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Parametric approach for Luenberger observers for descriptor linear systems

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Abstract. A complete parametric approach is proposed for the design of the Luenberger type function Kx observers for descriptor linear systems. Based on a complete parametric solution to a class of generalized Sylvester matrix equations, parametric expressions for all the coefficient matrices of the observer are derived. The approach provides all the degrees of design freedom, which can be utilized to achieve some additional design requirements. An illustrative example shows the effect of the proposed approach.

Key words: Descriptor linear systems, Luenberger function observers, Sylvester matrix equations, parametric approach.

1. Introduction

Since Luenberger observer theory was proposed in [1], it has attracted much attention of many investigators not only in the field of normal linear systems ([2] and [3]) but also in the field of descriptor linear systems ([4–6]). The Luenberger observers with the function of decoupling the unknown inputs in descriptor linear systems is investigated in [7] and [8]. For the solution of the Luenberger function Kx observers for descriptor linear systems, the method of singular value decompositions is adopted in [9], the technique of the generalized inverse of matrix is applied in [8] and [10]. However, the parametric expressions of all the coefficients matrices of the observer have not been established. Moreover, the restriction that the controlled plant is C-observable is required in the aforementioned references. In this note, under the condition that the considered system is R-observable, based on the explicit general solution to a type of generalized Sylvester matrix equations investigated in [11,12], a parametric approach is proposed for the design of the Luenberger function Kx observers for descriptor linear systems and the design algorithm is proposed. The proposed method gives the parameterizations of all the gain matrices of the observer. The approach offers all the degrees of design freedom by a group of free parameters which can be utilized to meet additional desired performances and specifications. An illustrative example shows the effect of the proposed approach.

2. Problem formulation

Consider the following descriptor linear system

$$\begin{cases} E\dot{x} = Ax + Bu \\ y = Cx \end{cases}, \tag{1}$$

where $x(t) \in \mathbf{R}^n, u(t) \in \mathbf{R}^r$ and $y(t) \in \mathbf{R}^m$ are the state, the control input, and the measured output, respectively; E, A, B and C are constant matrices of appropriate dimensions. For this system, the Luenberger type normal function observers take the following form:

$$\begin{cases} \dot{z} = Fz + Su + Ly \\ w = Mz + Ny \end{cases}, \tag{2}$$

where $z \in \mathbf{R}^p$ is the observer state vector. The design purpose is to seek the parameter matrices F, S, L, M and N such that

$$\lim_{t\to\infty} (Kx(t) - w(t)) = 0 \tag{3}$$

holds for some matrix $K \in \mathrm{R}^{q\times n}$ and arbitrarily given initial values $x(0), z(0)$ and control input $u(t)$.

Regarding conditions for the Luenberger type function observers in the form of (2), the following result holds [8].

Theorem 1. Assume that the regular system (1) is R-controllable, that is,

$$\mathrm{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n, \text{ for any } s \in \mathbf{C}$$

and system (2) is observable, that is,

$$\mathrm{rank} \begin{bmatrix} sI - F \\ M \end{bmatrix} = n, \text{ for any } s \in \mathbf{C}.$$

Then system (2) is a normal Kx observer for system (1) if and only if there exists a matrix $T \in \mathrm{R}^{p\times n}$ satisfying

1. $S = TB$
2. $TA - FTE = LC$
3. $K = MTE + NC$
4. All the eigenvalues of matrix F are in the left of complex plane.

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Due to the above theorem, design of the Luenberger observer (2) is converted into the problem of seeking the parameter matrices F, S, L, M and N satisfying the four conditions in the above theorem.

3. Parametric approach

In this section, we propose, based on the parametric solution to a type of generalized Sylvester matrix equation, a parametric approach to the design of the Luenberger type function Kx observers in the form of (2). Relevant work can be found in [11–16].

In order to derive the general parametric form of the Luenberger type function Kx observers for system (1), we introduce the following assumptions on system (1).

Assumption 1. $\operatorname{rank} B = r, \operatorname{rank} C = m$.

Assumption 2. The matrix triple (E, A, C) is R-observable.

3.1. Parametric expression for matrix *F*. The only requirement on the coefficient matrix F of the Luenberger function Kx observer (2) is that it has eigenvalues with negative real parts. For convenience and simplicity, let us restrict the matrix F to nondefective, it thus has a diagonal Jordan matrix. Therefore, a general form for the matrix F can be given, based on the Jordan form decomposition theory, as

$$F = Q^{-1}\Lambda Q, \Lambda = \operatorname{diag}\left[\, s_1\ s_2\ \cdots\ s_p \,\right], \tag{4}$$

where $s_i, i = 1, 2, \cdots, p$ are clearly the eigenvalues of the matrix F. They are self-conjugate and satisfy the following constraint.

Constraint 1. $\operatorname{Re} s_i < 0, i = 1, 2, \cdots, p$.

The matrix

$$Q = \left[\, q_1\ q_2\ \cdots\ q_p \,\right] \in C^{p\times p}, \tag{5}$$

is obviously the left eigenvector matrix of the matrix F, which satisfies

Constraint 2. $\det Q \neq 0$ and $q_i = \bar{q}_l$.

In the case of $p = 1$, this matrix Q may be chosen to be 1 without loss of generality.

3.2. Parametric expressions for the matrices *T* and *L*. The matrices T and L are determined by the second condition in Theorem 1, that is, the matrix equation

$$TA - FTE = LC. \tag{6}$$

Equation (6) is a generalized Sylvester matrix equation. It is in the dual form of the one considered in [11] and [12]. By applying the complete parametric solution proposed in [11,12] for generalized Sylvester matrix equations, the parametric expressions for the matrices T and L satisfying (6) can be readily obtained as

$$T = QV, L = -QW, \tag{7}$$

with

$$V = \left[\, v_1\ v_2\ \cdots\ v_p \,\right]^T, v_i = N(s_i)f_i, \tag{8}$$

$$W = \left[\, w_1\ w_2\ \cdots\ w_p \,\right]^T, w_i = D(s_i)f_i, \tag{9}$$

where $N(s)$ and $D(s)$ are a pair of right coprime polynomial matrices satisfying the following right coprime factorization:

$$(sE^T - A^T)^{-1}C^T = N(s)D^{-1}(s), \tag{10}$$

and $f_i \in C^m, i = 1, 2, \cdots, p$ is a group of design parameters satisfying the following constraint.

Constraint 3. $f_i = \bar{f}_l$ if $s_i = \bar{s}_l$

Remark 1. The right coprime factorization (10) performs an important role in the parameterization of the matrices T and L. For solution of the right coprime factorization (10), refer to [11,12].

Remark 2. When there are complex eigenvalues chosen for the matrix F, the matrices T and L are complex. This can be prevented by a well-known technique which can convert, at the same time, all the three matrices Λ, T, L into real ones. This technique is demonstrated as follows. Without loss of generality, let us assume that s_1 and $s_2 = \bar{s}_1 = \alpha + \beta\,\mathrm{j}$ are a pair of conjugate eigenvalues, all the other ones are real. In this case, there holds $v_1 = \bar{v}_2$ and $w_1 = \bar{w}_2$, and the real substitutions of these three matrices can be taken as

$$\Lambda = \begin{bmatrix} \alpha & \beta & & & \\ -\beta & \alpha & & & \\ & & s_3 & & \\ & & & \ddots & \\ & & & & s_p \end{bmatrix},$$

and

$$T = Q\left[\, \operatorname{Re}(v_1)\ \operatorname{Im}(v_1)\ v_3\ \cdots\ v_p \,\right]^T,$$

$$L = Q\left[\, \operatorname{Re}(w_1)\ \operatorname{Im}(w_1)\ w_3\ \cdots\ w_p \,\right]^T,$$

where the matrix Q is taken, in this case, a non-singular real matrix.

3.3. Parametric expressions for the matrices *N* and *M*. The matrices N and M are determined by the third condition in Theorem 1, that is, the matrix equation

$$K = MTE + NC, \tag{11}$$

which can be equivalently written as

$$K = \left[\, M\ N \,\right]\begin{bmatrix} TE \\ C \end{bmatrix}. \tag{12}$$

From (12), it follows that there exist a pair of matrices M and N satisfying (11) if and only if the following constraint is met.

Constraint 4. $\operatorname{rank}\begin{bmatrix} TE \\ C \end{bmatrix} = \operatorname{rank}\begin{bmatrix} TE \\ C \\ K \end{bmatrix}$.

Under the above constraint, the parametric solutions to the observer gain matrices M and N can be presented.

Lemma 1. Let Assumptions 1 and 2 be met, the matrix T be given by (7) and (8). Then when Constraint 4 is met, there exist a matrix $K_0 \in R^{r\times r^*}$ and nonsingular matrices P and Q_1 satisfying

$$P\begin{bmatrix} TE \\ C \end{bmatrix} Q_1 = \begin{bmatrix} T_0 & 0 \\ 0 & 0 \end{bmatrix}, KQ_1 = \begin{bmatrix} K_0 & 0 \end{bmatrix}, \quad (13)$$

where $T_0 \in R^{r^*\times r^*}$ is nonsingular, r^* is the common rank in Constraint 4. In this case, the matrices N and M are given by

$$\begin{bmatrix} M & N \end{bmatrix} = \begin{bmatrix} K_0 T_0^{-1} & N' \end{bmatrix} P, \quad (14)$$

with $N' \in \mathrm{R}^{r\times(m+p-r^*)}$ being an arbitrary real parameter matrix.

Proof. Under the condition of Constraint 4, there exist nonsingular matrices P and Q_1 satisfying (13). Post-multiplying both sides of (12) by Q_1, and using (13), gives

$$\begin{bmatrix} K_0 & 0 \end{bmatrix} = \begin{bmatrix} M & N \end{bmatrix} P^{-1} \begin{bmatrix} T_0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (15)$$

which is equivalent to

$$K_0 = \begin{bmatrix} M & N \end{bmatrix} P^{-1} \begin{bmatrix} T_0 \\ 0 \end{bmatrix}. \quad (16)$$

Let

$$\begin{bmatrix} M & N \end{bmatrix} P^{-1} = \begin{bmatrix} M' & N' \end{bmatrix}, N' \in R^{r\times(m+p-r^*)}, \quad (17)$$

then (16) becomes

$$K_0 = M'T_0. \quad (18)$$

Combining the above relation with (17) gives the parametric expression (14) for the matrices N and M.

By summarizing the above, the following theorem about the parametric solution of the Luenberger function Kx observers in the form of (2) can be given as follows.

Theorem 2. Let Assumptions 1 and 2 be met, then
(a) A Luenberger-type observer in the form of (2) with the matrix $F \in \mathrm{R}^{p\times p}$ nondefective, exists for system (1) if there exist parameters $s_i, q_i, f_i, i = 1, 2, \cdots, p$ and N' satisfying Constraints 1–4.
(b) When the above condition (a) is met, all the Luenberger observers in the form of (2) for system (1), with the matrix F nondefective, are parameterized by (4)–(9) and (13)- (14) with $s_i, q_i, f_i, i = 1, 2, \cdots, p$ and N' satisfying Constraints 1–4.

Based on the above theorem, we have the following algorithm for design of the Luenberger type of normal observers in the form of (2) for system (1).

Algorithm 1. Luenberger observer design
Step 1. Solve the pair of right coprime polynomial matrices $N(s)$ and $D(s)$ satisfying the right coprime factorization (10).
Step 2. Specify the observer order p,and obtain the specific forms for Constraints 1–4.
Step 3. Solve the matrices T_0, K_0 and P satisfying (13).
Step 4. Find a set of parameters $s_i, f_i, q_i, i = 1, 2, \cdots, p$ satisfying Constraint 1–4.
Step 5. Calculate the coefficient matrices F, T, L, N and M according to (4),(7)–(9) and (13)–(14).

Remark 3. In the case that the parameters $s_i, q_i, f_i, i = 1, 2, \cdots, p$ and N' satisfying Constraints 1–4 do not exist, the order of the observer may be increased to provide sufficient design freedom for a solution to exist.

Remark 4. An important advantage of the above algorithm is that it provides all the design freedom. These degrees of freedom can be further utilized to achieve additional system specifications.

4. An illustrate example

Consider a linear system in the form of (1) with the following parameters

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

We will design a Luenberger observer which tracks asymptotically the function Kx, with

$$K = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

which can stabilize the given system.

In the following, we will obtain the observer gain matrices using Algorithm 1
Step 1. By using a method given by ([11] [12]), the solution to the right coprime factorization (10) can be obtained as follows:

$$N(s) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ s & 0 \end{bmatrix}, D(s) = \begin{bmatrix} 0 & 5+s \\ -1 & 0 \end{bmatrix}.$$

Step 2. For this example, a first-order observer in the form of (2) is considered. In this case, the matrix Q can be chosen to be 1 without loss of generality. In order that Constraint C1 is met, the observer pole is restricted to be a negative real scalar, then the matrix $F = s$. Choosing

$$f = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix}^T,$$

then Constraint 3 holds automatically, and the matrices T, V and L are given, in view of (7)-(9), as

$$T = \begin{bmatrix} \alpha_1 & \alpha_2 & s\alpha_1 \end{bmatrix}, L = -\begin{bmatrix} (5+s)\alpha_2 & -\alpha_1 \end{bmatrix}.$$

With this matrix T, it is obvious that Constraint 4 is $\alpha_1 \neq 0$ and $r^* = 3$.
Step 3. Under the condition $\alpha_1 \neq 0$, the matrices P and Q_1 can both be chosen to be identity matrices when the matrices T_0 and K_0 are expressed as follows:

$$T_0 = \begin{bmatrix} \alpha_2 & \alpha_1 & s\alpha_1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, K_0 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

Step 4. Summing up the above, the design freedom involved in this design is composed of the closed-loop eigenvalues s and parameters α_1 and α_2.

Step 5. The matrices M and N can be obtained as

$$\begin{aligned}\left[M\ N\right] &= \left[1\ 1\ 1\right]\begin{bmatrix}\alpha_2 & \alpha_1 & s\alpha_1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}^{-1}\\ &= \left[\tfrac{1}{s\alpha_1}\ -\tfrac{\alpha_2}{s\alpha_1}+1\ -\tfrac{1}{s}+1\right].\end{aligned}$$

This gives

$$M = \frac{1}{s\alpha_1}, N = \left[-\tfrac{\alpha_2}{s\alpha_1}+1\ -\tfrac{1}{s}+1\right].$$

Specially choosing $s = -1, \alpha_1 = \alpha_2 = 1$ gives the following specific solution

$$F = 1, T = \left[1\ 1\ -1\right], L = \left[-4\ 1\right],$$
$$M = 1, N = \left[2\ 2\right].$$

5. Conclusion

Based on a parametric solution to a type of generalized Sylvester matrix equations, a parametric approach is proposed for the design of the Luenberger function Kx observer in descriptor linear systems under the condition that the considered descriptor linear system is R-observable. The proposed approach offers all the degrees of freedom which can be utilized to achieve additional performances and specifications.

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