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The L^1-impulse method as an alternative to the Fourier series in the power theory of continuous time systems

M. SIWCZYŃSKI* and M. JARACZEWSKI

Faculty of Electrical and Computer Engineering, Cracow University of Technology, 24 Warszawska St., 31-155 Cracow, Poland

Abstract. The Fourier series method is frequently applied to analyze periodical phenomena in electric circuits. Besides its virtues it has many drawbacks. Fourier series usually have slow convergence and fail for fast changing signals, especially for discontinues ones. Therefore they are suitable to describe only quasiharmonic phenomena.

For strongly nonsinusoidal signal analysis we propose the $\boldsymbol{L}^1$-impulse method.

The $\boldsymbol{L}^1$-impulse method consists in an equivalent notation of a function belonging to $\boldsymbol{L}^1$ as a sum of exponential functions. Such exponential functions have rational counterparts with poles in both sides of imaginary axis. With the $\boldsymbol{L}^1$-impulse functions we can describe periodical signals, thus we get the homomorfizm between periodical signals and a rational functions sets. This approach is especially adapted to strongly deformed signals (even discontinues ones) in linear power systems, and thanks to that we can easily calculate optimal signals of such systems using the loss operator of the circuit. The loss operator is exactly the rational function with central symmetry of poles [1].

In this paper the relation between the $\boldsymbol{L}^1$-impulse and the Fourier series method was presented.

It was also proved that in the case of strong signal deformation the $\boldsymbol{L}^1$-impulse method gains advantage.

Key words: periodically time-varying networks, operational calculus, stability, synthesis, optimization.

1. L^1-impulses and periodic signals

The $\boldsymbol{L}^1$-impulse is an absolutely summable signal

$$x : \int_{-\infty}^{\infty} |x(t)|dt < \infty$$

and its periodic extension is a T-period function

$$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} x(t + pT), \tag{1}$$

where $p \in \mathbb{Z}$ (integers).

Series (1) always converges. It results from the fact that every $\boldsymbol{L}^1$-impulse is majorised by an exponential signal:

$$y(t) = ae^{bt}\mathbf{1}(-t) + ce^{-dt}\mathbf{1}(t), \tag{2}$$

where a, b, c, d – positive numbers, $\mathbf{1}(t)$ – step function.

Applying the formula (1) to the (2) we get

$$\begin{aligned}\tilde{y}(t) &= \sum_{p=-\infty}^{\infty} y(t+pT) = a\sum_{p=-\infty}^{\infty} e^{b(t+pT)}\ \mathbf{1}(-t-pT) \\ &\quad + c\sum_{p=-\infty}^{\infty} e^{-d(t+pT)}\mathbf{1}(t+pT) \\ &= ae^{bt}\left(\sum_{p=1}^{\infty} e^{-bpT}\right) + ce^{-dt}\left(\sum_{p=0}^{\infty} e^{-dpT}\right) \\ &= \frac{ae^{-bT}}{1-e^{-bT}}e^{bt} + \frac{c}{1-e^{-dT}}e^{-dt},\end{aligned} \tag{3}$$

for $t \in [0, T)$.

The inner product of the $\boldsymbol{L}^1$-impulses is defined as follows

$$(x, y) = \int_{-\infty}^{\infty} x(t)y(t)dt \tag{4}$$

and the linear operator H

$$Hx(t) = \int_{-\infty}^{\infty} h(t, t')x(t')dt' \tag{5}$$

maps the $\boldsymbol{L}^\infty$ in itself only if [2]:

$$\bigwedge_{t\in\mathbb{R}} h(t, \bullet) \in L^1,$$

where $\boldsymbol{L}^\infty$ – space of bounded signals, R – real numbers.

The special case of the (5) operator is the convolution operator

$$h * x(t) = \int_{-\infty}^{\infty} h(t - t')x(t')dt', \tag{6}$$

which maps $\boldsymbol{L}^\infty$ into $\boldsymbol{L}^1$ only if impulse function

$$h \in \boldsymbol{L}^1.$$

It results that the sequence of two convolution operators act as $\boldsymbol{L}^\infty$ into $\boldsymbol{L}^\infty$ mapping, which means at the same time that

$$h_1 * h_2 \in \boldsymbol{L}^1 \quad \text{for} \quad h_1 \in \boldsymbol{L}^1,\ h_2 \in \boldsymbol{L}^1. \tag{7}$$

Thus the convolution of the $\boldsymbol{L}^1$ impulses produce also the $\boldsymbol{L}^1$ impulse.

*e-mail: e-3@pk.edu.pl

The H^* operator is the adjoint operator for linear operator H, which meets the condition

$$(Hx, y) = (x, H^* y), \tag{8}$$

for any x, y belonging to the $\boldsymbol{L}^1$ impulses.

It is easy to prove that the kernel $h^*(t, t')$ of the adjoint operator (5) meets the equation

$$h^*(t, t') = h(t', t). \tag{9}$$

Thus, for the convolution operator (6) we have

$$h^*(t) = h(-t). \tag{10}$$

The Fourier transform of any signal is given by the formula

$$Y(s) = \overline{y}(s) = \int\limits_{-\infty}^{\infty} y(t) e^{-st} dt. \tag{11}$$

The convolution operator is then written as follows

$$[Hx]^{-}(s) = H(s)X(s)$$

and the adjoint operator performs according to the formula [1]:

$$\begin{aligned} H(s) = \int\limits_{-\infty}^{\infty} h(t)e^{-st}dt &\rightarrow \int\limits_{-\infty}^{\infty} h(-t)e^{-st}dt \\ &= \int\limits_{-\infty}^{\infty} h(t)e^{-s(-t)}d(-t) \\ &= \int\limits_{-\infty}^{\infty} h(t)e^{-(-s)t}dt = H(-s). \end{aligned} \tag{12}$$

Thus the adjointing formula has form

$$H^*(s) = H(-s). \tag{13}$$

In the case of convolution operators, the adjoint operators (10) and (13) can be treated as *adjoint signals*.

The self adjoint linear operator R has a form

$$R = \frac{1}{2}(H + H^*) \tag{14}$$

and is called the loss operator of the H operator. This name derives from the fact that if H denotes the immitance operator then R determines the energy delivered to the two terminal receiver. This energy is given by the quadratic form [2] and [3]:

$$(Hx, x) = (Rx, x). \tag{15}$$

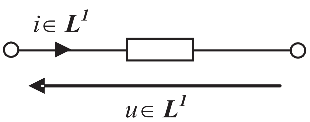

Fig. 1. The two terminal receiver which is supplied with energy by pair of the $\boldsymbol{L}^1$ impulse signals

According to the Parsevall's formula, the energy supplied to the receiver can be calculated as an inner product

$$\begin{aligned} (u, i) = \int\limits_{\infty}^{\infty} u(t)i(t)dt &= \frac{1}{2\pi j}\int_{\updownarrow} U(s)I(-s)ds \\ &= \frac{1}{2\pi j}\int_{\updownarrow} U(-s)I(s)ds. \end{aligned} \tag{16}$$

In the case of current input the formula becomes the quadratic form

$$\begin{aligned} (u, i) &= \frac{1}{2\pi j}\int_{\updownarrow} Z(s)I(s)I(-s)ds \\ &= \frac{1}{2\pi j}\int_{\updownarrow} R(s)I(s)I(-s)ds, \end{aligned} \tag{17}$$

where (see Fig. 1)

$$R(s) = \frac{1}{2}[Z(s) + Z(-s)], \tag{18}$$

is the loss operator of the two terminal network, and:
$Z(s)$ – stands for impedance operator of convolution type,
$\updownarrow$ – symbolize the path along imaginary axis toward its increasing values.

Ultimately, in the formula (17) we have the integral of an energy function

$$F(s) = R(s)I(s)I(-s), \tag{19}$$

witch is self adjoint function

$$F(-s) = F(s). \tag{20}$$

The integration formula (17) has special application when $F(s)$ is rational or quasi rational function. In that case it is convenient to apply the Jordan's lemma, according which we change the imaginary axis integral into the curvlinear integral along the simple closed curve (arc + imaginary axis) on left or right half plane

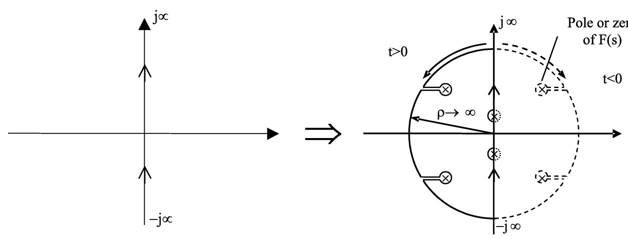

Fig. 2. The substitution of axis integral for the simple closed curve integral when ($\varrho \to \infty$)

This substitution, according to the Jordan's lemma, is possible when the arc integral with $\varrho \to \infty$ on '(' – left or ')' – right half plain converges to zero

$$\left[\int_{(\vee)} R(s)I(s)I(-s)ds\right]_{\varrho\to\infty} \rightarrow 0. \tag{21}$$

Actually it is enough to meets the condition (21) on only one contour because of central symmetry of $F(s)$ function (20).

Thus the inner product can be calculated only with contour integral

$$(u,i) = \frac{1}{2\pi j}\oint R(s)I(s)I(-s)ds = \frac{(-1)}{2\pi j}\oint R(s)I(s)I(-s)ds, \quad (22)$$

where: $\oint$ – left arc (with $\varrho \to \infty$) + imaginary axis contour integral, $\oint$ – right arc (with $\varrho \to \infty$) + imaginary axis contour integral .

To calculate (22) we use simply the Cauchy residue method.

The Jordan lemma can also be applied to calculate the inverse Laplace transform on the both sides of the time axis. In that way we can calculate *Ri* voltage signal ('active voltage') in t domain occurring in (15) or (17).

$$Ri(t) = \begin{cases} \dfrac{-1}{2\pi j}\oint R(s)I(s)e^{st}ds & \text{for} \quad t<0 \\ \dfrac{1}{2\pi j}\oint R(s)I(s)e^{st}ds & \text{for} \quad t>0 \end{cases}. \quad (23)$$

We should notice that in (23) the poles of the integrated function are placed on both half planes. Because of that we get the signals in the $\boldsymbol{L}^1$ impulse form. The poles and zeros of $R(s)$ has quadratic symmetry on complex plane and $I(s)$ can also have poles on both sides of complex plane.

2. The rational functions and the exponential functions factorization of L^1-impulses (isomorphism)

The very important issue will be discussed in this chapter – the isomorphism between the rational function set of complex variable and the exponential $\boldsymbol{L}^1$ impulse set of positive and negative time domain. Thanks to that we can create a special operational calculus in the $\boldsymbol{L}^1$ impulse class and also in its periodical extensions.

The following relation defines the isomorphism between right-hand (casual) exponential $\boldsymbol{L}^1$ impulse and the partial fraction of complex variable

$$e^{\sigma t}\mathbf{1}(t) \leftrightarrow \frac{1}{s-\sigma} \quad \text{for} \quad Re\sigma < 0. \quad (24)$$

The formula describing similar isomorphism, for left-hand (noncasual) $\boldsymbol{L}^1$ impulse has form

$$e^{\sigma t}\mathbf{1}(-t) = \left[e^{(-\sigma)t}\mathbf{1}(t)\right]^* \leftrightarrow \left[\frac{1}{s-\sigma}\right]^* = \frac{(-1)}{s-\sigma} \quad \text{for} \quad Re\sigma > 0. \quad (25)$$

Thus

$$\frac{1}{s-\sigma} = \begin{cases} e^{\sigma t}\mathbf{1}(t) & Re\sigma<0 \\ (-1)e^{\sigma t}\mathbf{1}(-t) & Re\sigma>0 \end{cases}. \quad (26)$$

For proper casual rational function with single pole the factorization is given by

$$H(s) = -\sum_{\sigma\in\mathcal{P}_+}\frac{a^+(\sigma)}{s-\sigma} + \sum_{\sigma\in\mathcal{P}_-}\frac{a^-(\sigma)}{s-\sigma} \quad (27)$$

where:
$\mathcal{P}_+$ – poles of $H(s)$ on the right half-plain,
$\mathcal{P}_-$ – poles of $H(s)$ on the left half-plai $a^+(\sigma)$, $a^-(\sigma)$ – residues in pole σ,
$a^+(\sigma) = -\left[H(s)(s-\sigma)\right]_{s\to\sigma} \quad \sigma\in\mathcal{P}_+$
$a^-(\sigma) = \left[H(s)(s-\sigma)\right]_{s\to\sigma} \quad \sigma\in\mathcal{P}_-$.

We assume that the function $H(s)$ has not essential singularities on imaginary axis, because otherwise it cannot be assigned to any $\boldsymbol{L}^1$ impulse.

From (26) and (27) results the existence of the isomorphism between proper rational function set of complex variable and the exponential $\boldsymbol{L}^1$ impulse set of positive and negative time domain:

$$H(s) = \sum_{\sigma\in\mathcal{P}_+} a^+(\sigma)e^{\sigma t}\mathbf{1}(-t) + \sum_{\sigma\in\mathcal{P}_-} a^-(\sigma)e^{\sigma t}\mathbf{1}(t) \in \boldsymbol{L}^1. \quad (28)$$

The T – periodic extension of $\boldsymbol{L}^1$ impulses we get using geometric series:

$$e^{\sigma t}\mathbf{1}(-t)\big|_{Re\sigma>0} \to \sum_{p=-\infty}^{\infty} e^{\sigma(t+pT)}\mathbf{1}(-t-pT) \to \sum_{p=1}^{\infty} e^{\sigma(t-pT)} = \frac{e^{-\sigma T}}{1-e^{-\sigma T}}e^{\sigma t} = \frac{e^{\sigma(t-T)}}{1-e^{-\sigma T}}, \quad (29)$$

$$e^{\sigma t}\mathbf{1}(t)\big|_{Re\sigma<0} \to \sum_{p=-\infty}^{\infty} e^{\sigma(t+pT)}\mathbf{1}(t+pT) \to \sum_{p=0}^{\infty} e^{\sigma(t+pT)} = \frac{e^{\sigma t}}{1-e^{\sigma T}}, \quad (30)$$

where $t\in[0,T)$.

It then results the homomorphism of rational function set and T-periodic functions (extension of the $\boldsymbol{L}^1$ impulse function):

$$H(s) \to \sum_{\sigma\in\mathcal{P}_+} a^+(\sigma)\frac{e^{\sigma(t-T)}}{1-e^{-\sigma T}} + \sum_{\sigma\in\mathcal{P}_-} a^-(\sigma)\frac{e^{\sigma t}}{1-e^{\sigma T}}, \quad (31)$$

where $t\in[0,T)$.

It is also easy to write the general form in the case of multi-pole $H(s)$ function. The multiple differentiation of the isomorphism (26) with respect to σ parameter gives:

$$\frac{1}{(s-\sigma)^g} \leftrightarrow \begin{cases} \dfrac{\boldsymbol{t}^{g-1}}{(g-1)!}\boldsymbol{e}^{\sigma t}\mathbf{1}(t) & Re\sigma<0 \\ (-1)\dfrac{\boldsymbol{t}^{g-1}}{(g-1)!}e^{\sigma t}\mathbf{1}(-t) & Re\sigma>0 \end{cases}. \quad (32)$$

Doing the same with homomorphism (29)–(30) we get

$$\frac{1}{(s-\sigma)^g} \to \begin{matrix} \dfrac{1}{(g-1)!}\dfrac{\boldsymbol{d}^{g-1}}{d\sigma^{g-1}}\left(\dfrac{e^{\sigma t}}{1-e^{\sigma T}}\right) & Re\sigma<0 \\ \dfrac{(-1)}{(g-1)!}\dfrac{\boldsymbol{d}^{g-1}}{d\sigma^{g-1}}\left(\dfrac{e^{\sigma(t-T)}}{1-e^{-\sigma T}}\right) & Re\sigma>0 \end{matrix}, \quad (33)$$

where $t\in[0,T)$.

The formula (33) describes the homomorphism between g-order partial fraction and T-periodic extension of the $\boldsymbol{L}^1$ impulse function.

3. The L^1-impulses method versus Fourier series method

The formula (1) maps the $\boldsymbol{L}^1$ impulse set into T-periodic signal ($\boldsymbol{P}^T$) set and defines *Poisson operator* (*PO*):

$$POx(t) = \sum_{p=-\infty}^{\infty} x(t+pT). \tag{34}$$

When acting with *PO*- operator on convolution operator we get:

$$\begin{aligned} PO(h*x)(t) &= \sum_{p=-\infty}^{\infty} \int_{-\infty}^{\infty} h(t+pT-\tau)x(\tau)d\tau \\ &= \int_{-\infty}^{\infty} \left[\sum_{p=-\infty}^{\infty} h(t-\tau+pT) \right] x(\tau)d\tau \\ & \sum_{p=-\infty}^{\infty} \int_{-\infty}^{\infty} h(t-\tau)x(\tau+pT)d\tau \\ &= \int_{-\infty}^{\infty} h(t-\tau) \left[\sum_{p=-\infty}^{\infty} x(\tau+pT) \right] d\tau \end{aligned}$$

thus

$$PO(h*x) = (PO\,h)*x = h*(PO\,x) \in \mathcal{P}^T \tag{35}$$

for $h \in L^1$ and $x \in L^1$.

Now we introduce the new term *segment* $\overset{\sqcap}{x} \in L^1$

$$\overset{\sqcap}{x}(t) = \begin{cases} PO\,x(t) & t \in [0,T) \\ 0 & t \notin [0,T) \end{cases}, \tag{36}$$

where $x \in L^1$ and it is obvious that

$$POx = PO\,\overset{\sqcap}{x} \tag{37}$$

thus we get another version of (35)

$$PO(h*x) = (h*PO\,\overset{\sqcap}{x}) = (PO\,h)*\overset{\sqcap}{x}. \tag{38}$$

The formula (38) stands for the cyclic convolution defined as

$$(PO\,h)*\overset{\sqcap}{x}(t) = \widetilde{h} \circledast \overset{\sqcap}{x}(t) = \int_0^T h(t\ominus\tau)x(\tau)d\tau, \tag{39}$$

where

$$\widetilde{h}(t) = (PO\,h)(t) = \sum_{p=-\infty}^{\infty} h(\tau+pT)$$

the $\ominus$ operation stands for subtraction modulo Ti.e.

$$t\ominus\tau = \begin{cases} t-\tau & t-\tau\ in[0,T) \\ t-\tau+T & t-\tau \notin [0,T) \end{cases}.$$

The explanation of formulas (35) and (38) is shown in Fig. 3 as the input/output block diagrams

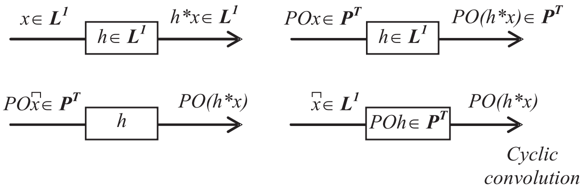

Fig. 3. The explanation of how signals of $\boldsymbol{L}^1$ and $\boldsymbol{P}^T$ class passes through the circuit of convolution type

From now the $\boldsymbol{L}^1$- impulses method and Fourier series method drift away. As to prove this one have to analyze the system with feedback delay element stimulated with a signal segment (Fig. 4).

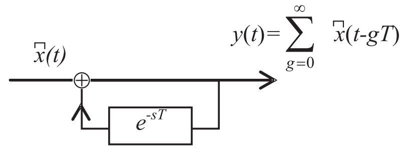

Fig. 4. The system with feedback element generating the casual and periodical signal

On the output we get casual periodic signal:

$$y(t) = \sum_{q=0}^{\infty} \overset{\sqcap}{x}(t-qT), \tag{40}$$

which transform is given by

$$Y(s) = \frac{1}{1-e^{-sT}} \overset{\sqcap}{X}(s), \tag{41}$$

where $\overset{\sqcap}{x}$ stands for transform of the periodic signal segment

$$\overset{\sqcap}{X}(s) = \int_0^T \overset{\sqcap}{x}(t)e^{-st}dt = \int_0^T PO\,x(t)e^{-st}dt. \tag{42}$$

The only pole set of function (38) is the countable- infinite set:

$$\begin{aligned} 1-e^{-sT} = 0 \to e^{-sT} = 1 \to \\ \ln^T\sqrt{1} =: \left\{ \frac{j2\pi}{T}n; n \in \{0,\pm1,\pm2,...,\} \right\}. \end{aligned} \tag{43}$$

The distribution of poles of (41) leads to the Fourier series

$$\begin{aligned} y(t) &= \left\{ \frac{s-\sigma}{1-e^{-sT}} \overset{\sqcap}{X}(s) \right\} e^{\sigma t}_{\ s\to\in\ln^T\sqrt{1}} \\ &= \frac{1}{T} \sum_{s\in\ln^T\sqrt{1}} \overset{\sqcap}{X}(s)e^{st}, \end{aligned} \tag{44}$$

for $t \in [0,T) : \overset{\sqcap}{x}(t) = y(t)$, thus the pair of transforms (42) and (44) makes the forward and inverse Fourier transform in discrete frequency domain. It agrees with Fourier series theorem.

The generalization of (42) and (44) on circuits with the transmittance $H(s)$ has form (see Fig. 5)

$$y(t) = \frac{1}{T} \sum_{s \in \ln^T \sqrt{1}} H(s) \overset{\sqcap}{X}(s) e^{st}$$
$$\text{where} \quad \overset{\sqcap}{X}(s) = \int_0^T \overset{\sqcap}{x}(t) e^{-st} dt. \tag{45}$$

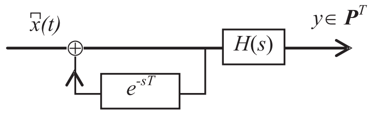

Fig. 5. The periodic system with the transmittance $H(s)$

It results from the multiplicative propriety of Fourier transform (3.9) over cyclic convolution:

$$\int_0^T \left[\int_0^T h(t \ominus t') x(t') dt' \right] e^{-st} dt$$
$$= \int_0^T \left[\int_0^T h(t \ominus t') e^{-st} dt \right] x(t') dt'$$
$$= \int_0^T \int_0^T h(t) x(t') e^{-s(t \oplus t')} dt \, dt'$$
$$= \left[\int_0^T h(t) e^{-st} dt \right] \left[\int_0^T x(t) e^{-st} dt \right],$$

where $s \in \ln^T \sqrt{1}$.

The performing diagram of $\boldsymbol{L}^1$ - impulse method and its T-periodic extensions ($\boldsymbol{P}^T$) in time and s domain is shown in Fig. 6.

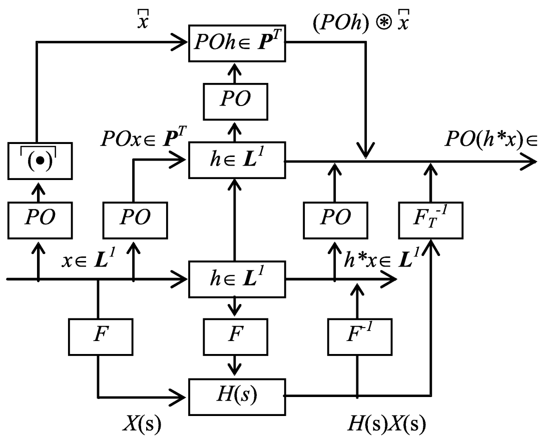

Fig. 6. The block diagram illustrating performance of the $\boldsymbol{L}^1$ - impulse method and its T-periodic extensions ($\boldsymbol{P}^T$) with the relation to the Fourier transform

In the above diagram the blocs F, F^{-1} terms the $\boldsymbol{L}^1$ – isomorphism between Fourier transform (27)–(28) with rational functions

$$\sum_{\sigma \ in \mathcal{P}_+} a^+(\sigma) e^{\sigma t} \mathbf{1}(-t) + \sum_{\sigma \in \mathcal{P}_-} a^-(\sigma) e^{\sigma t} \mathbf{1}(t)$$
$$\overset{F}{\underset{F-1}{\rightleftarrows}} - \sum_{\sigma \in \mathcal{P}_+} \frac{a^+(\sigma)}{s - \sigma} + \sum_{\sigma \in \mathcal{P}_-} \frac{a^-(\sigma)}{s - \sigma}, \tag{46}$$

for $t \in [0, T)$:

$$H(s)X(s) \overset{F_T^{-1}}{\rightarrow} \sum_{\sigma \in \mathcal{P}_+} a^+(\sigma) \frac{e^{\sigma(t-T)}}{1 - e^{-\sigma T}}$$
$$+ \sum_{\sigma \in \mathcal{P}_-} a^-(\sigma) \frac{e^{\sigma t}}{1 - e^{\sigma T}}, \tag{47}$$

where:
F_T^{-1} denotes $\boldsymbol{P}^T$ – Fourier homomorphism (31)
$a^+(\sigma) = -residue(H(s)X(s)) = -[H(s)X(s)(s - \sigma)]_{s \to \sigma}$
$a^-(\sigma) = residue(H(s)X(s))$.

From the Fig. 6 and formulas (41, 42) results that the $\boldsymbol{L}^1$ – impulse method acts on poles of both side of imaginary axis, when Fourier series method acts only on imaginary axis (on infinite set poles equal)

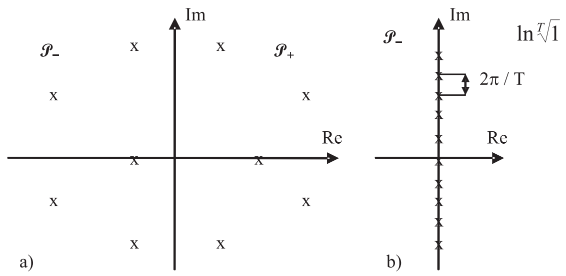

Fig. 7. Distribution of poles a) the $\boldsymbol{L}^1$-impulse method, b) the Fourier series method

Example. The use of the $\boldsymbol{L}^1$-impulse method will be now shown. As an example we take the source with finite duration $\boldsymbol{L}^1$-impulse and the inner impedance operator as an *RC* branch (Fig. 8). The source voltage square wave has 1V maximum value and τ duration.

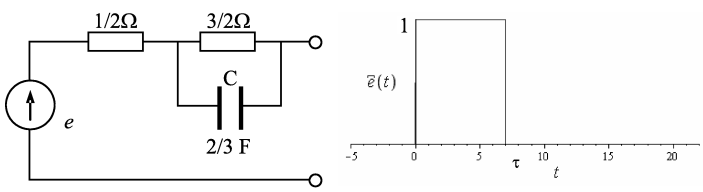

Fig. 8. Equivalent circuit of the power source and its voltage segment waveform

In Fig. 9 the T-periodic $\boldsymbol{L}^1$-impulse of source voltage with duration $\tau < T$ is shown, achieved using the Poisson's formula:

$$\widetilde{e}(t) = \sum_{p=-\infty}^{\infty} \overset{\sqcap}{e}(t + pT).$$

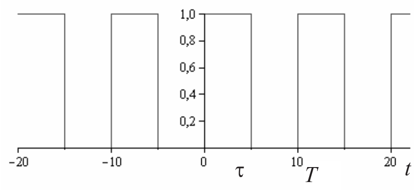

Fig. 9. Periodic $\boldsymbol{L}^1$-impulse of the source voltage

Fore the above circuit:

$$Z(s) = \frac{1}{2}\frac{(4+s)}{1+s}; \quad \overset{\sqcap}{E}(s) = \frac{1}{2}(1-e^{-s\tau}).$$

The non periodic current segment (i.e. an output for the voltage segment) is given by the formula

$$\overset{\sqcap}{I}(s) = \frac{\overset{\sqcap}{E}(s)}{Z(s)} = \frac{2(-e^{-s\tau}+1)(1+s)}{s(4+s)} \rightarrow$$

$$\overset{\sqcap}{i}(t) = \begin{cases} \frac{1}{2} + \frac{3}{2}e^{-4t} & 0 < t < \tau \\ \frac{3}{2}(1-e^{4\tau})e^{-4t} & \tau \le t \end{cases}$$

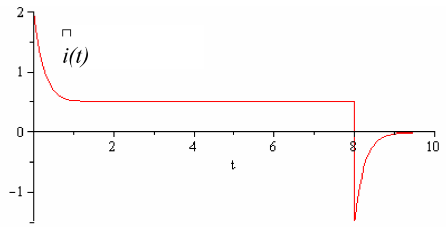

Fig. 10. $\boldsymbol{L}^1$-impulse of the current segment

According to diagram in Fig. 6 we can directly calculate the periodic current:

$$\tilde{i}(t) = \\ = \begin{cases} \frac{1}{2} + \frac{3}{2}\left(1 + \frac{e^{4\tau}-1}{1-e^{4T}}\right)(e^{-4t}) & 0 < t \quad \text{and} \quad t < \tau \\ \frac{3}{2}\frac{e^{4\tau}-1}{1-e^{4T}}(e^{-4t})e^{4T} & \tau < t \quad \text{and} \quad t < T \end{cases}.$$

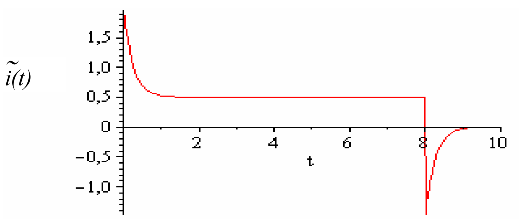

Fig. 11. Periodic $\boldsymbol{L}^1$-impulse of current

The periodic voltage and current transforms have forms

$$\tilde{E}(s) = \frac{1}{s}(1-e^{-s\tau})\frac{1}{1-e^{-sT}}$$

$$\tilde{I}(s) = \frac{\tilde{E}(s)}{Z(s)} = \frac{1(-e^{-s\tau}+1)(1+s)}{s(4+2)(1-e^{-sT})}.$$

From the Fourier inverse transform we get

$$\tilde{i}(t) = \frac{1}{2}\sum_{n=0}^{35} \underset{s=j\frac{2\pi}{T}\cdot n}{\text{res}} \left(\tilde{I}(s)\cdot e^{st} - \tilde{I}(-s)\cdot e^{-st}\right).$$

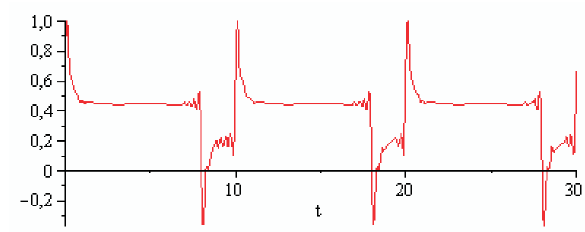

Fig. 12. Periodic source current achieved by the inverse Fourier transform

When we compare the periodic currents achieved using these two methods i.e. Fourier transform and Poisson formula of $\boldsymbol{L}^1$-impulse, we can see the difference (see Fig. 13).

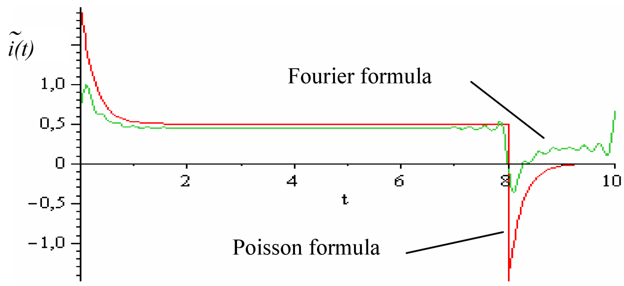

Fig. 13. Periodic source current achieved using two different methods

4. Conclusions

Presented here the $\boldsymbol{L}^1$-impulse method with its T-periodic extension is undoubtedly an alternative to the Fourier transform method. It distinguishes from the Fourier transform and resembles the classical operational calculus method. This method uses rational functions, partial factorizations, finite exponential distribution and curvelinear integrals of a complex variables. Similarly, as it is in operational calculus the $\boldsymbol{L}^1$-impulse method uses isomorphism between the set of complex variable and exponential functions from $\boldsymbol{L}^1$ and next by the use of the Poisson formula makes the periodic extension of signals (which is also described by the finite combination of exponential functions). Then the $\boldsymbol{L}^1$-impulse method is also suitable to describe the periodic steady states.

As opposed to the Fourier series method which uses the poles from imaginary axis the $\boldsymbol{L}^1$-impulse method uses poles distributed on both sides of a complex plain out of imaginary axis. This situation suits the optimization problems because the operator function of losses always has symmetrical (quadratic) distribution of poles [1]. In the time domain it results the $\boldsymbol{L}^1$-impulse time signals on both sides of time axis.

Thanks to its features the Fourier series method is suitable to analyze quasiharmonic signals and fails when describing fast changing signals (the Gibbs phenomenon). The $\boldsymbol{L}^1$-impulse method can deal even with discontinues signals and effectively solve the optimization problems. It can be used also to design filters used in the power electronics.

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