BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES
Vol. 57, No. 3, 2009

Production control in moving bottlenecks in conditions of unit and small-batch production

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Abstract. Since the production is aimed at fulfilling specific needs of demanding customers and not at filling warehouses, the production volume should reflect the volume of orders. In times of fight for the client every order has to be performed on time. What is more, in times of fight for shortening the delivery cycle, meeting safe deadlines, that is distant in time, is not enough. Companies are forced to meet short deadlines with keeping the product price competitiveness condition. It is hardly possible without a proper, APS (Advanced Planning System) class, advanced planning support system. Currently, advanced planning systems are coming into use, however their cost exceeds the possibilities of small and medium enterprises and algorithms used often require great customization to industries' needs and conditions of unit and small-batch production. The paper has been drawn on the basis of research on overloads of moving bottlenecks in conditions of unit and small batch production in real conditions having a big number of resources and tasks. The methods used so far are not capable of finding the global optimum of such big data ranges. At present few working enterprises in conditions of unit and small batch production, especially in small and medium-sized enterprises (SME), are exploiting techniques of the production process optimization. For this reason computer tools for applying to the industrial scale are needed. The above method basis on the data so far collected in computer systems. Results of preliminary research were introduced from applying the possibility of TOC (Theory of Constraints) to the industrial scale for reducing bottlenecks in unit and small batch production.

The authors built a heuristic algorithm which could find solution good enough and based on TOC assumptions and verification of assumptions using tests in real production systems. The above method found application to the industrial scale, as extension of the ERP class system.

Key words: unit and small batch production, Theory of Constraints, job shop scheduling, moving bottlenecks, heuristic algorithm.

1. Introduction

The guarantee of success on contemporary, more and more competence-driven and changeable market is fast and flexible implementation of production processes, which also assures immediate adjustment of production to changes both of the environment and more and more demanding customers. If the 70's were the times of costs reduction, the characteristic of the 80's was quality improvement, the 90's were focused on flexible production, the beginning of the 21st century is characterized by focus on customer's satisfaction. This trend translates into production of articles adapted to customer's needs and to shortening the availability of products – very often below the production cycle. To implement the tasks connected with production control in such conditions it is necessary to develop operational plans determining the order of production tasks performance by individual production sections. For the plans no to be a chance set of tasks it is necessary to order them properly and to optimize the course of processes. Production control in the moving bottleneck is particularly important. For the solution to this problem authors suggested method based on dynamic classifications and setting into groups organizationally similar.

2. Formal description of the issue

This paper contains some propositions regarding optimization of production plan (accepting the orders, sequencing of task) and a description of problems related to this issue in real conditions narrowed to above mentioned ones. Particular attention was given to issues of uncertainty and verification of algorithm assumptions using positive feedback in the plan. They are the key to obtaining a model which properly reflects the reality in which we often encounter vague or even incomplete information.

The job shop scheduling problem is a classic scheduling problem, where a set of n jobs $\{J_i\}_{i=1}^{n}$, must be processed or assembled on a set of m specified machines $\{M_j\}_{j=1}^{m}$. Each job J_i consists of a specific set of operations $\{w_{ik}\}_{k=1}^{n(i)}$, which have to be processed according to a given technical precedence order $n(i)$. The job shop scheduling optimization problem is to determine the optimal schedule for specifying the process sequence and the starting time of the jobs to be processed on the machines for optimizing the production targets. Job shop scheduling problem belongs to N^P-hard problems [1] and therefore the good enough solution was sought. According to TOC the method is based on focusing on the jobs of bottlenecks.

To find the solution of arranging task, measurements connected with the rate of utilization of machine job fund (Z_j) were used:

$$Z_j = \frac{H_j - \mathcal{J}_j}{w_j H_j}, \tag{1}$$

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where $\mathcal{J}_j$ – means the sum of jobs' process time on the machine j, H_j – standard working time of the machine j, w_j – rate of carry out the standard of the operation for the machine j. If $H_j < \mathcal{J}_j$ then machine j is overloaded.

The criterion of the optimization was formulated as

$$\sum_{j=1}^{m} -Z_j = \sum_{j=1}^{m} \frac{-(H_j - \mathcal{J}_j)}{w_j H_j} \rightarrow \min, \tag{2}$$

for every j where $H_j < \mathcal{J}_j$.

It is hardly possible to determine "a priori" fulfillment of the condition $H_j < \mathcal{J}_j$ so dp_aps_1 procedure (described in Subsec. 3.1) is committing preliminary selection of machines. More about tasks' optimization in an operational production plan based on dynamic classification can be found in [2].

2.1. Alternatives of the manufacturing process. The above-mentioned assumption should be supplemented with additional ones which will bring the issue closer to real conditions. The first assumption relates to alternatives of the manufacturing process. For some products and elements we possess a database of alternative routes. The routes of production process can basically be presented in the form of a graph.

The problem of arranging tasks was illustrated with example (Fig. 1). The presented problem concerns two tasks, each of them consists of three operations. Operations are being processed one by one by three different machines. Task J_1 consists of the following operations {1a, 1b, 1c}, task J_2 is a set of operations {2a, 2b, 2c}. Set of machines $M = \{M_1,\ M_2,\ M_3\}$, where machines execute the following operations: $M_1:\ \{1b,\ 2c\}$, $M_2:\ \{1a,\ 2a\}$, $M_3:\ \{1c,\ 2b\}$.

The acceptable answer was given as follows: $M1\colon \{1b \rightarrow 2c\}$, $M2\colon \{1a \rightarrow 2a\}$, $M3\colon \{1c \rightarrow 2b\}$.

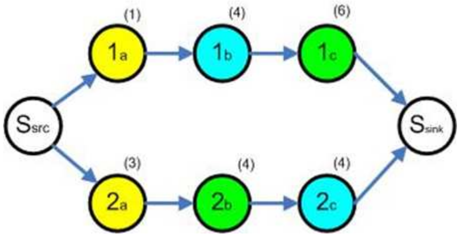

Fig. 1. Graph of model arranging tasks

From the practical point of view the above mentioned issue was broadened by alternatives of production process. Each task J_i is possible to be performed in alternative production processes. Task J_{ia} is a sequence $an(i)$ of operations $\{w_{ik}\}_{k=1}^{an(i)}$, for alternative $a(l)$, whose arrangement (order) is defined by a set of limitations usually described using a graph. Single performance of a task J_i in alternative a is enough for the task J_i to be performed. In the example (Fig. 2) for the task J_1 there are 3 variants a_{11}, a_{12}, a_{13} of the production process and for the task J_2 there are 2 variants a_{21}, a_{22}. The number of possible solutions of scheduling is growing rapidly. Model solution (see Fig. 2) is based on choosing the alternative a_{12} for task J_1 and a_{21} for task J_2. The schedule for the machines' tasks is as follows: $M1\colon \{1a \rightarrow 2c \rightarrow 1d\}$, $M2\colon \{2a\}$, $M3\colon \{2a \rightarrow 1b \rightarrow 1e\}$, $M4\colon \{1c\}$.

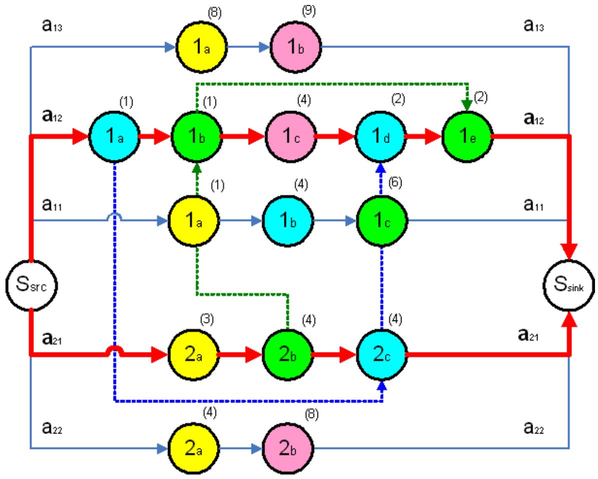

Fig. 2. Graph of the model solution for scheduling with alternatives of the manufacturing process

Practically the amount of alternatives is enormous. The amount of alternatives is limited by owned machines and machines of cooperators of the enterprise. In principle we can define the optimal variant in statistical terms (e.g. – without taking into account the availability of machines or the influence of this variant on other processes). The criterion of choice of this variant in statistical sense is usually costly but a large-area analysis can also be conducted. The main variant – optimal in the statistical sense, will play crucial part in searching for a global optimal solution. The phrase "searching for" is the key one here since finding such a solution in case of N^P hard problem is rather a chance event. Broader information on this subject can be found in [3].

2.2. Example illustrating the issue. Let's define the problem. In (Fig. 3) we can see 3 orders of different products realized at the same time. We can see conflicts (marked with dotted circle) on the yellow (y) and green resource (g) for order no 1 and no 2, yellow resource – for order no 2 and no 3 and on blue resource (b) – for order no 1 and no 3. In this case the time of the production will be longer.

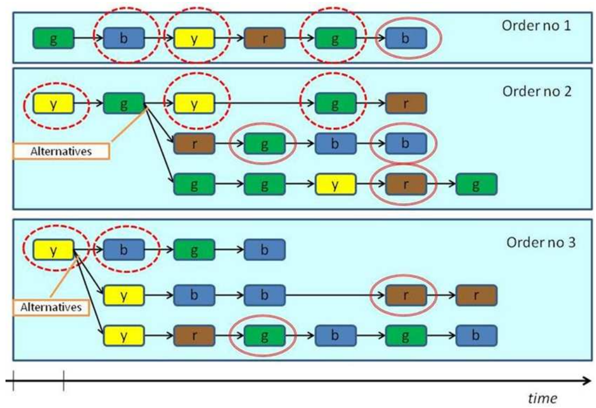

Fig. 3. Definition of the problem

But if we have alternative variants of the process we can reduce appearing of bottlenecks. In this case we eliminated some conflicts for order 1 and 2 but new conflicts appeared (marked with double circle).

Choice of proper variants of the process is a solution in given organizational conditions. In practice we are creating the optimal variant in static condition and we are creating substitute variants which can be optimal in specific organizational conditions (for instance: overloading resources).

2.3. Solutions already used. There are many algorithms used to solve problems of arranging tasks (scheduling), which can be divided into two main groups: optimizing (accurate) and approximating (rough). The first group is algorithms guaranteeing finding an optimal solution. Practically speaking, while solving problems of bigger scale only approximating techniques are used, which do not guarantee finding an optimum but they require fewer resources and are faster. The main problem in approximating algorithms is "being stuck" in one of local extremes [3]. The basic strength of this type of algorithms is finding acceptable, "good enough" solution. One of the main problems to be solved is the starting point determining the pace of reaching the aim function and the possibility of avoiding "being stuck" in the local extreme. The algorithm presented below can be also counted among approximating methods. This article is based on Robertson and Perera model of automated data collection for simulation [4] and ERP data and functions presented by Gupta and Kohli in [5]. This kind of problems are also solved using methods of engineering optimization [6].

3. Solution algorithm

The solution in the method above is based on assumptions of the Theory of Constraints formulated by Eliyahu M. Goldratt in business novel [7]. In the solution we will use those elements of the Theory of Constraints which refer to a bottleneck [8]. Since the flow of material stream is limited by the flow in the bottleneck, the profit of a company directly correlates with this flow. The theory, simple in its essence, will be strengthened by mathematical device and computer data container of ERP-class data system.

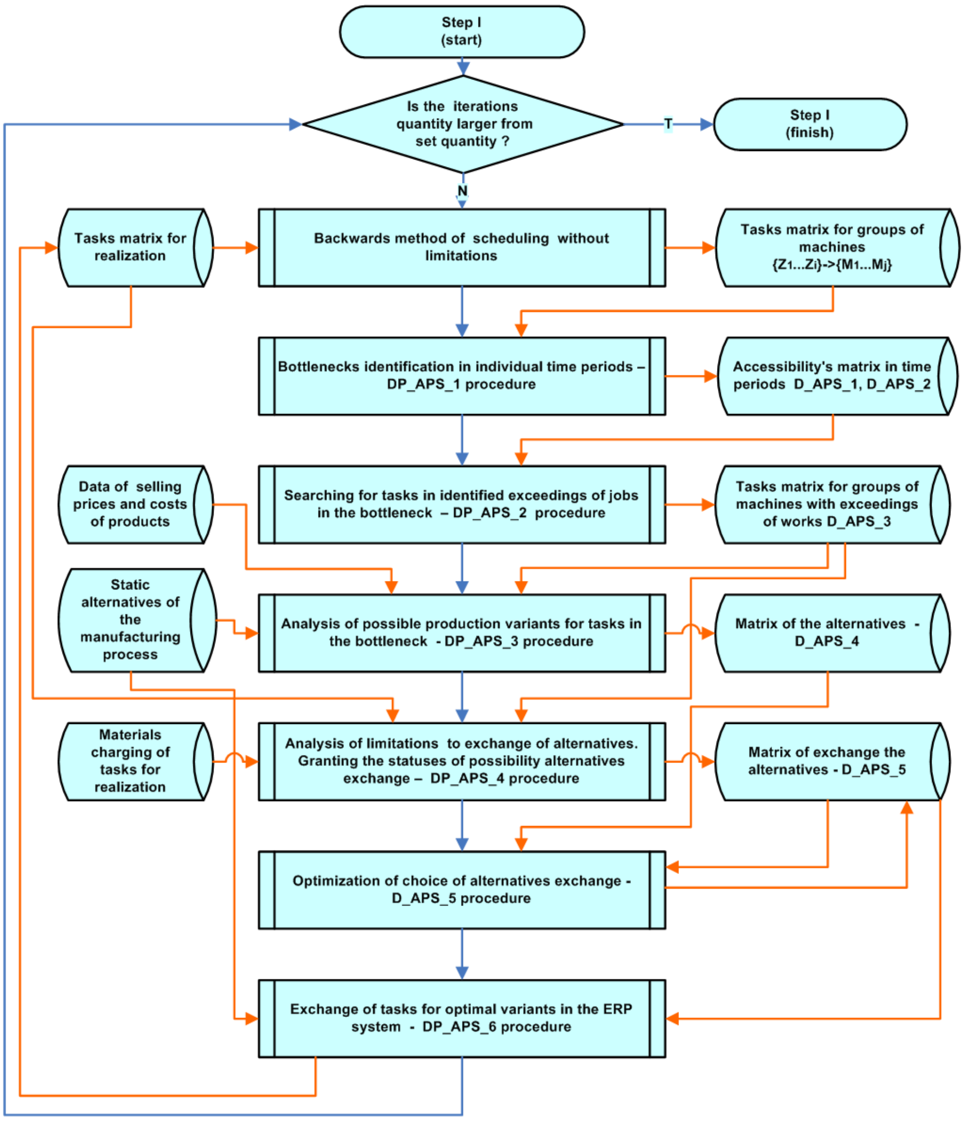

Fig. 4. General scheme of the optimization algorithm

Creating the algorithm (Fig. 4.) we focused on the bottleneck's work. The problem is however the fact that the bottleneck is moving, it appears periodically at some machines, while very seldom or never at others. In the first step of the algorithm we make standard backwards scheduling using the method without resources limitations. For production orders the work tasks have been originally generated in the variant described as the major one. It is usually the statistically optimal variant of the process. Assigning the reverse scheduling function to so prepared tasks allows us to perform tasks as late as possible. Limitless scheduling allows for identifying overloads of particular resources in specific periods of time. Additionally, the above method of scheduling enables calculating the normative length of the cycle – $\mathcal{F}_i^*$ and the sum of lengths of cycles being the basis for calculating – $\sum_{i=1}^{n} \mathcal{F}_i^*$, the rates ν_i of the lengthening of production cycle calculated according to $\nu_i = \dfrac{\mathcal{F}_i}{\mathcal{F}_i^*} \geq 1$, $V_{\max}$ – maximum of the lengthening of production cycle calculated according to $V_{\max} = \max_{0 \leq i \leq n} \nu_i$, $\overline{V}$ – medium of the lengthening of production cycle calculated according to

$$\overline{V} = \frac{\sum\limits_{i=1}^{n} \mathcal{F}_i}{\sum\limits_{i=1}^{n} \mathcal{F}_i^*}. \tag{3}$$

Since we only focus on critical resources of the whole range of tasks and from the resources we only pick up those for which the sum of tasks $\sum \mathcal{I}_k$ is greater than the accepted norm $\sum H_k$. The multiple overload will remain under the relation (2). The problem to be solved is still the density of time axis division. Generally, we dispose of daily density, weekly density or monthly density. Assuming monthly density is burdened with too great an error. In the sum of a month there may not be any excess; there can however appear heaps in particular days and weeks. On the other hand, assuming daily accuracy seems to be exaggerated. Optimization within daily plans should be left to be solved in the next optimization phases. The above assumptions have been verified by production practice.

3.1. Bottlenecks identification – DP_APS_1 procedure. The operation of the procedure for bottleneck identification DP_APS_1 consists in assigning aggregated overloads to weekly ranges and groups of positions. In the first step the availability of work positions is calculated on the basis of data of class ERP system. In class ERP system the availability of work positions, in other words job time fund follows from 3 basic attributes of positions group: the number of positions (machines), work calendar (working days, non-working days – planned renovations, failures, holidays etc.) and regulations of work scheme (1 – shift, 2 – shift, continuous work etc.).

The aim of this step is to create the matrix H_{kt} of job fund $H\colon \{1, 2, \ldots n\} \times \{1, 2, \ldots m\}$ aggregated to particular weeks of job fund, where $\{1, 2, \ldots, m\}$ is a set of machines groups and $\{1, 2, \ldots n\}$ is a set of numbers of week of the year. In the next step what is investigated is the sum $\sum \mathcal{I}_k$, of job intensity of particular terms ranges, in addition to which the terms of tasks performance follow directly from previously implemented scheduling function. In the next step we calculate the matrix of overloads PC while the element of matrix $pc_{kt} = (j_{kt} - h_{kt})$, where j_{kt} is the element of tasks matrix and a h_{kt} is the element of job fund matrix. The PC matrix is presented in D_APS_1 table. If $pc_{kt} > 0$ in order to speed up calculations the outcomes are written into a supporting table D_APS_2. In order to depict the functioning of the procedure, the achieved outcomes have been presented below on table (Fig. 5) and on graph (Fig. 6). Data originates from the process of the verification of assumptions in conditions of the production practice.

Overloading	Weeks							
Machines	2007/52	2008/01	2008/02	2008/03	2008/04	2008/05	2008/08	Sum
F02			6.80					6.80
F03	12.62	143.54	20.31					176.47
F04	19.41	14.91						34.32
F08		52.07	3.03					55.10
K01			5.44					5.44
M01					17.29		16.50	33.79
S01			0.11					0.11
SL01		135.00	201.35	79.98		58.40		474.73
SL02	18.82							18.82
SL03		30.68	100.00	57.35	2.88			190.91
T01	76.20	65.15	12.96	20.00				174.31
T03		13.86	28.54					42.40
W01			4.80					4.80
W02				53.33				53.33
Sum	127.04	455.21	383.33	210.65	20.17	58.40	16.50	1271.31

Fig. 5. Overloading of the work (of machines) in the week's aggregation

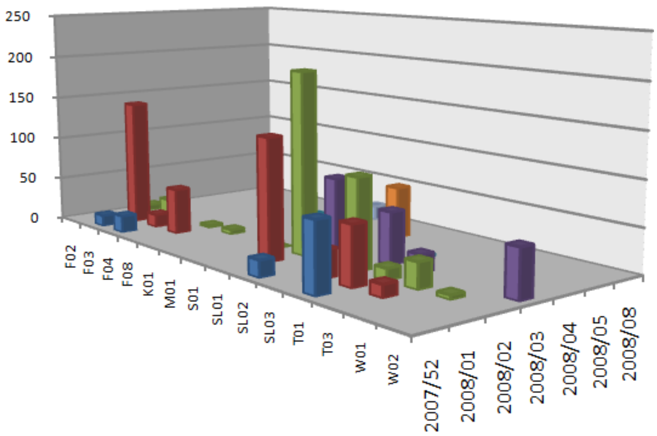

Fig. 6. Graph of tasks' overloading in the week's period

3.2. Searching for tasks from time ranges in the bottlenecks – DP_APS_2 procedure. The operation of the procedure for tasks searching in bottlenecks – DP_APS_2 consists in defining the area of possible exchanges of alternatives of production process. In order to do it, it is necessary to find all tasks $\{J_1, J_2, ..., J_n\}$ for which operations $\{w_{ac}, w_{ik}, ..., w_{az}\}$ in a weekly time range show overload $pc_{kt} > 0$. In order to speed up the calculations the outcomes are written down in the supporting table D_APS_3. Additionally, apart from the tasks list also the elements' code and number of units produced are searched for. The above information will be the input data to the procedure of searching for alternatives of a production process for the elements. Figure 7 is showing demonstration results of calculations.

TASK'S ID	ELEMENTS	AMOUNT
0102007002546006002	2837-1212V-4	5.000
0102007002602002002	36-755V2-P3	3.000
0102007002662001002	2V18-V842-P4	2.000
0102007002662001003	2V18-V842-P5	1.000
0102007002695001003	V182 1968 VV--V2	16.000
0102007002695001005	0182 1968 00--04	16.000
0102007002717009003	MX91C25385	8.000
0102007300491001002	40-751721-P14	1.000
0102007300491003001	40-75456	1.000
0102007300514004001	MX00X22074	6.000
0102007300514005001	MX00X22075	6.000
0102007300520004001	MX00X22074	5.000

Fig. 7. The set of tasks in the bottleneck from the overloaded period (F02-2008/02)

3.3. The analysis of possible production variants for tasks in the bottleneck – DP_APS_3 procedure. The operation of the procedure for tasks searching in bottlenecks – DP_APS_3 (Fig. 8) consists in the analysis of possibilities connected with the change of process variant into less overloading for the bottleneck. In order to do that it is necessary to build the matrix of possible solutions for elements from D_APS_3 matrix and for every a_i variant to calculate job intensity of a variant in the bottleneck and the rate of value stream flow through the bottleneck. The essence of the procedure is connected with the evaluation of value stream flow through the bottleneck. What was used in this case was the assumptions of Theory of Constraints connected with cost evaluation of the variant used to solving the traditional problem PQ.

What was used in this case was the assumptions of Theory of Constraints connected with cost evaluation of the variant used to solving the traditional problem PQ [9]. We calculate the rate of throughput per time of capacity constraint resources (CCR):

$$\xi_k = \frac{Tu_i}{\mathcal{I}_i}, \quad Tu_i = \mathrm{P} - \mathrm{TVC_i}, \tag{4}$$

where P – price of product, $\mathrm{TVC_i}$ – total variable cost in i variant.

Total variable cost is calculating according to Throughput Accounting (TA) [9]. In this case, equals material purchase costs), $\mathcal{I}_i$ denotes job intensity in the bottleneck (CCR) in the i variant. The higher this rate, the better is

$$\xi_{opt} = \max_{i \leq n} \xi_i.$$

In analyzed cases there appeared in some variants the cases of lack of work of the bottleneck.

Then $\xi_k = \infty$, which means optimal variant from the point of view of critical resource in the given time range (Fig. 9).

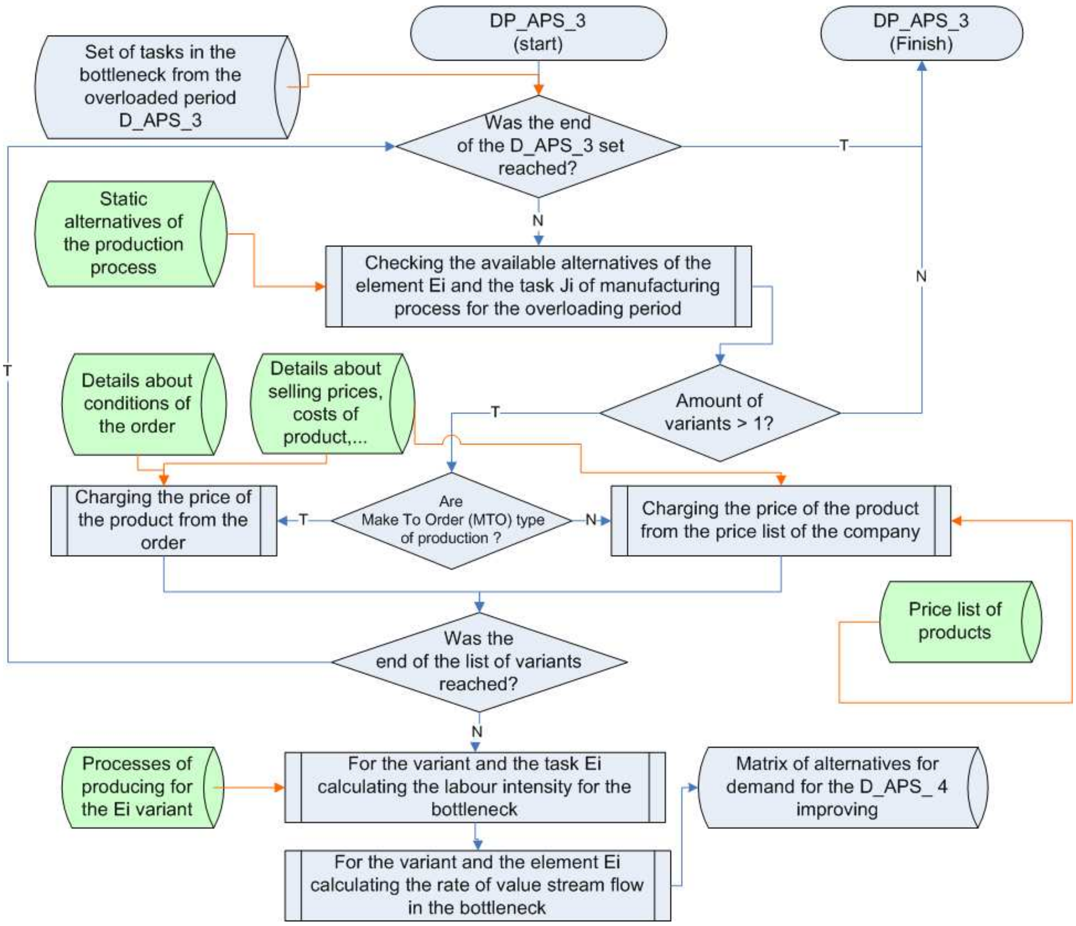

Fig. 8. Analysis of alternatives of processes of manufacturing for tasks in the bottleneck

BOTTLENECK	ELEMENTS	ALTERNATIVES	Tu_i	$\mathcal{I}_i$	ξ_i	AMOUNT
F02	2837-1212V-4	1	32.995	1.750	18.854	5.000
F02	2837-1212V-4	2	32.995	1.750	18.854	5.000
F03	V16V 1V63 VV	1	600.717	58.000	10.357	24.000
F03	V16V 1V63 VV	2	600.717	44.400	13.530	24.000
F03	V16V 1V63 VV	3	600.717	44.400	13.530	24.000
F03	V322 2V53 VV	1	256.849	29.500	8.707	16.000
F03	V322 2V53 VV	2	256.849	17.200	14.933	16.000
F03	V863 1213 VV	1	464.101	3.600	128.917	2.000
F03	V863 1213 VV	2	464.101	3.600	128.917	2.000
F03	V863 1213 VV	3	464.101	3.200	145.032	2.000
F03	V863 1213 VV	4	464.101	2.800	165.751	2.000
F03	142FA2	1	512.760		max	3.000
F03	142FA2	2	512.760	12.000	42.730	3.000
F03	28-VV3V43	1	525.346	8.500	61.805	3.000
F03	28-VV3V43	2	525.346	6.900	76.137	3.000
F03	28-VV3V44	1	527.317	6.900	76.423	3.000
F03	28-VV3V44	2	527.317		max	3.000
F03	28-VV3V44	3	527.317		max	3.000

Fig. 9. The rate of throughput per time of capacity constraint resources (CCR)

3.4. The analysis of limitations to exchange of alternatives for tasks matrix – DP_APS_4 procedure. The operation of the procedure for searching of limitations to exchange of variant DP_APS_4 (Fig. 10) consists in the analysis of limitations connected with:

- limitation of availability of alternatives number,
- limitation of material charging,
- limitation of advancement elements performance for tasks.

In case of appearing limitations for task $\mathcal{I}_i$ the limitation status is calculated and written down in the alternatives exchange table D_APS_5.

3.5. Optimization of choice of alternatives exchange – DP_APS_5 procedure. The operation of the procedure of searching for optimization of choice of alternative exchange DP_APS_5 generally consists in searching through the set of possible solutions of D_APS_5 taking into account the limitations analyzed in procedure DP_APS_4 in connection with the demand for decreasing overloads of the bottleneck from the set D_APS_2. Optimization consists in arranging exchange variants according to the cost criterion of the rate of ξ_k values stream flow to the moment when demand for job intensity after exchange of variants > 0. Additionally, there follows the checking of next limitation relating to conformity of material demand in optimal variant with a variant so far appearing in task J_k. As a result of procedure operation there are creat-

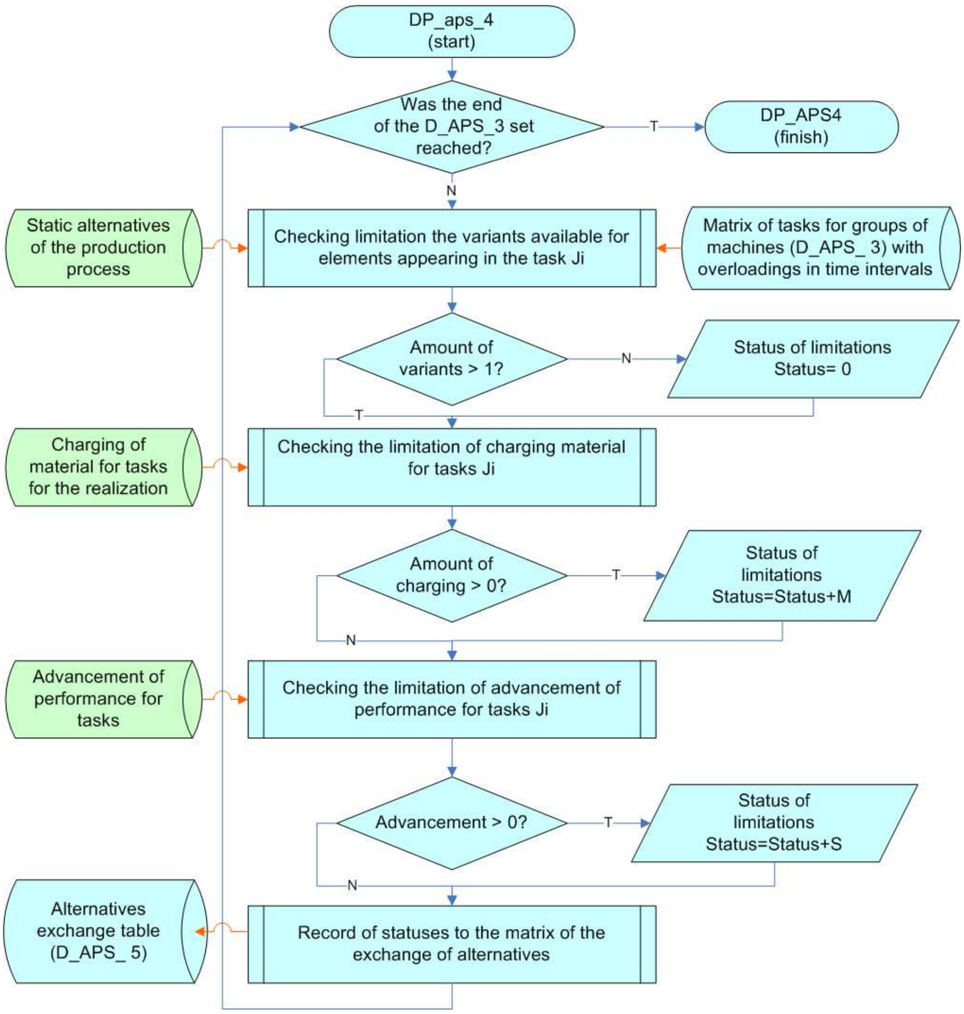

Fig. 10. Algorithm of analysis of limitations to exchange of alternatives for tasks matrix

ed lists of tasks for variants exchange divided into 3 groups, taking into account different limitation levels:

- group 1 – tasks having no limitations,
- group 2 – tasks having limitations connected with charging but material demand is compatible,
- group 3 – tasks having limitations connected with advancement but material demand is consistent,
- group 4 – tasks designed for variants exchange, eventually having considerable limitations (e.g. advancement > 0). Their exchange is connected with considerable organization costs and requires manual interference. Exchange of variant is not recommended in this group.

The outcome of procedure's operation is defined by the possibility of variants' exchange (Fig. 11). Basically the exchange is applicable to group 1 and group 2.

MACHINES	WEEK	OVERLOADING	I GROUP	IV GROUP	III GROUP	II GROUP
F02	2008/01	19.650	18.850	0.750	18.850	0.750
F02	2008/02	27.850	27.850	25.700	27.200	25.700
F03	2007/52	12.617	12.617	12.617	12.617	12.617
F03	2008/01	123.643	123.643	103.223	123.643	103.223
F03	2008/02	28.350	27.430	24.930	27.430	24.930
F04	2007/52	19.411	19.411	19.411	19.411	19.411
K01	2008/02	5.240	5.240	5.240	5.240	5.240

Fig. 11. Division of tasks into groups of the possibility variants' exchange (fragment)

3.6. Exchange of tasks for optimal variants – DP_APS_6 procedure.

The operation of the procedure of exchange of tasks for optimal variants DP_APS_6 consists in interference into the schedule of tasks performance and exchange in the ERP system data base of the tasks list into an optimal list according to the DP_APS_5 procedure and of outcomes saved in matrix D_APS_5 and D_APS_2. After conducting DP_APS_6 procedure we perform another iteration starting from scheduling with backwards method. In practical conditions, after undergoing three iterations there was neither improvement nor further decrease of overloads noticed.

4. Experimental research

The research was done on 6 undertakings marked A-G, of different production characteristics (Fig. 12). Appropriate samples concerning production system were taken from the companies. Input data come from accumulated data bases of the REKORD.ERP system.

FIRMS		AMOUNT OF ORDERS	AMOUNT OF JOBS	AMOUNT OF TASKS	AMOUNT OF MACHINES	SUM OF THE TASKS' WORKING TIME (IN HOURS)	SUM OF THE CYCLES' LENGTH (IN DAYS)	SUM OF THE OVERLOADING (IN HUORS)	AMOUNT OF ELEMENTS
A	1	803	26729	116652	178	222174.24	62726.25	33062.42	10576
	2	821	27973	123003	179	231508.96	65697.86	33429.60	10828
	3	827	28017	121482	177	223398.92	66117.97	32335.44	11115
B	1	364	2195	19182	29	88361.93	11621.05	2490.52	1617
	2	356	2576	25616	30	115101.59	17210.21	4495.61	1717
C	1	466	5099	16318	38	29275.61	4505.45	1838.00	4101
	2	365	3976	14079	40	23747.51	6313.22	729.32	3140
D	1	1081	3008	18865	127	51296.30	62120.92	14044.83	1588
	2	834	2303	12820	125	32363.26	55673.33	7702.45	1445
E	1	1937	3468	11480	153	16967.56	1952.12	4879.67	2250
F	1	131	50102	59789	33	5870.15	764.67	417.00	2579
G	1	60	60	500	22	8891.48	940.35	0.00	59

Fig. 12. Input data for researches algorithm's usefulness

4.1. Operation time of optimization procedures.

The scope of research included a number of iterations of optimization procedures, from 11 in the first case to 1 in the case of G and F companies. In the method above is to find a practical application, it should achieve effectively in finite time of 2 to 3 hours. The authors assume daily activation of several iterations. The number of iterations has been defined experimentally. Quite considerable differences appeared between execution times of optimization procedures (e.g. Fig. 13 and Fig. 14) nevertheless summary times didn't exceed acceptable values. Summary times of individual courses have been presented below (Fig. 15).

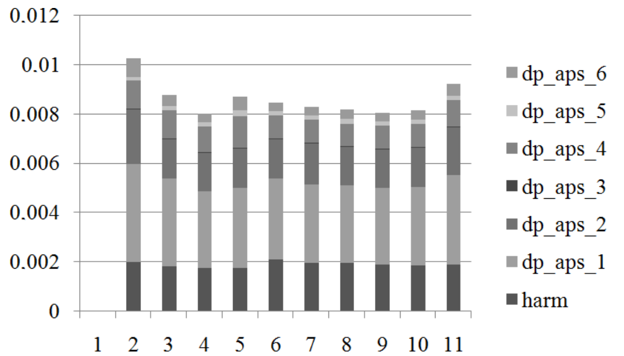

Fig. 13. Demonstration operation times of optimization procedures (A1) (in minutes)

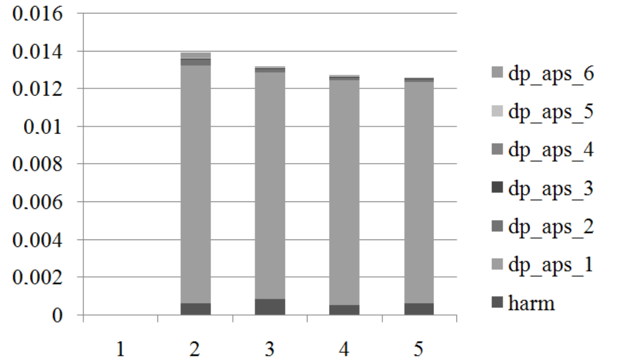

Fig. 14. Demonstration operation times of optimization procedures (E1) (in minutes)

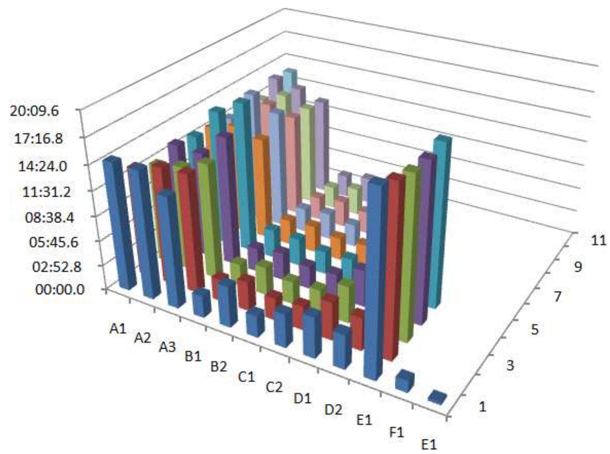

Fig. 15. Sums of operation times of optimization procedures, in iterations (in minutes)

The longest time of one iteration operation did not exceed 20 minutes, which in courses of 3–6 iterations used, gives the time of 60–120 minutes. The achieved time results allow for practical application of the above method.

4.2. Exemplary analysis of research outcomes – Sample A. Production company A is a company with the most complicated structure of the production, with a large number of production resources, biggest both job intensities and lengths of cycles. In research 3 samples of the state of the production system were taken, in 2 week's periods. The plan of research assumed performing such an amount of the iteration of courses, which the system will achieve stable value, optimal and further routes after won't cause the decrease in the sum of overloading.

Sample A1

As a result of the operation of procedures (Fig. 16) an amount of orders and groups of machines in the operation production plan kept steady. Reducing amount of jobs, from initial amount 26729 up to minimum into 11 iterations 26343 (reducing about 1.4%), it proves about choosing by the system cooperative variants, in addition their amount wasn't significant. The most interesting characteristics of the rate of overloading were shown below (Fig. 17).

NUMBER OF THE ITERATION	AMOUNT OF ORDERS	AMOUNT OF JOBS	AMOUNT OF TASKS	AMOUNT OF MACHINES	SUM OF THE TASKS' WORKING TIME (IN HOURS)	SUM OF THE CYCLES' LENGTH (IN DAYS)	SUM OF THE OVERLOADING (IN HUORS)	AMOUNT OF ELEMENTS	AMOUNT OF ELEMENTS (I GROUP)	AMOUNT OF ELEMENTS (II GROUP)	AMOUNT OF ELEMENTS FOR EXCHANGE
1	**803**	**26729**	**116652**	**178**	**222174.24**	**62726.25**	**33062.42**	**10576**	**5485**	**5149**	**20647**
2	803	26595	115937	178	220588.19	62465.43	32129.15	10561	3529	4406	20090
3	803	26510	115488	178	219129.09	62177.11	30754.17	10546	2424	3564	19976
4	803	26452	115324	178	219166.09	62233.22	30898.04	10537	2056	2990	19446
5	803	26425	115149	178	218396.76	62056.97	30646.49	10538	1856	3219	19708
6	803	26405	115070	178	218409.33	62085.72	30636.06	10538	1806	3223	**19650**
7	803	26381	115035	178	218110.54	62032.94	30331.89	10534	1763	2993	19628
8	803	26374	114920	178	217841.74	61962.79	**30095.82**	10533	1629	**2892**	19572
9	803	26357	114892	178	**217704.54**	**61916.94**	30317.59	10533	1833	2963	20057
10	803	26358	**114869**	178	217807.09	61961.54	30459.11	10533	**1587**	3029	19731
11	803	**26343**	114911	178	217905.99	61982.51	30108.69	**10532**	1652	2970	20202

Fig. 16. Results of the optimization procedure operating in the A1 sample

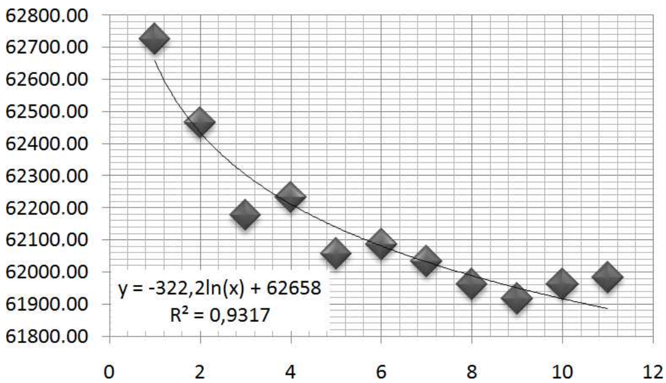

Fig. 17. The profile of the rate the sum of overloading in A1 sample in iterations (in hours)

After the initial considerable decrease in the rate in three first iterations, the more further decrease is smaller or even in next iterations it is growing. The amount of available variants doesn't let for more further reducing the indicator, what's more in iterations from 4 to 11 the indicator is fluctuating around constant value. Exchanging variants by the system to different only transferring overloading the around one bottleneck causes to different, not submitting than new. In this case applying next iterations won't bring significant benefits and he will increase only a computation time. In absolute value reducing the sum of overloading achieved value 2966 of the hours, and in percentage terms 8.9%.

As a result of algorithm operation the summary job intensity also decreased together with summary length of cycles, while there can be observed oscillation around constant value (see Fig. 18 and Fig. 19).

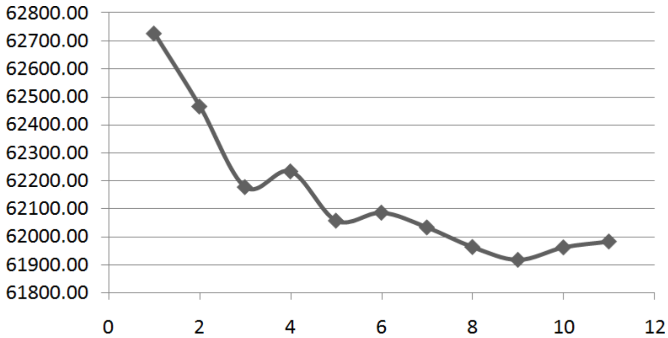
Fig. 18. The profile of the rate of summary job intensity in A1 sample (in hours)

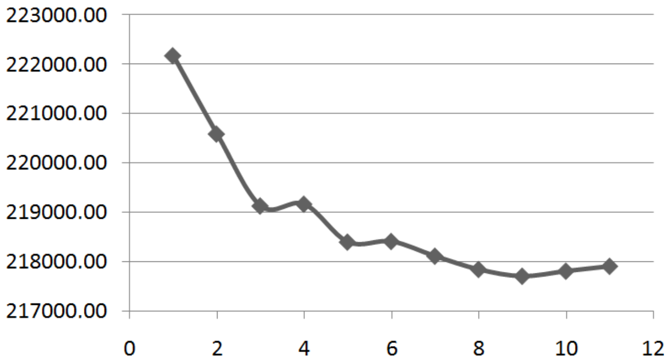
Fig. 19. The profile of the rate of summary length of cycles in A1 sample (in days)

Both graphs are similar to oneself, but not always the gradient of reducing the sum of the job intensity is transferred for reducing the sum of the length of cycles. It is connected with extended structures of the production and a possibility of parallel producing elements up to needs for the assembly. A rate of preparing the company to the work in alternative processes of producing is a next important parameter. It is associated with a possibility of the exchange of variants. If the small group of elements has alternative processes, the effectiveness of the algorithm will be limited. Value of the above rate is shown below (Fig. 20). Above rate, in spite of relatively of maximum values from about 10000 in the first route to 5000, however doesn't allow for complete reducing overloading.

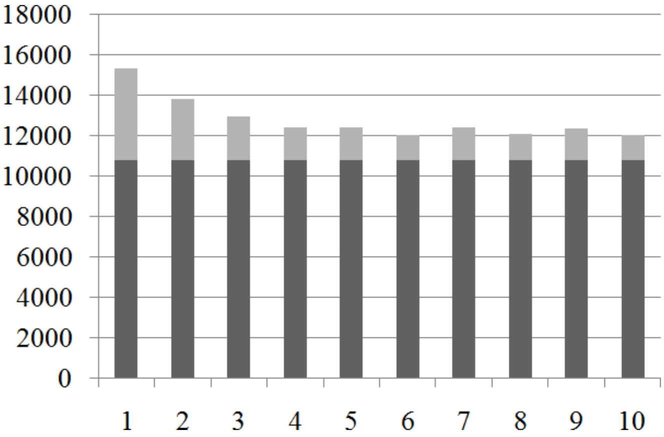

Fig. 20. Characteristics of the rate of the exchange of elements in A1 sample

4.3. Exemplary analysis of research outcomes – Sample B2. The objective of examination of the second sample was to define both the degree of method's usefulness and necessary number of iterations and to confirm the outcomes of sample 1 examination. In order to do that, 10 iterations in each sample were conducted (Fig. 21).

NUMBER OF THE ITERATION	AMOUNT OF ORDERS	AMOUNT OF JOBS	AMOUNT OF TASKS	AMOUNT OF MACHINES	SUM OF THE TASKS' WORKING TIME (IN HOURS)	SUM OF THE CYCLES' LENGTH (IN DAYS)	SUM OF THE OVERLOADING (IN HUORS)	AMOUNT OF ELEMENTS	AMOUNT OF ELEMENTS (I GROUP)	AMOUNT OF ELEMENTS (II GROUP)	AMOUNT OF ELEMENTS FOR EXCHANGE
1	356	2576	25616	30	115101.59	17210.21	4495.61	1717	5	1420	2755
2	352	2536	24527	30	107586.76	16280.19	3129.82	1695	12	383	2306
3	352	2526	24409	30	106729.51	16181.50	2683.65	1691	5	400	2395
4	352	2523	24385	30	106722.55	16193.80	2684.03	1688	11	232	2194
5	352	2523	24397	30	106735.14	16192.41	2648.64	1688	11	249	2315
6	352	2523	24384	30	106702.89	16190.84	2657.67	1688	9	248	2185
7	352	2523	24389	30	106608.51	16179.76	2582.47	1688	5	241	2301
8	352	2523	24392	30	106841.64	16203.76	2787.31	1688	11	293	2187
9	352	2523	24389	30	106594.04	16177.04	2580.07	1688	5	281	2308
10	352	2523	24383	30	106722.86	16193.81	2683.97	1688	12	251	2193

Fig. 21. Results of the optimization procedure operating in the B2 sample

The phenomenon of bottlenecks moving was confirmed also in sample 2. In individual iterations, while exchanging variants, to unload the overloads in one place, in one group of machines, the system can move them to the other machines. Let us check the conduct of a group of millers. Presented below is the configuration of overloads on the weekly axis with details of overloads after each of key iterations. Sample 2 was taken in the time of intensive growth of the order portfolio. It has a greater normative summary length of cycles, the number of tasks and job intensity in comparison with B1. The above relationship is a result of a production workshop assuming a bigger number of tasks in relation to its realizations in the preceding period. The number of orders has also increased together with the sum of overloads reaching the value of 4495.61 h. The number of groups of machines taking part in the operation plan has slightly increased (from 29 to 30). In the research 10 iterations were inducted.

As a result of procedures operation the number of orders changed (falling by 4). It is connected with choosing the variants of full cooperation. The relations of the number of jobs change similarly to sample B1, deceasing by 50 items. The characteristic of tasks number in individual iterations also tends to decrease, while it also stabilizes at a certain level (Fig. 22). The numbers of tasks after a rapid jump in the first iteration (sending tasks to cooperation) are gradually decreasing and stabilizing in subsequent courses. The characteristic of a rate which is most interesting for us has been shown above (Fig. 23). It behaves quite predictably and similarly to sample B1. After initial considerable decrease of the rate in the first three iterations the further decrease was observed not until iteration 6. The number of available variants does not allow for further decreasing of the rate, what is more, even in iterations from 4 to 10, the rate oscillates around a certain value (2700 h). In the later phase, while exchanging some variants into others the system only causes transfer of the overload form one bottleneck to the other, not contributing anything new.

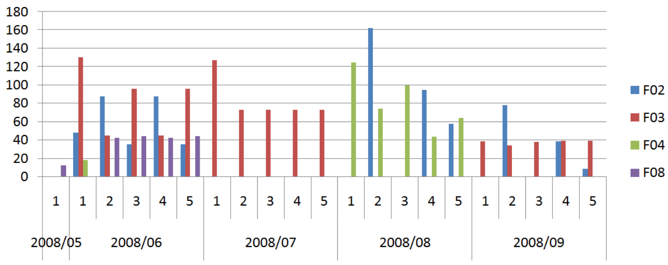

Fig. 22. Layout's graph of overloading after individual iterations in the period – in the B2 sample (F02, F04, F08 groups)

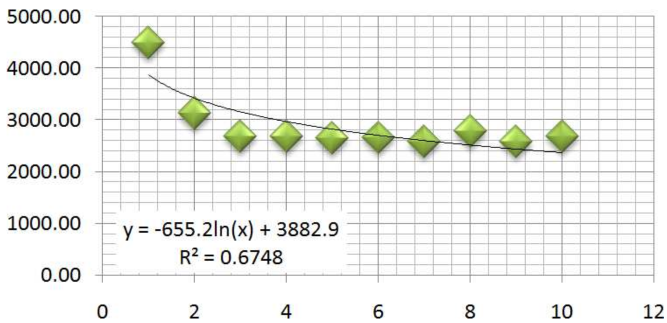

Fig. 23. The profile of the rate the sum of overloading in B2 sample (in hours)

As can be seen in this case using subsequent iterations (after 5) will not bring much profit but will only lengthen the time of calculations. The system starts „to vibrate" and the amplitude of "vibrations" of the system is running at around 100 h (Fig. 23). In absolute numbers, the decrease of sum of overloads amounted to 1811.64 h, and in percentages it reached over 40% – while the maximum value was observed in iteration 9. The above result was reached using to the full the production process alternatives.

As a result of algorithm operation the summary job intensity also decreased together with summary length of cycles, while there can be observed oscillation around certain value (Fig. 24 and Fig. 25).

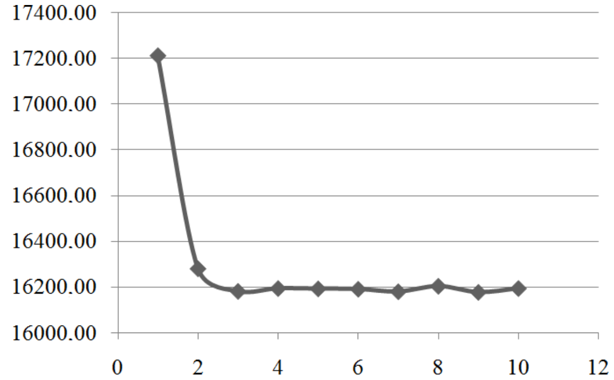

Fig. 24. The profile of the rate of summary job intensity in B2 sample (in hours)

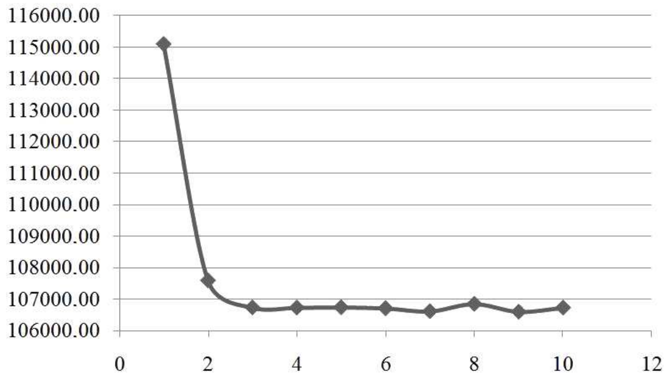

Fig. 25. The profile of the rate of summary length of cycles in B2 sample (in days)

Similarly to A1, A2, A3 the graphs have similar characteristics. Also the rate of company's preparation to working in alternative production processes looks similar to a B1. The above rate of relatively high values in absolute system, from around 1400 in the first course to 250 in courses 6–10 allows for a considerable reduction of overloads. It happens so thanks to great saturation with alternative variants – the overload has improved by nearly 40%. It is worth noticing the fact of growth in the number of group 1 elements (Fig. 26).

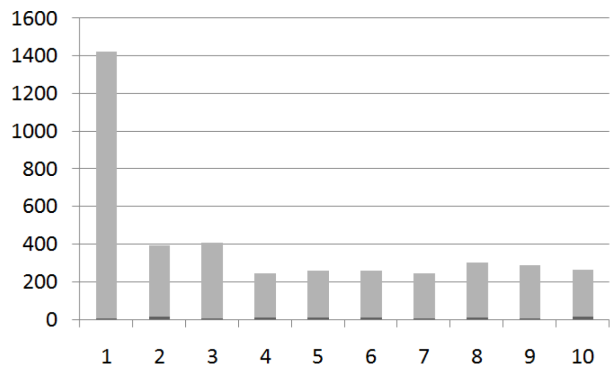

Fig. 26. Characteristics of the rate of the exchange of elements in B2 sample (in hours)

4.4. Exemplary analysis of research outcomes – F. Production company F is a company with the extended structure of the production, with not very large number of production resources, in addition it produces mainly not very complicated products from simple elements. Manual processing is dominating. In research 1 sample of the state of the production system was taken. The plan of examinations assumed performing such an amount of the iteration of routes, which the system will achieve stable value, optimal and more distant routes after won't cause the decrease in the sum of overloading. In spite of overloading unfortunately the company didn't have alternative variants. Results were shown below (Fig. 27). From that reason next iterations wasn't also made.

NUMBER OF THE ITERATION	AMOUNT OF ORDERS	AMOUNT OF JOBS	AMOUNT OF TASKS	AMOUNT OF MACHINES	SUM OF THE TASKS' WORKING TIME (IN HOURS)	SUM OF THE CYCLES' LENGTH (IN DAYS)	SUM OF THE OVERLOADING (IN HUORS)	AMOUNT OF ELEMENTS	AMOUNT OF ELEMENTS (I GROUP)	AMOUNT OF ELEMENTS (II GROUP)	AMOUNT OF ELEMENTS FOR EXCHANGE
1	131	50102	59789	33	5870.15	764.67	417.00	2579	0	0	875

Fig. 27. Results of the optimization procedure operating in the F sample

4.5. Exemplary analysis of research outcomes – G. The production company G is a company of an automotive, producing simple and on average assembled elements of plastics. In research 1 sample of the state of the production system was taken. The plan of examinations assumed performing such an amount of the iteration of routes, which the system will achieve stable value, optimal and more distant routes after won't cause the decrease in the sum of overloading. Arranging tasks showed, that overloading didn't appear and further research didn't make sense (Fig. 28).

NUMBER OF THE ITERATION	AMOUNT OF ORDERS	AMOUNT OF JOBS	AMOUNT OF TASKS	AMOUNT OF MACHINES	SUM OF THE TASKS' WORKING TIME (IN HOURS)	SUM OF THE CYCLES' LENGTH (IN DAYS)	SUM OF THE OVERLOADING (IN HUORS)	AMOUNT OF ELEMENTS	AMOUNT OF ELEMENTS (I GROUP)	AMOUNT OF ELEMENTS (II GROUP)	AMOUNT OF ELEMENTS FOR EXCHANGE
1	60	60	500	22	8891.48	940.35	0,00	59	0	0	0

Fig. 28. Results of the optimization procedure operating in the G sample

4.6. Summary of the experimental research results. Summary of the experimental research was shown below (Fig. 29).

FIRMS AND SAMPLES		SUMMARY OF THE OVERLOADING (START OF OPTIMIZATION)	SUMMARY OF THE OVERLOADING (FINISH OF OPTIMIZATION)	SIZE OF REDUCED OVERLOADING (IN HOURS)	SIZE OF REDUCED OVERLOADING (IN PERCENT)	AMOUNT OF ELEMENTS FOR EXCHANGE	SUMMARY OF REDUCED THE CYCLES' LENGTH (IN DAYS)	SUMMARY OF THE REDUCED WORKING TIME (IN HOURS)	TOTAL OPERATING TIME
A	1	33062.42	30095.82	2966.60	8.97	10634	809.31	4469.70	2:17:26
	2	33429.60	31334.17	2095.43	6.27	10185	610.16	3222.19	2:15:44
	3	32335.44	30405.81	1929.63	5.97	9136	593.41	3017.13	2:14:11
B	1	2490.52	1885.23	605.29	24.3	870	797.64	5287.23	0:27:52
	2	4495.61	2580.07	1915.54	42.6	1425	1033.17	8507.55	0:33:46
C	1	1838.00	732.27	1105.73	60.1	4	12,12	62.95	0:26:16
	2	729.32	721.36	7.96	1.09	6	10.62	1.13	0:28:39
D	1	14044.83	13449.53	595.30	4.24	3933	203.74	1175.34	0:38:59
	2	7702.45	7477.96	224.49	2.91	699	146.06	745.54	0:19:23
E	1	4879.67	4423.40	456.27	9.35	385	46.47	490.44	34:03.6
F	1	417.00	417.00	0.00	0.00	0.00	0.00	0.00	00:01.1
G	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	00:00.0

Fig. 29. Overall comparison of findings

While examining the above enterprises it has been found that:

- Among examined companies the usefulness of the above method is visible in companies which have for a long time

been using alternative processes in which the attained decrease of overloads reaches 60%.
- Efficiency of the above method does not cause problems in practical implementation.
- In case of companies with elaborate products structures usefulness is particularly visible.
- In companies which lack overloads (G1) using of the method is groundless.

There exists the whole range of companies not prepared for using this method (e.g. sample F).

What is new in that approach? Considerable an advantage of the method is automated data collection for simulation in conditions of unit and small batch production. In conditions of unit and small batch production collection of the data is an extremely time consuming process predominantly because the task is manually orientated. Hence, automating this process of data collection would be extremely advantageous.

This paper presents one of examples how simulation could utilize the ERP system as the simulation data source. It may be one of steps for Digital Factory creating [10]. The concept of automatic data collection through an interface between the simulation model and ERP class system could not be a distant (Fig. 30). It is a problem demanding separate consideration.

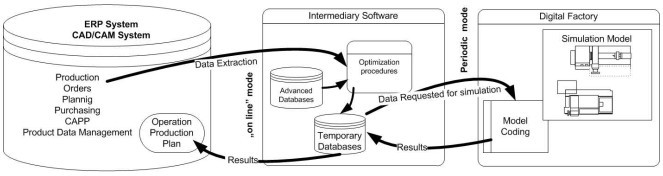

Fig. 30. General concept of presented solution

5. Conclusions

In research papers there can be found descriptions of many test problems of tasks ordering. It is difficult to find an example of a problem solved in real conditions of such a number of tasks and job resources. Therefore, the authors have presented the analysis of the problem of tasks ordering on real data in a broad spectrum of many production companies. The authors' aim is not to prove superiority of this method over others. The task was to state usefulness of the method of process alternatives exchange in real conditions. The results below refer to states before optimization and after its application. Providing the above results helped to define the rim conditions of companies in which usefulness of this method is sufficient. Heuristic algorithms cannot be proven using mathematical methods. A number of tests on real data have been carried out to prove this method.

The above method can be called the simulation "on line". This method found application to the industrial scale, as extension of the ERP class system.

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