p\left(l_\eta+l_\xi\vartheta_v-l_\zeta\varphi_v\right)\left(\ddot{y}_v+g\right)=P_{ss},\end{aligned} \tag{4}$$

where:

$$\begin{aligned}&\lambda_{v11}=y_v+y_{vst}+l_1\left(\vartheta_v+\vartheta_{vst}\right)-d_1\left(\varphi_v+\varphi_{vst}\right)-y_{01},\\&\lambda_{v12}=y_v+y_{vst}+l_1\left(\vartheta_v+\vartheta_{vst}\right)+d_2\left(\varphi_v+\varphi_{vst}\right)-y_{02},\\&\lambda_{v13}=y_v+y_{vst}-l_2\left(\vartheta_v+\vartheta_{vst}\right)-d_1\left(\varphi_v+\varphi_{vst}\right)-y_{03},\\&\lambda_{v14}=y_v+y_{vst}-l_2\left(\vartheta_v+\vartheta_{vst}\right)+d_2\left(\varphi_v+\varphi_{vst}\right)-y_{04},\\&\dot{\lambda}_{v11}=\dot{y}_v+l_1\dot{\vartheta}_v-d_1\dot{\varphi}_v-\dot{y}_{01},\\&\dot{\lambda}_{v12}=\dot{y}_v+l_1\dot{\vartheta}_v+d_2\dot{\varphi}_v-\dot{y}_{02},\\&\dot{\lambda}_{v13}=\dot{y}_v-l_2\dot{\vartheta}_v-d_1\dot{\varphi}_v-\dot{y}_{03},\\&\dot{\lambda}_{v14}=\dot{y}_v-l_2\dot{\vartheta}_v+d_2\dot{\varphi}_v-\dot{y}_{04},\\&l_\xi=\cos\vartheta_{pv}\cos\psi_{pv},\\&l_\eta=\sin\vartheta_{pv},\\&l_\zeta=-\cos\vartheta_{pv}\sin\psi_{pv},\\&l_{x1}=-\cos\vartheta_{pv}\sin\psi_{pv},\\&l_{x2}=\cos\vartheta_{pv}\cos\psi_{pv},\\&l_{y1}=\sin\vartheta_{pv}\sin\psi_{pv},\\&l_{y2}=-\sin\vartheta_{pv}\cos\psi_{pv},\\&l_{z1}=\cos\psi_{pv},\\&l_{z2}=\sin\psi_{pv}.\end{aligned}$$

3. Sample parameters of the launcher-missile system

The excitations generated during the missile launch are transmitted from the launcher to the gyroscopic system, which is a mechanical element of the coordinator unit. It is required that the electronic system be able to convert signals in the first feedback loop under disturbance conditions. The converted signals should permit tracking the line of sight through the gyroscope axis. The photoelectric resistor converts electromagnetic radiation in the infrared range into electric voltage pulses using a modulating shield. These signals are transmitted to the pre-amplifier and then to the carrier frequency amplifier. The noises produced by the photoelectric resistor and the electronic elements are eliminated, and the carrier frequency signal is separated and amplified. The signal is then sent to the envelope of the amplifier, which is tuned to amplify the frequency of the gyroscope rotational velocity and to separate the first harmonic of the envelope of pulse packages. The signal in the form of a harmonic function is transmitted to the correction amplifier. The signal is amplified to correct the gyroscope measurements and sent to the autopilot. As a result, the first feedback loop includes the opto-electro-mechanical target coordinator.

The following are parameters selected for the mathematical model (1)–(4).

1. Parameters describing the inertia elements of
   - the launcher

$$\begin{aligned}m_v&=m_w+m_{pr}\\I_{vx}&=\left(I_{w\xi'_v}+I_{pr\xi_{pv}}\cos^2\vartheta_{pv}+I_{pr\eta_{pv}}\sin^2\vartheta_{pv}\right)\cdot\\&\cdot\cos^2\psi_{pv}+\left(I_{w\zeta'_v}+I_{pr\zeta_{pv}}\right)\sin^2\psi_{pv},\\I_{vz}&=\left(I_{w\xi'_v}+I_{pr\xi_{pv}}\cos^2\vartheta_{pv}+I_{pr\eta_{pv}}\sin^2\vartheta_{pv}\right)\cdot\\&\cdot\sin^2\psi_{pv}+\left(I_{w\zeta'_v}+I_{pr\zeta_{pv}}\right)\cos^2\psi_{pv},\\I_{vxz}&=\left(I_{w\xi'_v}+I_{pr\xi_{pv}}\cos^2\vartheta_{pv}+I_{pr\eta_{pv}}\sin^2\vartheta_{pv}-\right.\\&\left.-I_{w\zeta'_v}-I_{pr\zeta_{pv}}\right)\cos\psi_{pv}\sin\psi_{pv},\end{aligned}$$

   - the turret

$$m_w=50\text{ kg},\qquad I_{w\xi'_v}=10\text{ kgm}^2,$$
$$I_{w\eta'_v}=7\text{ kgm}^2,\qquad I_{w\zeta'_v}=12\text{ kgm}^2,$$

   - the guide rail

$$m_{pr}=30\text{ kg},\qquad I_{pr\xi_{pv}}=0.6\text{ kgm}^2,$$
$$I_{pr\eta_{pv}}=4\text{ kgm}^2,\qquad I_{pr\zeta_{pv}}=3.5\text{ kgm}^2$$

2. Parameters describing the inertia elements of the missile

$$m_p=12\text{ kg},\qquad I_{px_p}=0.01\text{ kgm}^2,$$
$$I_{py_p}=2\text{ kgm}^2,\qquad I_{pz_p}=2\text{ kgm}^2,$$

3. Parameters describing the non-inertia elements of the turret suspension:

$$k_{v11}=300000\text{ N/m},\qquad c_{v11}=150\text{ Ns/m},$$
$$k_{v12}=300000\text{ N/m},\qquad c_{v12}=150\text{ Ns/m},$$
$$k_{v13}=300000\text{ N/m},\qquad c_{v13}=150\text{ Ns/m},$$
$$k_{v14}=300000\text{ N/m},\qquad c_{v14}=150\text{ Ns/m},$$

4. Geometric characteristics

$$l_1=0.3\text{ m},\qquad l_2=0.3\text{ m},$$
$$d_1=0.2\text{ m}\qquad d_2=0.2\text{ m},$$
$$l_p=1.6\text{ m}\qquad l_{sp}=0.8\text{ m},$$

5. Thrust of the missile booster

$$P_{ss}=4000\text{ N}.$$

Kinematic relations. The position of the missile mass centre, S_p, in the $0_v x_v y_v z_v$ coordinate system:

$$\vec{r}_p \left(r_{px_v}, r_{py_v}, r_{pz_v}\right), \tag{5}$$

$$\begin{aligned} r_{px_v} &= \xi_{pv} l_\xi - \xi_{pv} l_\eta \vartheta_v, \\ r_{py_v} &= \xi_{pv} l_\xi \vartheta_v - \xi_{pv} l_\zeta \varphi_v + \xi_{pv} l_\eta + y_v, \\ r_{pz_v} &= \xi_{pv} l_\eta \varphi_v + \xi_{pv} l_\zeta. \end{aligned}$$

The coordinates of the velocity vector of the missile mass centre in the $0_v x_v y_v z_v$ coordinate system:

$$\vec{V}_p \left(V_{px_v}, V_{py_v}, V_{pz_v}\right), \tag{6}$$

$$\begin{aligned} V_{px_v} &= \left(l_\xi - l_\eta \vartheta_v\right) \dot{\xi}_{pv} - l_\eta \xi_{pv} \dot{\vartheta}_v, \\ V_{py_v} &= \left(l_\eta + l_\xi \vartheta_v - l_\zeta \varphi_v\right) \dot{\xi}_{pv} + l_\xi \xi_{pv} \dot{\vartheta}_v - l_\zeta \xi_{pv} \dot{\varphi}_v + \dot{y}_v, \\ V_{pz_v} &= \left(l_\zeta + l_\eta \varphi_v\right) \dot{\xi}_p + l_\eta \xi_{pv} \dot{\varphi}_v. \end{aligned}$$

The module and the direction angles of the velocity vector of the missile mass centre, S_p:

$$V_p = \sqrt{V_{px_v}^2 + V_{py_v}^2 + V_{pz_v}^2}, \tag{7}$$

$$\tan\gamma_p = \frac{V_{py_v}}{\sqrt{V_{px_v}^2 + V_{pz_v}^2}}, \quad \tan\chi_p = -\frac{V_{pz_v}}{V_{px_v}}. \tag{8}$$

The coordinates of the angular missile velocity vector in the $S_p x_p y_p z_p$ coordinate system:

$$\vec{\omega}_p \left(\omega_{px_p}, \omega_{py_p}, \omega_{pz_p}\right), \tag{9}$$

$$\begin{aligned} \omega_{px_p} &= \dot{\varphi}_v \cos\vartheta_{pv}, \cos\psi_{pv} - \dot{\vartheta}_v \cos\vartheta_{pv} \sin\psi_{pv} \\ \omega_{py_p} &= \dot{\vartheta}_v \sin\vartheta_{pv} \sin\psi_{pv} - \dot{\varphi}_v \sin\vartheta_{pv} \cos\psi_{pv}, \\ \omega_{pz_p} &= \dot{\vartheta}_v \cos\psi_{pv} + \dot{\varphi}_v \sin\psi_{pv}. \end{aligned}$$

Equations of equilibrium.

$$\begin{gathered} k_{v11} a_{v11} + k_{v12} a_{v12} + k_{v13} a_{v13} + k_{v14} a_{v14} + \\ + \left(m_v + m_p\right) g = 0, \end{gathered} \tag{10}$$

$$k_{v11} l_1 a_{v11} + k_{v12} l_1 a_{v12} - k_{v13} l_2 a_{v13} - k_{v14} l_2 a_{v14} = 0, \tag{11}$$

$$\begin{gathered} k_{v12} d_2 a_{v12} - k_{v11} d_1 a_{v11} - k_{v13} d_1 a_{v13} + \\ + k_{v14} d_2 a_{v14} = 0, \end{gathered} \tag{12}$$

where

$$\begin{aligned} a_{v11} &= y_{vst} + l_1 \vartheta_{vst} - d_1 \varphi_{vst}, \\ a_{v12} &= y_{vst} + l_1 \vartheta_{vst} + d_2 \varphi_{vst}, \\ a_{v13} &= y_{vst} - l_2 \vartheta_{vst} - d_1 \varphi_{vst}, \\ a_{v14} &= y_{vst} - l_2 \vartheta_{vst} + d_2 \varphi_{vst}. \end{aligned}$$

4. A numerical simulation of the motion of the launcher-missile system

The launcher configuration for $t = 0\,s$ is defined by the initial position of the turret and the guide rail: $\psi_{pv} = 45$ deg and $\vartheta_{pv} = 45$ deg. During the launch, the missile moves along the guide rail. Any undesirable load related to the launch is taken by the turret [4, 5]. The variation in the turret linear acceleration shown in Fig. 4 was used to determine the missile launch time.

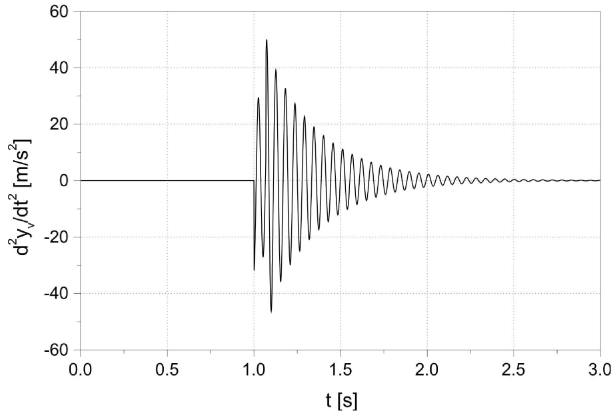

Fig. 4. Linear acceleration of the turret

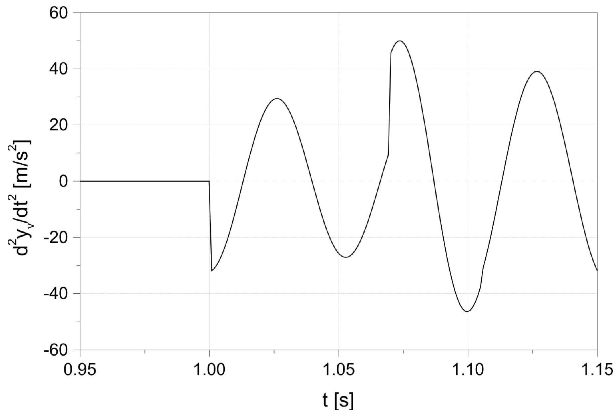

Fig. 5. Linear acceleration of the turret during the missile launch

The guide rail and the missile constitute a kinematic pair. The missile motion produced by the booster affects the guide rail motion, and accordingly the turret motion. At $t = 1$ s the booster is fired and the missile begins its motion along the guide rail. A change in the missile position along the guide rail results from the presence of thrust. The launch generates disturbances that are transferred onto the turret. The variation in the linear acceleration characterizing the turret response to the excitation caused by the launch is discontinuous (Fig. 5). The jump discontinuities are related to the changes in the missile velocity. This results from the physical nature of the processes occurring at those moments. The characteristic points are moments when the booster is fired or burns out and when the missile leaves the launcher. The presence of thrust causes an increase in the turret linear acceleration at $t = 1$ s. The thrust generates the missile motion along the guide rail. Thus, the launcher-missile system is a time-dependent system. The movement of the missile with a pre-determined mass along the guide rail leads to a change in the distribution of the mass in the turret-missile system. Owing to the couplings between the missile motion and the turret performance, the generated disturbances affect the variation in the linear acceleration. When the booster burns out, the missile is no longer loaded by the thrust. A reduction in the thrust to zero is responsible for an increase in the turret linear acceleration at $t = 1.07$ s. The turret-missile system is not only a system with different distribution of mass but also a system with different mass values. At $t = 1.07$ s the booster burns out and the missile uses the acquired kinetic energy. However, when the missile leaves the guide rail at $t = 1.10530$ s, the mass of the turret-missile system changes only once and in a discrete way. The new system no longer constitutes a kinematic pair as there is a change in the system structure, and accordingly, in the order of its objects. On the one hand, it describes the end of the missile motion along the guide rail; on the other hand, it determines the initial conditions of the missile motion after the launch. The missile launch is responsible for both vertical and angular vibrations of the turret. Using the variation in the turret angular acceleration, one is able to determine the moment of missile launch with ease, Figs. 6 and 8.

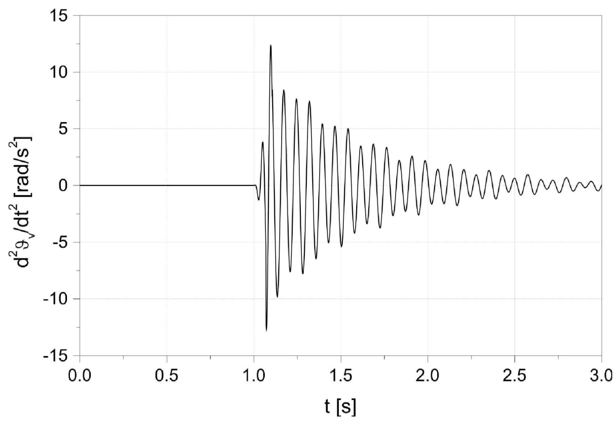

Fig. 6. Angular acceleration of the turret in the pitch motion

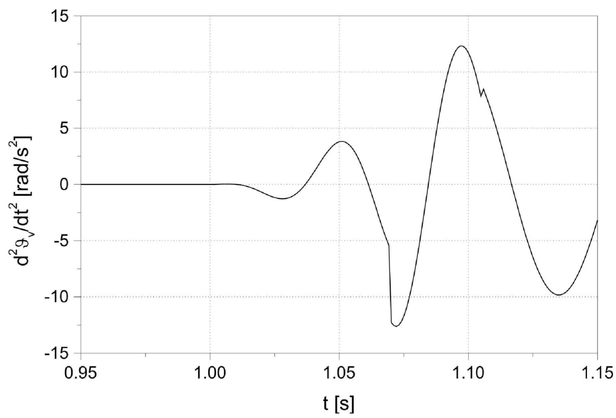

Fig. 7. Angular acceleration of the turret in the pitch motion during the missile launch

The variation in the angular acceleration in the pitch and roll motion characterizing the turret response to an excitation caused by the missile launch is a discontinuous function, Figs. 7 and 9. The missile and the guide rail constitute a kinematic pair, which is responsible for the motion of the two elements. Figure 10 presents the time-dependent variation in the linear acceleration of the missile. This acceleration the acceleration characterizes the missile motion along the guide rail. Before the launch, the missile is rigidly connected with the guide rail. The missile does not move along the guide rail until $t = 1$ s, thus the relative acceleration is equal to zero. At $t = 1$ s thrust is applied to the missile body, the occurrence of thrust results in a sudden increase in the acceleration. The missile starts moving along the guide rail. The change in the missile position is attributable to thrust. The launch generates disturbances that are transferred onto the other elements of the unit. As an integral element of the anti-aircraft system, the missile is exposed to a feedback of the system in the form of an excitation generated by the guide rail.

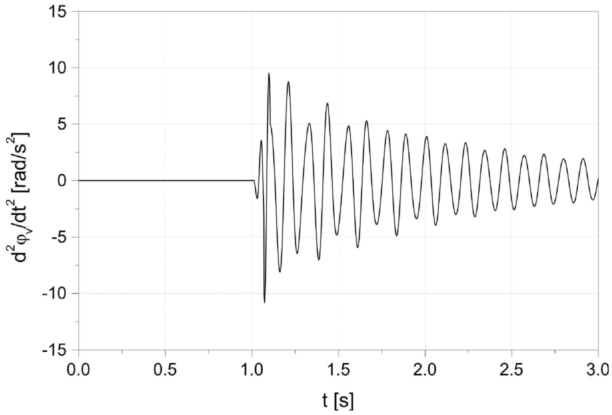

Fig. 8. Angular acceleration of the turret in the tilt motion

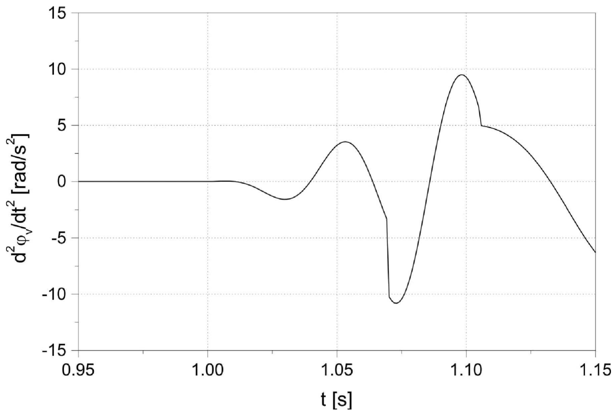

Fig. 9. Angular acceleration of the turret in the tilt motion during the missile launch

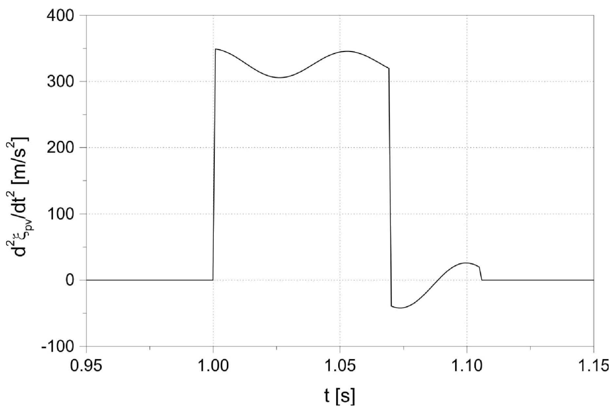

Fig. 10. Linear acceleration of the missile in the motion along the guide rail

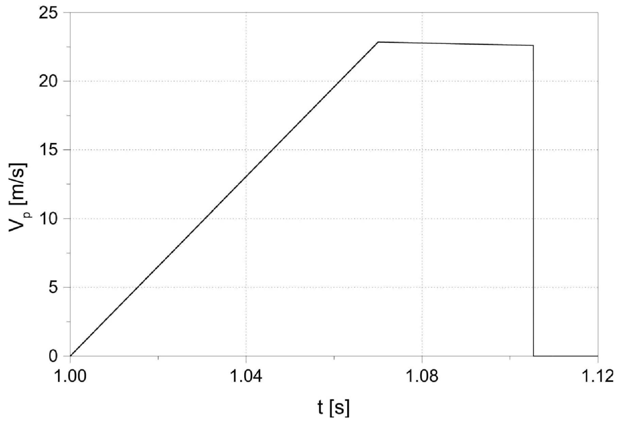

Fig. 11. Linear velocity of the missile during a launch

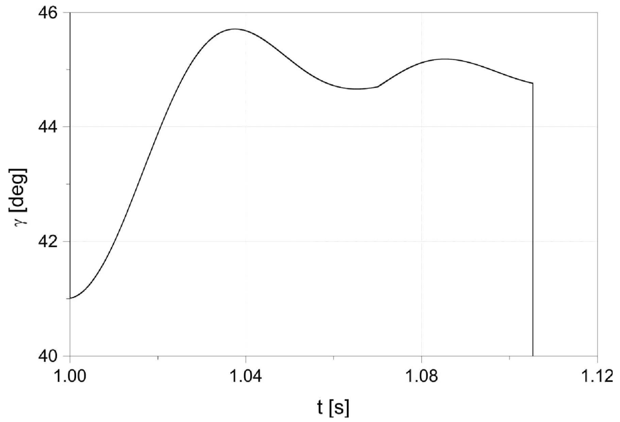

Fig. 12. Direction angle of the missile linear velocity vector γ during a launch

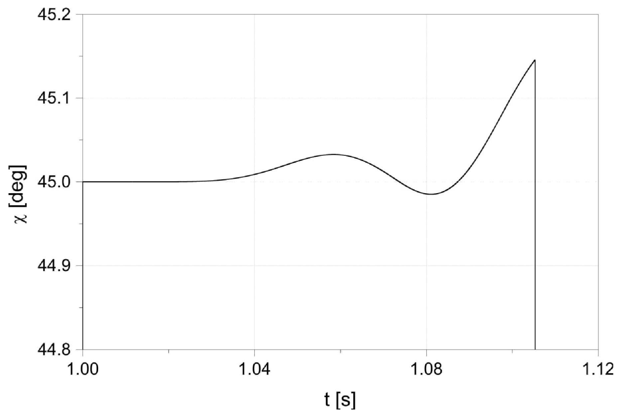

Fig. 13. Direction angle of the missile linear velocity vectorχ during a launch

As a result, the relative acceleration changes until $t = 1.07$ s. At $t = 1.07$ s, the booster burns out and the missile body is no longer loaded with thrust. The reduction in thrust to zero leads to a sudden increase in acceleration. Due to various interactions, the acceleration is not equal to zero. When $t = 1.07$ s the missile moves using the acquired kinetic energy. The relative linear acceleration changes in time.

Figure 11 shows the time-dependent variation in the missile linear velocity. Firing the booster causes loading the missile with thrust. The thrust is high enough to push and accelerate the missile to a considerable velocity over a very short distance in a fraction of a second. The time the thrust acts on the missile is 0.07 s. The thrust is constant in time and causes the motion of the missile along the guide rail. The velocity rises until $t = 1.07$ s, when the booster burns out and the missile body is affected by some load. As there is no thrust, the missile linear velocity fluctuates around a constant value. The fluctuations are due to the disturbances generated in the missile launch system.

The direction angle γ of the missile linear velocity vector fluctuates around 45 deg (Fig. 12). A change in this angle is due to the disturbances generated in the missile launch system. A change in the direction angle χ of the missile linear velocity vector slightly departs from 45 deg (Fig. 13).

5. Conclusions

The results show that the missile moving along the guide rail causes vibrations of the system. The vibrations, in turn, constitute kinematic excitations for the drive of the target coordinator control, i.e. the mechanical gyroscope. It is necessary to analyze the effects of the excitations on the operation of the gyroscope, because its accuracy is responsible for the effective performance of the self-guided missile, i.e. the destruction of a moveable aerial target.

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