BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 58, No. 3, 2010
DOI: 10.2478/v10175-010-0041-3

Cellular automata model of self-organizing traffic control in urban networks

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Abstract. A model of city traffic based on Nagel-Schreckenberg cellular automaton (CA) model is presented. Traffic control is realized at intersections with two conflicting streams each (at any time at most one stream can have "green light" assigned to it). For simple and regular lattice-like networks which are considered, it is easy to find optimal switching periods giving maximum possible flow rates. These optimal strategies are compared with a self-controlling approach proposed by [1], which has not been implemented in a CA model until now. Previous work proved that generally this method gives superior results when compared to classical methods. In this paper we show that for deterministic scenario such control leads to self-organization, and that the solution always quickly converges to the optimal solution which is known in this case. Moreover, we consider also non-deterministic case, in the sense that possibility of turning with given probability is allowed. It is shown that the self-controlling strategy always gives better results than any solution based on fixed cycles with green waves.

Key words: cellular automata model, self-organizing traffic control, urban networks.

1. Introduction

Many real complex systems such as vehicular traffic or production networks are characterized by complicated dynamics of the underlying transportation processes. Undoubtedly, optimization in terms of time and cost is of vital importance in such systems. However, due to highly complicated dynamics it is not an easy task. Lack of the efficient optimization can be seen, for example, in everyday life when spending hours waiting in traffic jams. It is difficult to reasonably calculate economical costs connected with vehicle delays, nevertheless, one can be sure that they must be huge. For example, in Copenhagen the economic loss due to vehicle delays is about 750 millions per year [2], while in the entire Germany the damage is estimated to be of order $100 billion each year [3]. Emissions of gases are significant and can be compared with industrial pollution. All these problems are especially burdensome in large cities and agglomerations.

Flow of vehicles in an urban street network is almost entirely controlled by traffic lights. Consequently by choosing signal control schemes one has a large impact on average fuel consumption and travel times. One of the most popular ways of optimizing traffic is to choose pre-calculated schemes, which are aimed at synchronizing green times along a one-way or two-way main arterials. In principle such methods force the traffic flow to comply with previously designed patterns in order to minimize travel times. However, since traffic demand varies, there is a need for some responsiveness to the current traffic state. The simplest way is to use pre-calculated green times for different times of a day and for different weekdays – traffic varies significantly between Friday afternoon and Sunday night. Nevertheless, any deviation of traffic intensity from its averaged values for which the scheme was calculated must inevitable lead to some inefficiency. Such deviations are always present in any real traffic system.

In order to improve efficiency of control methods it is necessary to implement an on-line optimization techniques based on real time traffic intensity observations. This can be done in a centralized system, in which there exists a central unit possessing all information concerning current state of the traffic and it calculates optimal control schemes. However, there are many problems with this approach. Firstly, all the measuring devices must be connected to the central unit, secondly it is not easy to find the optimal solution: in general the problem can be stated as NP-hard [4] and significant amount of computational time is required. Moreover, the solution is found for averaged flow rates from the past which certainly will not be repeated in the future exactly. Therefore there is a recent trend towards decentralized and self-organizing optimization techniques [5–7] which instantly respond to the current traffic state (known, e.g., from vehicle detectors mounted at some distance before an intersection).

In this paper we shortly discuss methods used in traffic modeling and some important features of vehicular flow. Then we present our cellular automaton city traffic model which is essentially similar to that by [8] and apply it to flows in the simplest possible networks. Finally self-organizing controlling strategy proposed by [1] is implemented on the top of the CA model.

1.1. Traffic models. Movement of vehicles is an example of a self-driven many-particle system driven far from equilibrium. There are many different approaches for modeling such systems, for excellent reviews see [3, 9]. Roughly we can divide them into two categories: microscopic and macroscopic. In the former attention is paid to each individual vehicle repre-

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sented by a particle. Interactions among the particles depend on the way the vehicles influence each other. Macroscopic models describe collective vehicle dynamics in terms of the spatial vehicle density per lane ρ and the average velocity V as a function of the location x and time t. They are often suitable for analytical investigation, ensure simple treatment of inflows, enable simulations of several lanes by effective one-lane models with certain probabilities of overtaking.

The microscopic models include:

- Follow-the-leader models in which it is assumed that the acceleration is determined be vehicles in front of the driver (e.g., intelligent driver model);
- Coupled-map lattice models in which dynamical equations for individual vehicles are formulated as discrete dynamical maps that relate states at time t and $t+1$ (velocity and acceleration are continuous variables);
- Cellular automata (CA) models in which each vehicle is represented by an occupied cell in a CA model (e.g., the Nagel-Schreckenberg models and its variants).

The macroscopic models include, among others:

- Fluid dynamical models (kinematic waves, incompressible Navier-Stokes-like momentum equations);
- Gas-kinetic models (based on an equation for the phase-space density $\rho(x, v, t)$).

First models for traffic flows appeared already in 1950s, today there are tens of variations of them [3, 9]. Each model of vehicular traffic should resemble flow phenomena observed in different circumstances: transitions from one dynamical phase to another (generally there are three dynamical phases of the flow: free-flow, synchronized flow, stop-and-go flow), criticality and self-organized criticality, metastability and hysteresis, phase-segregation, etc.

2. City traffic model

The city traffic model used in this paper is essentially similar to the one presented in [8] (BBSS). There are N^2 nodes (intersections) $I_{i,j}$, $i = 1, \ldots, N$, $j = 1, \ldots, N$, which form a square lattice. Each node has two incoming links: one from west-side and one from south-side, and two leaving links: one towards east-side and one towards north-side. Each node makes a decision which traffic stream should be served (i.e., decide which stream gets "green light"): the one from west towards east or the one from south towards north. Additionally a setup time τ can be specified. This is the amount of time which must pass when switching between streams. During the setup time all the streams have "red light" (or "orange light") and in reality this stage fulfills safety requirements and allows vehicles to leave the intersection. A sample 4×4 network is depicted in Fig. 1.

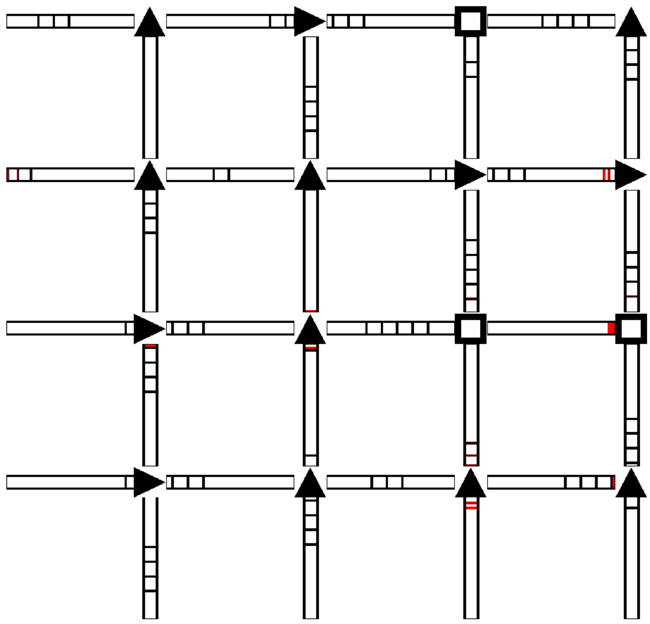

Fig. 1. A sample 4×4 net with link length $C = 50$. Vehicles leaving at east and north side are placed back on beginning of links at the west and south side. At the nodes: triangles represent flow direction ("green light" for each stream), rectangles identify nodes during setup time. Vehicles on links are represented by small rectangles

Through this paper we assume periodic boundary conditions. This means that vehicles leaving nodes placed at east and north boundaries, $I_{i=N,j}$ and $I_{i,j=N}$, will be placed again at corresponding links at west and south boundaries which are connected to $I_{i=1,j}$ and $I_{i,j=1}$ respectively. Therefore the total number of vehicles in such network remains constant and depends solely on initial conditions.

2.1. A single link. Each link in a network represents a single-lane street which is a one-dimensional cellular automaton with C cells. An occupied cell n symbolizes a single vehicle, therefore number of cells per lane should be chosen in such a way that the physical size of a cell is about the size of the vehicle. A discrete, integer variable v_n corresponding to the vehicle velocity is associated with each occupied cell. At each discrete time step $t \to t+1$ the state of automaton is updated according to certain rules. Let the maximum allowed velocity be $v_{\max}$ and the distance to the next vehicle d_n, then in the classical model by [10], the four *consecutive* steps for parallel updating [1] are:

1. Acceleration: $v_n \to \min(v_n + 1, v_{\max})$,
2. Breaking: $v_n \to \min(v_n, d_n - 1)$,
3. Randomization with probability P: $v_n \to \max(v_n - 1,\ 0)$,
4. Vehicle movement: $x_n \to x_n + v_n$.

All steps of this is basic model are necessary to reproduce the basic features of real traffic flow, like, e.g., the fundamental diagram. Step 1 represents driver tendency to drive as fast

[1] Parallel updating essentially means that all these steps are applied for *all* the vehicles at the same time. Note that these rules guarantee that the system is accident-free. In contrast to parallel updating, there is also possibility to do it in random sequential manner which gives different results and, e.g., does not lead to spontaneous traffic jam formation [2].

as possible, step 2 is necessary to avoid collisions and step 3 introduces random perturbations necessary to trigger spontaneous jam formation (which is a real phenomenon in traffic dynamics due to random changes of vehicle velocity in regions with high density). Finally, in step 4 the vehicles are moved according to the new velocity calculated in steps 1–3. There exists many modification of the Nagel-Schreckenberg model (NaSch), like incorporating cruise control where the fluctuations are turned off for $v_n = v_{\max}$ [11] or implementing slow-to-start rule [12] and many others.

Through this paper we use $v_{\max} = 5$, which should be equivalent to about 50 km/h in a real city traffic flow. Then, assuming that a single cell corresponds to a real size of 7.5 m (a vehicle length with safety distance in front and behind it), each step is about 2 seconds in real time. All the results presented below are calculated for automata with $C = 100$ or $C = 50$ cells, all vehicle velocities are 0 and vehicles are placed at random positions in a link in the initial state.

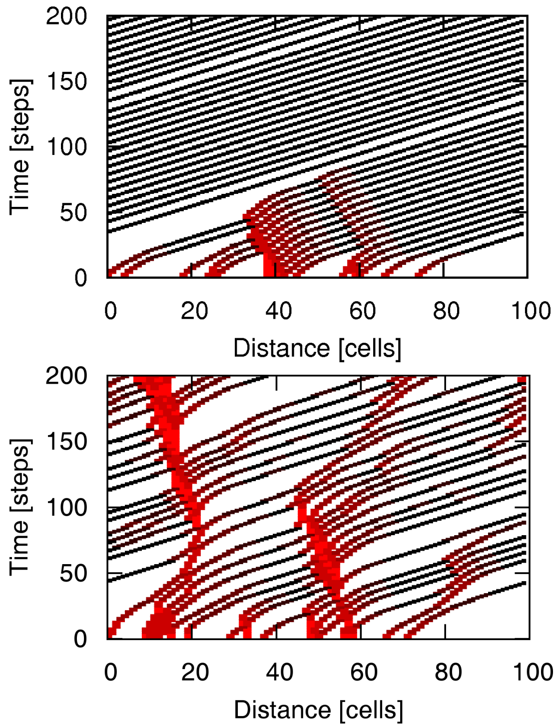

Fig. 2. Phase diagram showing movement of vehicles in a cellular automaton with NaSch rules. Each line represents a single vehicle, the maximum velocity is $v_{\max} = 5$, there are 100 cells, black color corresponds to the maximum velocity. Top: $\rho = 0.16$, $P = 0.0$, the density is close to $\rho_{\max} = 0.17$, there are no fluctuations and resulting flow $J = 0.8$ is near its maximum ($J_{\max} = 0.83$ for $\rho_{\max} = 0.17$). Bottom: due to the presence of fluctuations, $P = 0.25$, traffic jams are spontaneously formed and the mean flow is significantly reduced, $J = 0.503$

Figure 2 shows simulation of classical NaSch cellular automaton, i.e., a single link of our network. Again, the boundary conditions are periodic, i.e., the end of the street is connected with its beginning. The density $\rho = m/C$ is simply the number of vehicles m in the initial state divided by total number of cells in the link, C. For the density $\rho = 0.16$ (which means that 16 cells are occupied in an automaton consisting of 100 cells) and $P = 0$ the flow is in its free-flow state, that is, all the vehicles quickly reach their maximum velocity $v_{\max}$ without any stops. However, when fluctuations are introduced, $P = 0.25$, one can see spontaneous traffic jam formation.

Of course, for given $v_{\max}$ there exists a maximum density for which all the vehicles can move freely with $v_{\max}$, $\rho_{\max} = v_{\max}^{-1}$. If this density is exceeded, there exists at least one vehicle which has less than $v_{\max}$ occupied cells in front of it, and therefore is forced to slow down.

Relation between mean flow (number of vehicles leaving a street per unit time) and density, $J(\rho)$, is known as the fundamental diagram. The diagram for the NaSch model is presented in Fig. 3. Maximum value of flow J determines the critical density $\rho_{\max}$ above which the flow is no longer in free-flow state. Notice that including fluctuations decreases flow rate significantly.

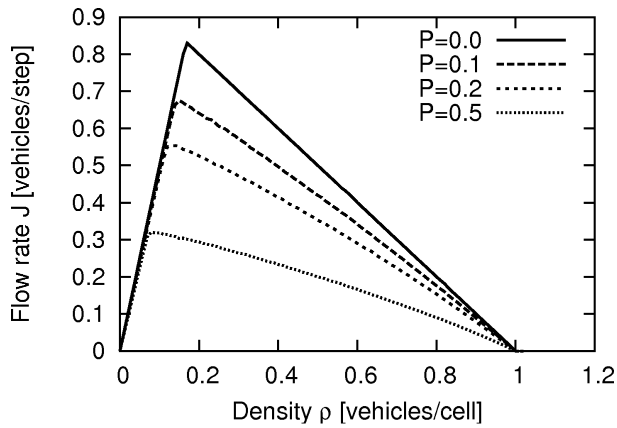

Fig. 3. Fundamental diagram for NaSch CA model. Mean flow rate $J(\rho)$ against vehicle density is shown. Notice the influence of different fluctuation probabilities P ($J(\rho)$ for $P = 0$ has a simple exact analytical solution)

In deterministic limits $P = 0$ and $P = 1$ it is possible to find dependence $J(\rho)$ exactly. For $P = 0$, in free flow regime, we have simply $J = \rho v_{\max}$, if $\rho > \rho_{\max}$ the average headway distance is $1/\rho - 1$, giving average flux $1 - \rho$, so for $P = 0$, $J(\rho) = \min(\rho v_{\max}, 1 - \rho)$. On the other hand, for $P = 1$, the flow rate is always zero, $J(\rho) = 0$ since vehicles are not able to accelerate. Although in this case for $\rho < (v_{\max} - 1)^{-1}$ it is possible to obtain a metastable state in which all vehicles travel with velocity $v_{\max} - 1$, such flow breaks down if any perturbations are present [9].

All results presented below are for the deterministic limit $P = 0$. This is somewhat artificial assumption, however it makes possible to fully define self-controlling nodes described in Subsec. 3.2.

2.2. Intersections. As already mentioned above, each link connects to a node which represents intersection of two streams. Each intersection gives "green light" to either west-east or south-north streams. There is an intermediate period between switching from "red" to "green" and "green" to "red", the setup time which takes τ steps. During the setup time no vehicle is allowed to pass the intersection.

In order to implement these rules into the model, we change rule 2 of the automata according to the BBSS model into:

2. Breaking:
   - Traffic light at the intersection to which the link is connected is "red" or the intersection is in setup time: $v_n \to \min(v_n, d_n - 1, s_n - 1)$
   - Traffic light is "green": if two cells behind the intersection are occupied: $v_n \to \min(v_n, d_n - 1, s_n - 1)$, otherwise $v_n \to \min(v_n, d_n - 1)$,

where s_n is the distance between vehicle n and its next intersection. The main difference between the model applied here and the BBSS model is introduction of the setup time. The nonzero case $\tau \neq 0$ does not change general features of the flow, however, it makes possible to relate flow rates directly with these for networks with self-controlling nodes (such nodes require $\tau \neq 0$). Throughout this paper we always use $\tau = 2$ (steps).

3. Controlling strategies

3.1. Periodic switching. The simplest possible strategy for control in nodes is to use cycle-based switching. For each node the cycle is

- "red light" for T steps
- setup time for τ steps
- "green light" for T steps
- setup time for τ steps

giving $2T + 2\tau$ steps in total. Additionally we allow phase shifts T^{ϕ} for different nodes in network. This means that the first step of the cycle is realized at time step $t + T^{\phi}$.

3.2. Self-controlled nodes. In contrast to the imposing cycle-based control process described above, we will consider a responsive self-controlling strategy proposed by [1] (LH). Here we only briefly sketch the general approach, for more detailed information see the paper by Lämmer and Helbing (the symbols used here are the same as in the cited work).

Let σ denote the stream which should be open for service (i.e., should get "green light"),

$$\sigma = \begin{cases} \text{head } \Omega & \text{if } \Omega \neq \emptyset \\ \arg\ \max_i \pi_i & \text{otherwise,} \end{cases}$$

where Ω is an ordered set containing stream indices which should be served in order to maintain stability, π_i is a priority index for the corresponding stream i. The controller realizes combination of two control strategies. One is called stabilization strategy which assures that each stream will be served at least once in $T_{\max}$ period and, on average, once in T_{avg}. If a stream i should be served in order to fulfill these requirements, its index is placed into the Ω set.

If set Ω is empty, $\Omega = \emptyset$, a stream with the highest priority index π_i is chosen for serving. The priority indices π are chosen in such a way that the total *expected* waiting time for vehicles is minimized. This control regime works well for small densities and consequently $\Omega = \emptyset$ when ρ is small.

The priority index for stream i, provided that currently served stream is σ, is defined as

$$\pi = \frac{\hat{n}_i}{\tau_{i,\sigma}^{\text{pen}} + \tau + \hat{g}_i},$$

where $\hat{n}_i$ is number of vehicles expected to be served in time $\tau + \hat{g}_i$ for the stream i, τ is the remaining setup time, $\hat{g}_i$ is time required to clear existing queue at the intersection *and* all vehicles arriving just after clearing, provided that they arrive with the maximum flow rate (i.e., as a platoon traveling with $v_{\max}$), $\tau_{i,\sigma}^{\text{pen}}$ is the additional penalty term for switching from stream σ to i.

Let us consider two streams: σ (with "green light") and i (with "red light") for flows with densities $\rho < \rho_{\max}$ (the same for each stream). The prioritization strategy will determine wheter to continue serving σ or start serving i in such a way that the expected waiting time for all vehicles is minimized. For example, if σ is being open, it may be more efficient to leave it open in order to serve approaching platoon instead of switching to i (where vehicles queue grows). Moreover, each switching is penalized due to presence of setup times.

However, if densities are large enough switching from σ to i could never occur (for example if queue being cleared at σ is infinite). This is when the second strategy takes over, $\Omega \neq \emptyset$, so each stream is served at least once in $T_{\max}$ steps.

This LH controlling strategy has been implemented in our cellular automata city traffic model. Here we consider only deterministic limit $P = 0$, so that it is possible to calculate exact number of approaching vehicles to an intersection $I_{i,j}$. Therefore appropriate priority indices can be found for each node. Note that such controller at each node uses only information from two links which are connected to it.

4. Results for regular networks

4.1. Single node case, N=1. The simplest possible network is, of course, when $N = 1$ (Fig. 4). First, let us consider periodic cycle-based switching strategy. Then, for given initial density ρ, P and τ, the flow rate through this single node depends solely on T. Such dependence is depicted in Fig. 5. Detailed discussion concerning dynamical phases of the flow for different ρ and T is presented in [8].

Naturally, strategy based on regular cycles imposes certain dynamical situation rather being responsive to the current traffic state. When T is properly adjusted vehicle platoons which are formed usually get "green light" giving maximum possible (optimal) flow rate J. If density is small enough, i.e., platoon length $\rho v_{\max} C$ per link is shorter than $C/2 - \tau v_{\max}$, that is

$$\rho < \frac{1}{2} v_{\max} - \tau / C \equiv \rho_{\text{crit}},$$

then there exists a cycle for which vehicles can move without stopping and the resulting mean flow is maximal, $J = J_{\max} =$

$\rho v_{\max}$ (similar relation holds for networks for $N > 1$ as well). However, since vehicle acceleration is finite, the condition $\rho < \rho_{\rm crit}$ does not guarantee that a state with maximal mean flow will be achieved for all initial conditions (in the fact, such state must be explicitly designed when ρ is close to $\rho_{\rm crit}$).

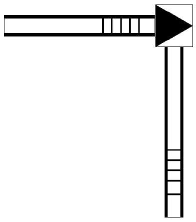

Fig. 4. Platoons of vehicles formed by the cycle-based intersection. T is chosen in such a way that all vehicles always get a "green light". This is possible, since the initial density for all links was small enough $\rho = 0.05$, $P = 0$, $C = 100$

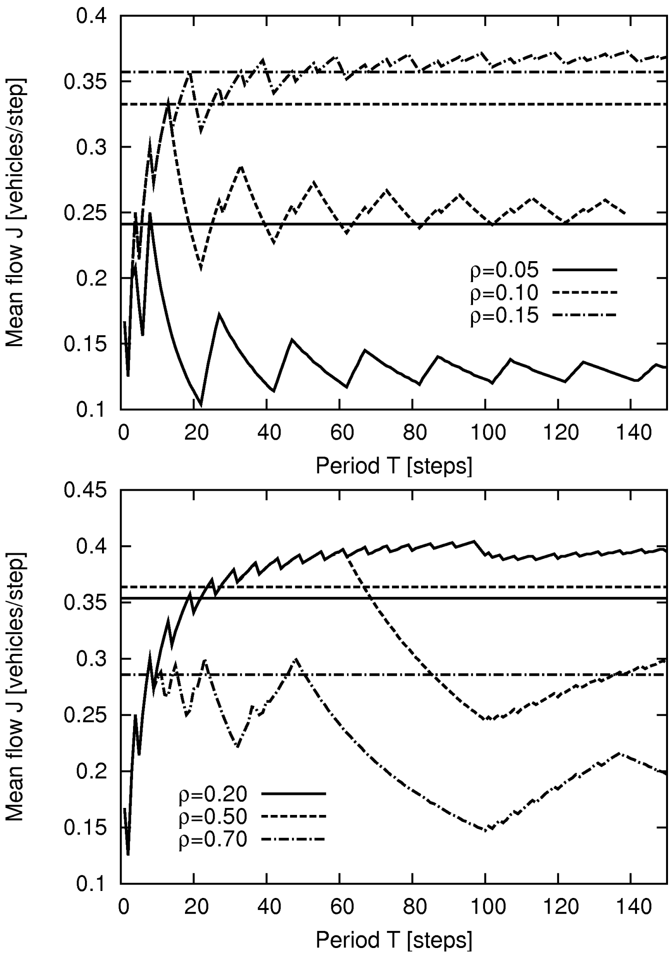

Fig. 5. Dependence of the mean flow J on green-time period T for a crossing of two streets with periodic boundary conditions for different densities. Top: $\rho < \rho_{\max} = 0.17$, bottom: $\rho > \rho_{\max}$, $v_{\max} = 5$. The horizontal lines denote J for a network with self-controlled nodes. In the free-flow regime the LH strategy performs very well and the mean flow converges closely to the optimal value. For densities $\rho > \rho_{\max}$ the stabilizing strategy with $T_{\rm avg} = 50, T_{\max} = 100$ takes over and the controller is forced to switch periodically with $T = T_{\rm avg}/2 - \tau$

On the other hand, for some values of T platoons are always stopped when arriving to the intersection. Consequently one can observe significant variations in $J(\rho)$ especially for smaller densities, $\rho < \rho_{\max}$, where is the largest potential for optimization. Note that by adjusting the green times it is possible to vary the mean flow by almost 100%.

If the cycle-based strategy is replaced by the LH self-controlling one, the situation is different. The only adjustable parameters are $T_{\max}$ and $T_{\rm avg}$ which are relevant only in the stabilization regime, i.e., for large densities. For $\rho < \rho_{\max}$ the strategy based on priority indices defined above quickly and automatically converges to the optimal solution found for the cycle-based method. The resulting flow rate (horizontal lines in Fig. 5) for the LH controller is near the maximum possible value.

The two working regimes: optimizing with priority indexes and stabilizing, can be clearly identified in Fig. 6. The vertical line denotes $\rho_{\max}$ above which the stabilizing strategy is dominant.

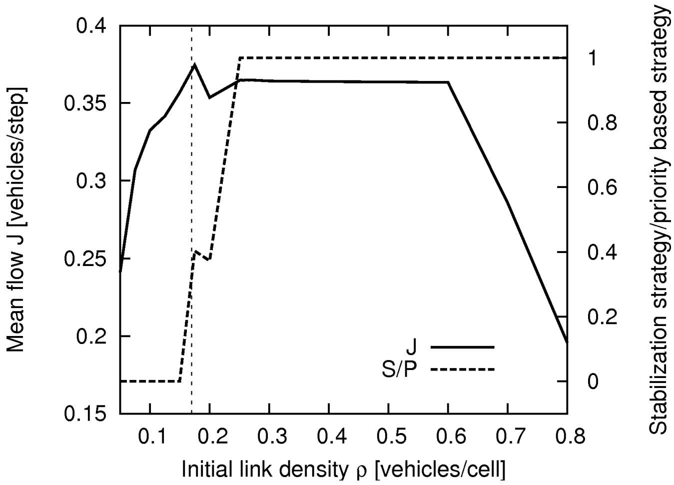

Fig. 6. $J(\rho)$ for LH self-controlling node and ratio stabilizing/prioritizing strategy, S/P. If S/P is close to 1 then stabilizing strategy dominates. This is clear for densities $\rho > \rho_{\max}$ ($\rho_{\max}$ is labeled with the vertical line)

4.2. 16 Nodes case, N=4. Naturally, even if the control at each intersection is optimal, the dynamic coupling of intersections in the network can make the overall flow inefficient or even unstable (meaning that queue length grow infinitely). In reality even very small and simple networks with simple switching rules can produce complex and chaotic dynamics, and generally it is impossible to predict the evolution of the system over longer time horizons. However, for the case of regular lattice-like network with one-way links and periodic boundaries, finding strategies giving good results is straightforward.

Consider now a network with $N = 4$ and cycle-based switching strategies in each node. In order to find maximum average flow (i.e. sum of average flow through all the nodes divided by the number of nodes) a green-wave optimization is applied. Such scheme is easy to find, because distance $C = 100$ between all the nodes is the same.

Let all the nodes have the cycle period set to T. At the time t, light at the intersection $I_{1,1}$ turns "green", and at the same time a vehicle approaches this intersection from the west side with maximum velocity. It takes $T_{\mathrm{delay}} = C/v_{\max}$ steps for the vehicle to get to the next intersection at the node $I_{2,1}$. Consequently the cycle in this intersection $I_{2,1}$ should be delayed by T_{delay}: $T^{\phi}_{(2,1)} = T_{\mathrm{delay}}$. Assuming that for $I_{1,1}$, $T^{\phi}_{(1,1)} = 0$, the phase shift for all the other nodes is then

$$T^{\phi}_{(i,j)} = (i + j - 2)T_{\mathrm{delay}} \bmod (2T + 2\tau).$$

Again, for cycle-based switching the flow rate depends only on the initial density ρ (the same for all the links) and cycle period T. Such dependence is shown in Fig. 7.

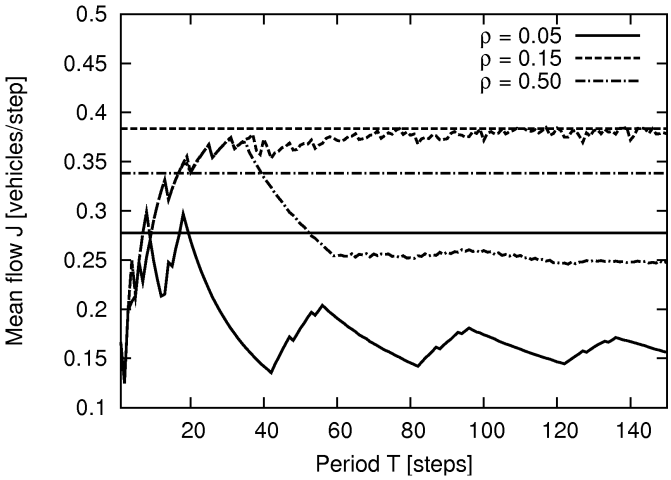

Fig. 7. $J(T)$ for cycle-based controllers in the $N = 4$ network for different densities. Horizontal lines mark mean flow values achieved when the LH self-controlling were used instead of the cycle-based

Analogously like for $N = 1$ case, simulations with self-controlled LH nodes were performed for different initial densities. The results are represented in Fig. 7 as horizontal lines. It is clear that also in this case, proper switching periods and phase shifts are found and the resulting mean flows are close to the maximum possible values. Note, that this is achieved without any parameterization (except, of course, the stabilization regime where $T_{\mathrm{avg}} = 50$ and $T_{\max} = 100$ were specified).

4.3. N=4 with turning allowed. Apart form the initial state, all the results presented above were fully deterministic ($P = 0$ with random vehicle positions in the first step $t = 0$). Let us introduce now an important feature of any real traffic flow, namely possibility of turning with given probability P_{turn}. This means that vehicles traveling along west-east (south-north) direction can enter an intersection provided that there is "green light" assigned to the appropriate stream. However, these vehicles do not have to follow theirs straight routes, but are allowed to turn to south-north (west-east) link with probability P_{turn}.

This is a significant change, since any strategy based on precalculated parameters relevant to certain flow characteristics (like the density which is obviously related to inflow rates) undoubtedly will perform worse if $P_{\mathrm{turn}} \neq 0$. The question is to what extend. In reality this is a general problem for any controlling strategy based on some flow observations from the past, because traffic conditions never occur in future exactly with the assumed, averaged parameters. There are always unpredictable events like traffic collisions, road works, etc., leading to significant discrepancy between expected and real traffic properties.

Figure 8 shows mean flow rates in the $N = 4$ network with turning probability $P_{\mathrm{turn}} = 0.2$ for cycle-based controller with period T and phase shifts resulting in the green wave scheme. When comparing this figure with Fig. 7, one can notice significant degradation in terms of the mean flow values J.

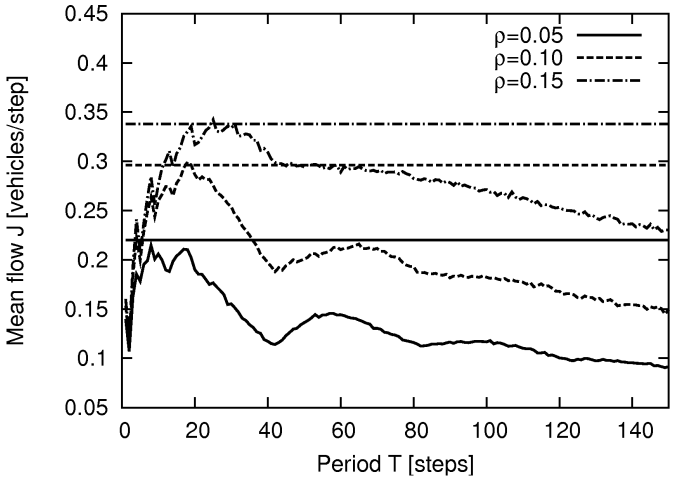

Fig. 8. As in Fig. 7 but here vehicle turning with probability $P_{\mathrm{turn}} = 0.2$ in each direction was allowed. Again, the horizontal lines denote average performance of the self-controlled LH strategy which gives very good results

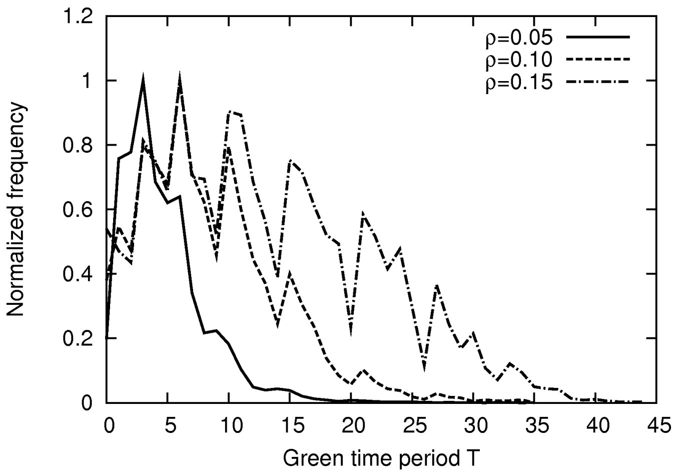

Fig. 9. Normalized histogram showing frequency for "green" time periods in 10000 steps for simulation $N = 4$ network with $P_{\mathrm{turn}} = 0.2$ and self-controlling LH nodes

As before, we have also performed simulations with self-controlled nodes (horizontal lines in Fig. 8). As one can see, the resulting mean flow is very large and, for $\rho = 0.05$, even larger than the maximum values for the cycle-based strategy.

This means that irregular switching cycles are present which, however, give superior overall performance.

In order to investigate what cycles are selected by the LH strategy in this scenario, we plot a histogram with frequencies for "green light" periods during simulation, see Fig. 9. We note that, for small density $\rho = 0.05$, very short "green time" periods are dominating. That means that switching occurs frequently in order to serve approaching vehicles. The situation is different for larger densities where many different "green time" periods are present.

5. Conclusions

In this paper we demonstrated two possible strategies for controlling flow in a simple and regular city-like traffic network with use of cellular automata model. One strategy is based on periodic cycle-based switching mechanism in which each of two streams is granted "green" light for T steps. It is easy to find optimal value T for which the resulting mean flow in the network is maximal, by performing simulations for all relevant values of T.

It is evident that the most promising regime for optimization is for low density flows, $\rho < \rho_{\max}$. This is understandable since for congested traffic where for each stream a large queue is present, there are no switching schemes which would outperform others significantly.

As the second control method we implemented a self-controlling strategy proposed by [1]. It has been shown that this methods quickly converges to the best solution (i.e., giving maximum possible mean flow), constructed with cycle-based scheme with optimal periods T and phase-shifts T^{ϕ} leading to the green wave formation. In other words, green waves emerged spontaneously without any parameterization. Such synchronization was possible because the strategy makes use of knowledge concerning approaching vehicles and, therefore, information can be propagated between nodes simply by vehicles.

The discussed autonomous controller has some features related to self-organized systems: lack of any central control unit, openness, scalability, failure tolerance. For example, if a single controller fails (i.e., all conflicting streams get "flashing orange light" and vehicles follow traffic signs), cycles at neighbouring intersections will be modified in order to adapt to the new situation. In contrast to the traditional cycle-based control there is no distinction between controlling and controlled elements: traffic lights control vehicles which in turn influence the lights. This approach leads to efficient utilization of the network for varying conditions.

Influence of such varying conditions was presented in Subsec. 4.3. It is clear that after introducing a possibility of turning with random parameter P_{turn}, the self-controlling strategy gives much better results. Naturally, even in this case the mean flow values are smaller when compared to the fully deterministic limit $P_{\mathrm{turn}} = 0$. However, still the performance is superior to that with regular, imposed cycles.

The next question which needs to be answered is how the discussed LH strategy will perform in networks with more complicated topologies with comparison to optimal (or nearly optimal) solution. Moreover, when using the CA traffic model, the non-deterministic $P \neq 0$ version needs to be applied.

There are recent realistic simulations of the LH strategy for a specific network of 13 intersections in the center of Dresden by [13]. The authors of [13] show that it possible to reduce waiting times by 56% for public transport vehicles, 9% for regular cars and by 36% for pedestrians and bicycle riders. The large differences in improvement are due to assignment different of weights for buses and trams when calculating priority indices, see Subsec. 3.2. In this way there is a possibility to prioritize certain vehicles in order to promote public transport in the city.

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